# Global grids of gravity anomalies and vertical gravity gradients at 10 km altitude from GOCE gradient data 2009-2011 and polar gravity

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**Abstract**: The GOCE satellite measures gravity gradients which are filtered and transformed to gradients in an Earth-referenced frame by the GOCE High Level processing Facility.

More than 32,000,000 data with 6 components are available from the period 2009-2011. IAG Arctic gravity project data was used north of 83 deg., while data from Antarctica was not used due to bureaucratic restrictions by the data-holders.

Subsets of the data have been used to produce gridded values at 10 km altitude of gravity anomalies and vertical gravity gradients in  $20 \times 20$  deg. blocks with 10' spacing. Various combinations and densities of data were used to obtain and compare values in areas with known gravity anomalies. The (marginally) best choice was vertical gravity gradients selected with an approximate 0.125 deg. spacing.

Using Least-Squares Collocation, error-estimates were computed and compared to the difference between the GOCE-grids and grids derived from EGM2008 to deg. 512. In general a good agreement was found, however with some inconsistencies in certain areas.

The computation time on a usual server with 24 processors was typically 50 minutes for a block with generally 40,000 GOCE vertical gradients as input. The computations will be updated with new Wiener-filtered data in the near future.

#### 1. Introduction

In order to promote the use of the GOCE gravity gradient data, new gridded products are being produced. The GOCE High Level processing Facility (HPF) has already delivered to ESA grids at ground level of gravity anomalies and geoid heights based on the Earth spherical harmonic global models (EGM). However, the original GOCE data are expected to contain information in certain areas with large gravity variations which are representing higher frequencies than those included in the GOCE EGM models. Note that the existing grids at ground level are computed without taking into account the topographic masses.

Currently we have available gravity gradients which are filtered and transformed to an Earth (terrestrial) fixed frame (TRF) by HPF. We have selected the data from 2009-2011 which consist of more than 32,000,000 gradients, spaced along-track with approximately 7.5 km between the measurement points. These data cover the area between -83.36 deg. and 83.36 deg. latitude. Therefore ground gravity was to be used above and below these latitudes. For the Arctic the IAG has made available a dense data-set, which we used. Data are also available for the Antarctic, but they are not yet released to the scientific community.

In order not to have to take the topography into account, it was decided to create the grids at the altitude of 10 km. The grids will consist of gravity anomalies ( $\Delta g$ ) and vertical gravity gradients ( $T_{zz}$ ) and their error-estimates and have an equi-angular spacing of 10' or 0.1666 deg. Grids are produced for 20 × 20 deg. blocks with a 2.5 deg. overlap border, i.e. having an extent of 25 × 25 deg. There are 162 of these blocks.

The method used to produce the grid-values was Least-Squares Collocation (LSC), since it permits the combination of gradient and gravity information and enables the estimation of errors; see Torge and Müller (2012). Also excellent results have been reported by e.g. Tscherning and Arabelos (2011).

In the following we describe the preparations for the use of LSC and the production and quality test of the gridded values.

## 2. Preparations for the use of LSC

In order to eliminate long-wavelength variations and to reduce the mean values of the data in the blocks, the remove-restore procedure (Forsberg and Tscherning, 1981) was used. The ITG-Grace2010s EGM (Mayer-Guerr et al., 2010) was used for this purpose to degree 36. Its contribution was subtracted from all gravity and gravity gradient data-sets and subsequently restored when necessary. The chosen maximum degree is consistent with the block size. Since no corrections are expected for the very long wavelengths from GOCE data, this implicitly means that this remove-restore procedure gives rise to a combination between GRACE and GOCE information.

Gravity anomaly data were computed in a 0.166 deg. grid at ground level from EGM2008 (Pavlis et al., 2012) and the contribution from the ITG-Grace model was subtracted. This data-set was used just for covariance computation. Simultaneously data-sets of gravity anomalies and gravity gradients were created in points along the orbit as close as possible to an equi-angular grid of 0.125 deg. and 0.166 deg. spacing.  $T_{zz}$  and  $T_{yy}$  values were synthesized for the 0.125 deg. spacing, and  $T_{zz}$  and gravity anomalies selected with the 0.166 deg. spacing.

## 2.1 Covariance estimation and fitting with analytic models

The (reduced) EGM2008 generated ground gravity and the  $T_{zz}$  GOCE gradients were used for the estimation of the empirical covariance function, and subsequently fitted with an analytical model (Knudsen, 1987). The results using the gravity anomalies were in some cases not satisfactory (the distance to the Bjerhammarsphere too large), so the gradient data were used instead. See Figs. 1 and 2. Also Fig. 2 illustrates the different result obtained when using gravity anomalies and  $T_{zz}$  for the covariance estimation.

