Validation of GOCE global gravitational field models in Norway

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Abstract

Numerous global gravitational field models have been derived from the GOCE satellite gravitational gradiometry mission. They differ by the harmonic analysis approach and the time span covered by the measurements. The quality of individual solutions is assessed by validating the global gravitational field models with respect to independent datasets.
Global gravitational field models based on the time-wise and the direct approach are validated in this contribution. All five releases are compared to height anomalies, free-air gravity anomalies, and deflections of the vertical over the continental part of Norway. The spectral enhancement method is applied to overcome the spectral inconsistency between the gravitational models and the terrestrial data. The three terrestrial datasets indicate comparable performance of the latest GOCE models with respect to EGM2008 up to degree and order 220 in the studied local area.

1. Introduction

The satellite gradiometry mission GOCE (ESA, 1999) was mapping the static part of the Earth's gravitational field in a homogeneous and nearly global way for more than four years. The mission main goals were very challenging. It was expected that GOCE would deliver a geoid model with an accuracy of 1-2 cm and free-air gravity anomalies with an accuracy of 1 mGal, both at the spatial resolution of 100 km.

Global gravitational field models (GGFMs) are the main results of the GOCE mission. Such models are used in realizing vertical reference frames in geodesy, exploring the interior of the Earth in geophysics and geology, studying the behavior of currents in oceanography, and detecting sea level rise and ice-melting in climatology.

Numerous GGFMs have already been derived from GOCE observations. Different strategies have been employed. They differ by a-priori information, stochastic modeling of the gravitational gradients, numerical technique for evaluation of the spherical harmonic coefficients, and combining various observation types with different time span (Pail et al., 2011, Brockmann et al., 2014, Bruinsma et al., 2014). Assessing quality of the individual solutions is therefore of crucial importance for both modelers and users.

In this study, validation of GGFMs based on the time-wise and the direct approaches is performed. We note that numerous validation studies of GOCE GGFMs have already been performed, e.g., Gruber et al. (2011), Hirt et al. (2011), Janák and Pitoňák (2011), Šprlák et al. (2012). They reveal a consistent representation of the Earth's gravitational field by the different analysis approaches. Successive improvements of the GOCE models for higher degree and order (d/o) spherical harmonic coefficients have been reported for models implementing more data. Moreover, it has been documented that GOCE models perform better with respect to EGM2008 in areas lacking previous gravity field observations such as Africa, Himalayas and South America. In this article, we extend the studies of Šprlák et al. (2012, 2015) and Gerlach et al. (2013) where various GOCE models up to the third release were validated with various datasets over the continental part of Norway.

For the validation purpose we exploit independent datasets of height anomalies derived from Global Navigation Satellite Systems (GNSS) and leveling data, free-air gravity anomalies, and deflections of the vertical over the continental part of Norway. The terrestrial datasets are described in Section 2. The spectral enhancement method (SEM) is applied to overcome the spectral inconsistency between the models and the terrestrial data. The accuracy of SEM and the results of the validation are presented in Section 3. The outcomes of this study are given in the conclusions.

In this paper we make use of the following nomenclature: low frequencies of the gravitational field quantities correspond to spherical harmonic d/o 2-100, middle frequencies are those between d/o 101-250, high frequencies between d/o 251-2190, and very-high frequencies are above d/o 2190.

2. Description of the terrestrial datasets

2.1 GNSS/leveling points

Traditional leveling is used to determine the vertical distance of objects relative to the geoid or quasigeoid as reference surface. It is employed for geodetic tasks where physical heights are required. Time efficiency and cost has favored the use of GNSS for precise determination of geodetic positions. The height component measured by GNSS is purely geometrical and related to a reference ellipsoid. The differences between GNSS and leveling observations allow geoid/quasigeoid heights to be derived. These quantities may be used to test GGFMs.

We have employed recent GNSS/leveling data compiled by the Norwegian Mapping Authority. The leveled heights are listed in the vertical system NN2000 (Lysaker and Vestøl, 2012), which is a zero-tidal vertical reference system tied to Normaal Amsterdams Peil at epoch 2000.0. In the NN2000 system, normal heights have been adopted. Thus the GNSS/leveling dataset provides observed height anomalies which we will refer to in the following.

The GNSS observations are referred to the epoch 1995.665. Conventionally, the ellipsoidal heights are provided in the tide-free system. Due to the different reference time epochs and tide systems adopted, the postglacial uplift as well as the tidal correction have to be applied, see section 3.1 for details.

There are 1340 GNSS/leveling stations in Norway. The stations are distributed along leveling lines, see Fig 1a. The observed height anomalies vary from 18.58 m in the eastern part to 47.60 m in the western part. The accuracy of the observed height anomalies is 2-3 cm (Omang, 2014, private communication).

