Geopotential Models as a Tool for Densifying Gravity and Orthometric Correction Computation

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Abstract:

In the current study, the global geopotential harmonic models are used to densify gravity values along spirit leveling routes, and hence to minimize the discretization error in computing the orthometric correction and represent an economical alternative. Furthermore, these models are checked regarding their capability of being the only source of gravity information along spirit leveling lines. The orthometric corrections, computed for two test links, are compared with those resulting from using purely observed gravity. Based on the obtained results, it is recommended to use the geopotential harmonic models as an economical source of gravity information along leveling routes. Moreover, it is recommended to investigate the application of the remove-restore technique of geopotential models to the computation of orthometric corrections.

Keywords: Orthometric correction, geopotential models, spirit levelling.

1 Introduction

Among the gravity field related height systems, the orthometric height is frequently used, since it has both a geometric and natural meaning [7]. The orthometric height (OH) can be obtained by spirit leveling. However, height differences from leveling must be corrected for the non-parallel equipotential surfaces using the orthometric correction (OC) in order to obtain OHs [3], [4].

It is common practice to evaluate the precision of a gravimetric geoid model by comparing modeled geoidal height with those computed from the difference between GPS-determined ellipsoidal heights and the OHs obtained from leveling. Since recent progress in both theory and numerical technique has greatly improved the precision of geoid modeling, an inaccurate orthometric height will make such geoid model evaluation unreliable [5].

Recently, the computation of OH has been revisited by many researchers. The aim of such works has been the development of rigorous computational methods for assessing the orthometric correction, e.g., [1], [5], [12] and [13]. These studies concentrated on the topographic and /or density effect on the OC. Such trend was also motivated by the fact that a so computed OH height would be consistent with a gravimetric geoid that already takes into account a terrain correction.

Rigorous OC computation is expensive because it requires observed gravity values at benchmarks along the leveling route [2], [8]. Different methods for assessing OC may yield different OHs and the differences can reach several centimeters. This implies that OHs from leveling may mismatch the true orthometric heights by several centi-meters if the OC computation is not sufficiently accurate [5].

Motivated from above facts, one agrees that gravity observations along leveling lines essentially have long-wavelength components that could be evaluated by geopotenial harmonic models. Such low frequency features constitute the major trends of level surfaces. Therefore, the aim of the current study is to investigate the ability of global geopotential models to densify gravity acceleration along leveling lines, thus minimizing the discretization error in the computation of OC and saving the cost of gravity measurements. Moreover, such models will be tested against their capability to be the only source of gravity along the leveling lines. Such investigations will be compared with the results of using observed gravity data and with an approximate method for computing the OC. The investigations encountering observed and harmonic models-derived gravity will be performed using two rigorous formulae [4], [5]. Conversely, the approximate formula, [8], leans only on normal gravity.

2 Basic concepts

The OH is the height above the geoid measured along the curved plumb line. Leveling alone will yield a geometric height difference between two consecutive benchmarks, which in turn yields OH differences that are dependent on the leveling route. Thus, the OC plays a critical role in obtaining unique OHs from leveling. By definition, the OH (H) at a benchmark is the ratio between its geopotential number (C) and its mean gravity along the plumb line ($\hat{g}$) between the surface and the geoid. Thus, for two benchmarks A and B [4] , Figure (1)

$$OC_{AB} = \Big[\sum_{i=1}^{k}[\,(g_i - \gamma_0) / \gamma_0\,]\,.\,\delta n_i\Big] + \Big[\,(\hat{g}_A - \gamma_0) / \gamma_0\Big]\,H_A - \Big[\,(\hat{g}_B - \gamma_0) / \gamma_0\Big]\,H_B, \qquad (1)$$

where

δn_i : is the geometric height difference at the i^{th} leveling section,

k : the total number of leveling sections,

g_i : the observed gravity relevant to the i^{th} section,

γ_0 : the normal gravity at geodetic latitude 45° on the reference ellipsoid (taken WGS-84),

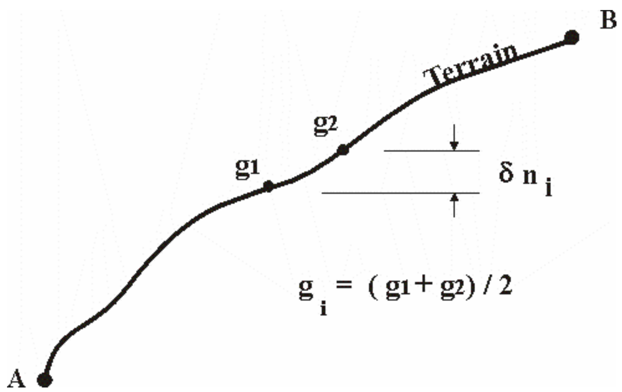

Figure (1): Surface gravity at the i^{th} leveling section

H_A & H_B :the orthometric height of A and B, which can be approximated by their leveled heights,
$\hat{g}_A$ & $\hat{g}_B$: the mean gravity values along plumbline at A and B, respectively.