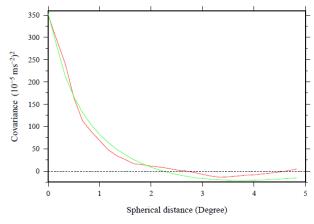
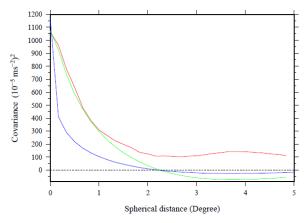


Fig.1. Example of a good fit between a local empirical (red) and an analytical (green) model.



**Fig. 2**. Empirical (red), analytical derived from  $T_{zz}$  (blue) and analytic from gravity anomalies (green) covariance function for block 97 (western half of Australia).

The important parameters are the depth to the Bjerhammar-sphere and the variance of the (reduced) ground gravity anomalies. These values varied between 0.3 km and 50.7 km and 20 mgal<sup>2</sup> and 5055 mgal<sup>2</sup>.

# 2.2. Control of data-error estimate

Associated with the TRF gradient data are error estimates, which for the  $T_{zz}$  values are of the order of 0.01 EU ( $10^{-9}$  s<sup>-2</sup>). The data are in reality along-track filtered values, but they are here used as if they were only associated with a point in space. So in order to "fix" the most appropriate error-estimate, prediction tests (comparison with EGM2008 values at 10 km) in an equatorial block (73) were performed with a sequence of different error-values, see Table 1 (the square of the noise standard deviation is simply added to the diagonal of the covariance matrix, and the prediction is repeated).

**Table 1**. Prediction results for different values of data  $T_{zz}$  noise standard deviation and comparable values for the difference with respect to the GOCE DIR-R2 model (Bruinsma et al., 2010).

Noise (EU)	Δg (mgal)		T <sub>zz</sub> (EU)	
	Observed-predicted	Error estimates	Observed-predicted	Error estimates
	Standard deviation	Mean value	Standard deviation	Mean value
0.030	7.41	7.67	3.28	3.06
0.020	7.30	7.22	3.26	2.99
0.010	7.44	6.52	3.29	2.88
0.008	7.62	6.32	3.33	2.84
0.005	8.40	5.75	3.52	2.76
0.003	9.28	5.40	3.77	2.65
EGM08 – DIR-R2	7.58		3.53	

This made us select the value of 0.02 EU as the data-error, hereby implicitly taking into account the representation error (but not the error-correlation).

### 2.3 Selection of data-type and distribution

In the above mentioned block, prediction tests were performed using the 0.125 deg. and the 0.166 deg. spacing with  $T_{zz}$  as observables, and  $T_{zz}$  and  $T_{yy}$  simultaneously with a 0.166 deg. spacing, see Table 2.

**Table 2.** Result of prediction test in the block bounded by latitude -10 deg. and 10 deg., longitude 0 deg. and 20 deg. (block 73). 441 anomalous (reduced) gravity values spaced 1 deg. used for comparison generated from EGM2008-ITG-Grace. Unit is mgal.

Number of data	Spacing (degree)	Data type	Mean difference	Standard deviation	Mean error estimate
22464	0.166	$T_{zz}$	-0.5	9.73	6.92
44929	0.166	$T_{zz}+T_{yy}$	-0.4	9.65	6.85
37971	0.125	$T_{zz}$	-0.5	9.16	6.79

This made us select the  $T_{zz}$  with 0.125 deg. spacing as our basic dataset to be used to predict the grid values. However, the differences are so small, that a sequence of computations also were made with the  $T_{zz}$  data with 0.166 deg. spacing. Using this data-set reduced the computation time considerably.

# 2.4 Quality check

Predictions were made both in a 1 deg. grid and in a 0.166 deg. grid. In the 1 deg. grid comparisons were made with EGM2008 generated values and error-estimates were performed too. The results of the comparison using the data spaced approximately 0.166 deg. is for each grid block shown in Figs. 3 and 4. The error-estimate is not shown, since it was quite close to the standard deviation of the difference between predicted and EGM2008-ITG-Grace generated values.