Assuming a constant topographic mass density of 2.67 g/cm^3, according to Prey's reduction, one obtains [7]

$$\hat{g}_A = g_A + 0.0424 \; H_A,$$
$$\hat{g}_B = g_B + 0.0424 \; H_B, \qquad (2)$$

where g_A and g_B are the observed gravity (in mgal) at A and B, respectively. It should be mentioned that a leveling section simply consists of selected accumulated leveling setups, since it would be very time consuming to measure gravity at each level setup [1], [2] and [8].

The magnitude of the OC can be thought of as a measure of the convergence of equipotential surfaces [1]. It is clear that the dominant factors that judge the OC magnitude are the spirit-leveled height differences (δn_i), the deviation of observed gravity from normal gravity and the average elevation of the leveling link. Recently, a new formula for assessing OC has been derived by [5], namely,

$$OC_{AB} = \left[\sum_{i=1}^{k} [(g_i - \hat{g}_B) \, . \, \delta n_i \right] / \hat{g}_B + \left[(\hat{g}_A / \hat{g}_B) - 1 \right] H_A \qquad (3)$$

As they take into account the observed gravity values along the leveling lines, Eqs. (1) and (3) are said to be rigorous formulae for determining the OC.

For the sake of comparison, an approximate formula will also be considered. Such method is based on the normal potential of the reference ellipsoid [2], [8]. Particularly, taking WGS-84 as a reference ellipsoid, for a leveling line AB, one obtains

$$NOC_{AB} \approx -1.542 \times 10^{-6} \; H_m \, . \, \sin 2\varphi_m \, . \, \Delta\varphi_{AB} \,, \qquad (4)$$

where

NOC_{AB} the normal orthometric correction,
φ_m the mean latitude of the leveling link AB,
H_m the mean elevation of the leveling line,
$\Delta\varphi_{AB}$ = (φ_B - φ_A), in arc-minutes.

While the above approximate formula takes into account the general systematic features such as the mean elevation and the north-south extent of the leveling line, it does not consider the random effect of surface gravity variation along the leveling lines.

3 Data

It is worth mentioning that the purpose of the current investigation is to study the ability of harmonic models-derived gravity to fully or partially determine the OC for leveling links in Egypt which are assumed to be spirit leveled. In particular, the investigated data is two series of adjacent stations (two lines) with known latitude, longitude, elevation and observed surface gravity. In fact, the elevations under study are either trigonometrically levelled or interpolated from topographic maps. Since the levelling and gravity observations of the Egyptian first-order levelling networks were not available, the purpose of the current study is not to find the OCs along an actual leveling net. So, it could be considered as a simulated case study.

In particular, an insight into Eqs. (1) and (3) shows that the precisions of the leveled height differences and the OHs of the end points do not significantly affect the uncertainty of the computed OC. This is also supported by the fact that the used point data are topographically located, such that remarkable values for OC arise. Moreover, the actual spirit leveling data are rather sparse and located in lower areas that yield very small OC. While the first link lies in the Western Desert, the second one runs along the Nile Valley. Figure (2) illustrates the geographical location of both lines, which were selected for the current study.

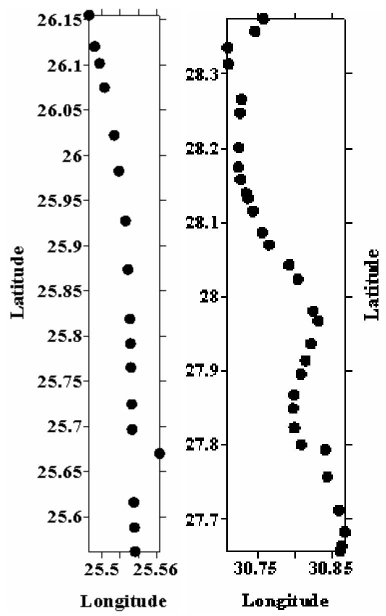

Figure (2): Post maps for the test lines (I) & (II)

In addition, Table (1) summarizes the main features of these lines.

Table (1): Summary for the two elevation links

Link	No. of points	Average spacing between points (km)	Route length (km)	Mean Elevation (m)	Elevation difference (m)
I	17	4.14	66.31	352.868	-86.940
II	31	2.86	85.73	42.633	-9.720

4 Methodology

Mainly for both links, two sets of OCs were computed according to the two rigorous formulae, Eqs. (1) and (3). Moreover, for each line an approximate value for the OC was assessed according to Eq. (4). Tables (2) and (3) summarize all computed values for the OC. The two sets computed by Eqs. (1) and (3) utilize as gravity information:

- observed gravity along all leveling sections,
- observed gravity and gravity derived from geopotential models in a staggered manner, Figure (3),
- only gravity values derived from geopotential models.