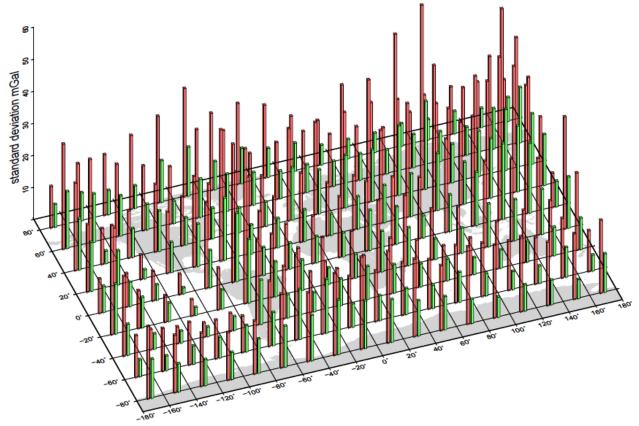
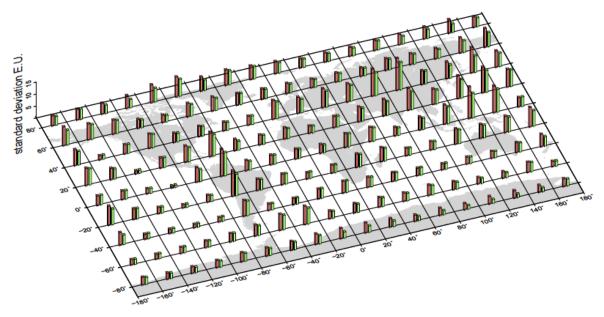
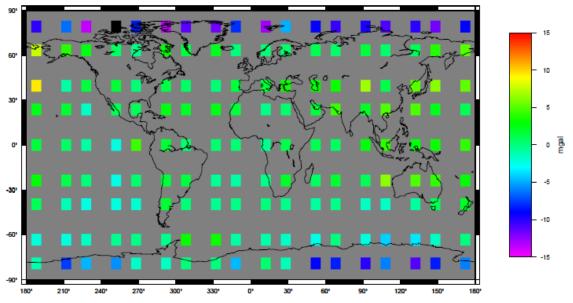


Fig. 3. Standard deviation of gravity differences (green), and of gravity (red).



**Fig. 4.** Standard deviation of differences  $T_{zz}$  EGM2008 minus predicted (green) and standard deviations of the EGM2008 generated values (red). The large differences are caused by the use of EGM2008 to degree and order 512 as a reference.

Comparison was also made with values generated from the GOCE DIR-R2 solution. The values predicted directly from the gradients had in 57% of the blocks a smaller "error" for  $T_{zz}$ , while in 45% of the blocks the gravity anomaly error had the smallest error when compared to EGM2008. The differences are shown in Fig. 5 for gravity anomalies. Note the superior agreement using LSC at the poles.



**Fig. 5.** Difference between standard deviations of (predicted-EGM2008) and standard deviations of (DIR-R2 –EGM2008) gravity anomalies at 10 km in  $20 \times 20$  blocks.

## 3. Computations.

The software used (geocol19, see <a href="http://cct.gfy.ku.dk/software/geocol19.pdf">http://cct.gfy.ku.dk/software/geocol19.pdf</a>) has been upgraded using multiprocessing, see Kaas et al. (2013). The processing time depends very much on how the processors are utilized, and this again depends on how the normal-equation (upper-triangular) matrix is divided in so-called chunks. The following table illustrates the differences.

**Table 3**. Total processing time in seconds (s) (including error-estimation), block 73 for different chunk block sizes (CH), number of data (N) and processors.

Processors	22	22	4
N	37971	22464	22464
СН	S	S	S
05		10407	41832
10	6764	2709	8381
15	7898	2962	7793
20	6966	2642	7469
25	7221	2836	7748
30	7476	2894	8300

We are in the process of implementing Message Passing Interface (MPI), so that several computers (with several processors) can be used simultaneously.

#### 4. Conclusion and further work

The intention was originally to use filtered gradient data using an advanced procedure developed at POLIMI (Reguzzoni and Tselfes, 2009), but these data have not yet become available. We expect that the POLIMI data will enable us to produce improved grids, also because these values will have associated estimates of the error-correlation. Furthermore the GOCE satellite has in 2013 while still collecting data decreased its altitude (and finally ended its life).

The present results are satisfactory in the sense that we for the majority of the blocks obtain a better agreement with EGM2008 of  $T_{zz}$  at 10 km when comparing with GOCE DIR-R2 values. For gravity anomalies we expect that using the POLIMI data, we can obtain a similar good agreement. Already now the differences between the results are very small, see Fig. 5. Improved results are also expected when using more data at middle latitudes.

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