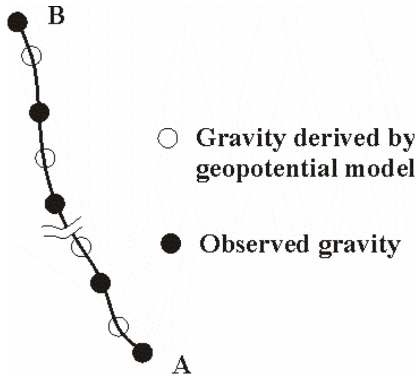

Figure (3): Using geopotential models to densify surface gravity values

The purpose of computing the second test is to investigate the ability of harmonic models to densify gravity values along leveling routes, whereas the third test should check the ability of geopotential models to replace observed gravity. In particular, three harmonic models were used. These models are EGM2008, EIGEN-CG01C and PGM2000A [6], [10], [11] and [9]. Given the geodetic latitude, longitude and elevation of the relevant benchmarks, the geopotential model-derived gravity anomaly can be computed at the earth's surface as follows [14]

$$\Delta g_{PM} = (kM / r^2) \sum_{n=2}^{nmax} (n\text{-}1)\,(a/r)^n \sum_{m=0}^{n} (\bar{C}^*_{nm} \cos m\lambda + \bar{S}_{nm} \sin m\lambda)\, \bar{P}_{nm}(\cos\theta), \qquad (5)$$

with

n_{max} the max. degree of the geopotential model,
kM the geocentric gravitational constant,
r the geocentric radius,
a the equatorial radius,
θ the geocentric co-latitude,
λ the geodetic longitude,
$\bar{C}^*_{nm}$ & $\bar{S}_{nm}$ the fully normalized spherical harmonic coefficients of degree n and order m, reduced for the even zonal harmonics of the WGS-84 reference ellipsoid,

$\bar{P}_{nm}(\cos\theta)$ the fully normalized associated Legendre function of degree n and order m.

In order to compute the surface gravity values relevant to the geopotential model, the normal gravity were firstly computed on the WGS-84 ellipsoid as follows

$$\gamma_0 = (a\, \gamma_e \cos^2\varphi + b\, \gamma_p \sin^2\varphi) / \sqrt{(a^2 \cos^2\varphi + b^2 \sin^2 \varphi)}, \qquad (6)$$

where

γ_e & γ_p the WGS-84 normal gravity at the equator and pole, respectively,

a & b the semi-major and semi-minor axis of the WGS-84 ellipsoid.

Then, the telluroid normal gravity, γ, relevant to every benchmark can be computed as follows

$$\gamma \approx \gamma_0 \,[1 - 2/a\,(1 + f + m - 2f \sin^2 \varphi)\, H + (3 /a^2)\, H^2], \qquad (7)$$

where f is the geometric flattening of the ellipsoid, H the elevation and m $(=\omega^2 a^2 b/kM)$ is the geodetic parameter of WGS-84. Finally, the geopotential model-derived surface gravity for each station is computed as

$$g_{PM} = \gamma + \Delta g_{PM} \qquad (8)$$

5 Results

From Tables (2) and (3), it is clear that in general the two rigorous formulae (Eqs. (1) & (3)) give almost the same values for OC, which differ from those obtained by the approximate formula. This is expected because this formula ignored the effect of local gravity information. Regarding the roles of harmonic models, the two tables show that the OCs computed from the partial and full use of such models are closer to those computed from observed gravity values. In particular, the EIGEN-CG01C and PGM2000A models give the closest results.

Table (2): Comparison among the OCs computed for the test line (I) (unit: mm)

Source of gravity	Harmonic model	Eq. (1)	Eq. (3)	Eq. (4)
Observed		**-25.012**	**-25.055**	-15.208
Staggered (Observed + harmonic models derived)	EGM2008	-19.987	-20.022	
	EIGEN-CG01C	-24.859	-24.902	
	PGM2000A	-26.657	-26.703	
Only derived from harmonic models	EGM2008	-20.317	-20.352	
	EIGEN-CG01C	-25.476	-25.519	
	PGM2000A	-27.210	-27.256	

Table (3): Comparison among the OCs computed for test line (II) (unit: mm)

Source of gravity	Harmonic model	Eq. (1)	Eq. (3)	Eq. (4)
Observed		**-2.807**	**-2.811**	-2.347
Staggered (Observed + harmonic models derived)	EGM2008	-2.779	-2.783	
	EIGEN-CG01C	-2.778	-2.782	
	PGM2000A	-2.804	-2.808	
Only derived from harmonic models	EGM2008	-2.528	-2.532	
	EIGEN-CG01C	-2.363	-2.366	
	PGM2000A	-2.670	-2.673	

6 Concluding remarks

Based on the obtained results, one would conclude that the geopotential harmonic models could be used as an economical tool for densifying/replacing gravity data along spirit leveling routes. This could be true, specially in regions with smooth gravity field natures. So it is recommended to apply such an approach in the treatment of precise leveling observations. In particular, positions of temporary benchmarks can be derived using hand-held GPS. This yields a 1 arc-second precision horizontal positioning which is sufficient for geopotential model computation. Moreover, such single GPS positioning accuracy safely lies within the relatively arbitrary spacing between gravity benchmarks along the levelling lines.

Finally, based on the current study, it is recommended to investigate the application of the remove-compute-restore technique of geopotential model to the computation of OC. Such technique could hopefully improve the results, since the contribution of a geopotential model to OC would represent the dominant component.

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