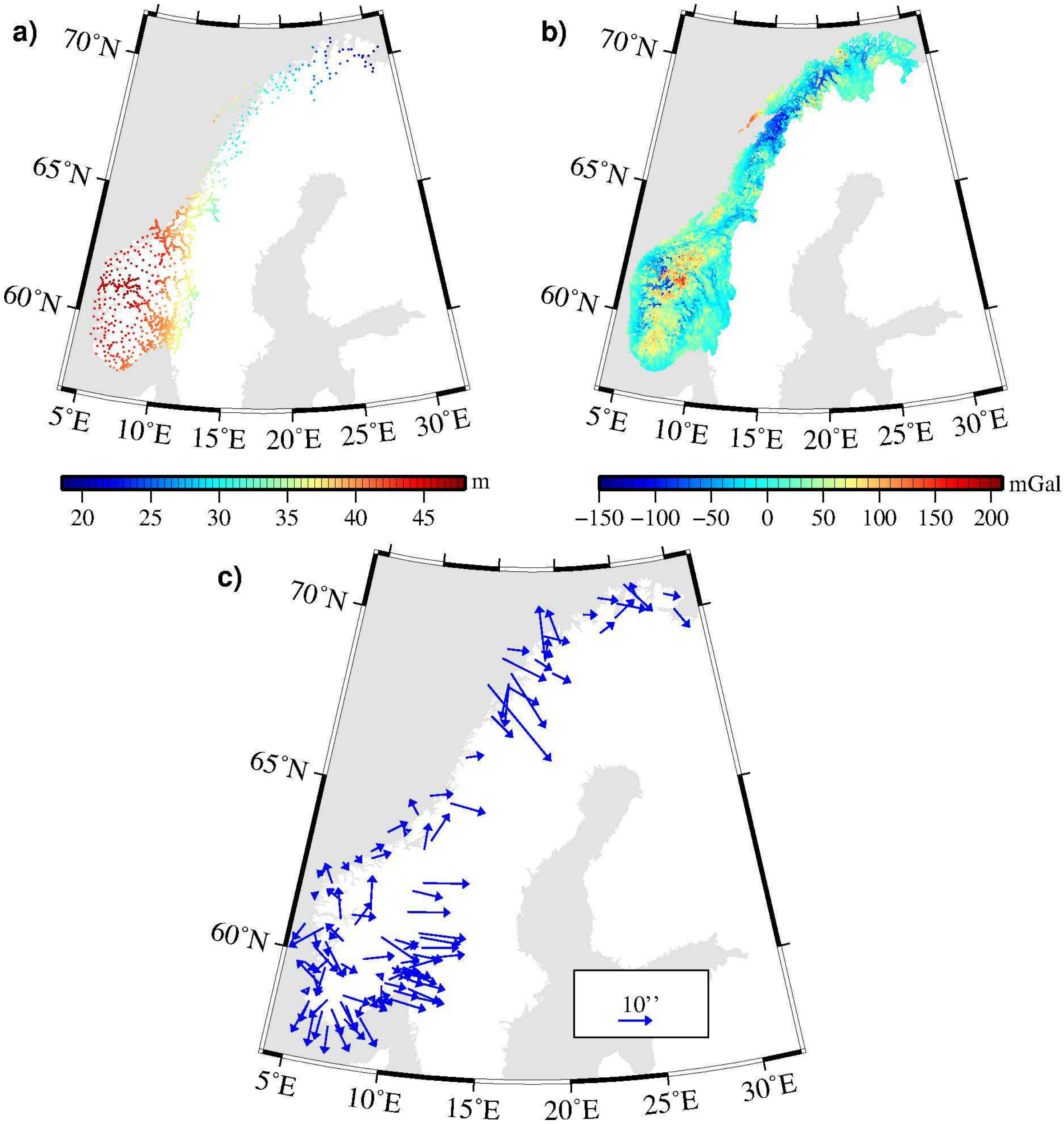

Fig. 1: Spatial distribution and magnitudes of the terrestrial datasets: a) Height anomalies, b) Free-air gravity anomalies, c) Deflections of the vertical (the arrows indicate the total deflection of the vertical).

2.2 Terrestrial free-air gravity anomalies

Terrestrial gravity data, observed by relative and absolute instruments, form the base for geoid/quasigeoid determination. The gravity reference frame in Norway (Harsson, 1978) consists of 286 primary stations, connected by relative gravimeters in early 1970s. Four stations (Hammerfest, Bodø, Trondheim, and Oslo) were observed with FG5-226 between 2005 and 2011. Absolute gravity values were derived from these time series and referred to September 2011. At this time an observing campaign at 21 primary stations were also made with A10-020 absolute gravimeter.

Attached to the primary stations are loops of 7791 secondary gravity stations, measured in the 1970s and early 1980s. The entire dataset was readjusted with respect to the absolute gravity values obtained with the FG5 and A10 instruments. In this way, the absolute level of all observations is provided by the new FG5 and A10 observations. The epoch of the newly adjusted network is 2011. Thus, comparison with GOCE observations does not require reductions for post-glacial uplift. The gravity values and also the observed free-air gravity anomalies are available in the zero tide system.

In addition to the geodetic gravity reference frame there exist a large number of relative gravimetric observations obtained for other purposes than geodesy. These terrestrial free-air gravity anomalies have been collected into a national gravity database, administered by the Geological Survey of Norway (Gellein et al., 1993). The database contains 69773 gravity stations.

Except for occasional data gaps in the central and the northern parts of Norway and very dense data along the coast in the south and south-east, the gravity stations are distributed regularly over the continental part of Norway, see Fig. 1b. The average distribution of gravity stations is 1 point per 5 km^2.

The observations were made over a time span of several decades using a variety of instruments. The estimated accuracy of the observed gravity is at the level of 0.2 mGal (Gellein et al., 1993). The free-air gravity anomalies range from -141.3 mGal to 205.5 mGal, with a standard deviation of 37.4 mGal and a mean value of 9.6 mGal, see Fig. 1b.

2.3 Deflections of the vertical

We have also compared GOCE models to deflections of the vertical at more than 100 observing stations in Norway. The astronomical coordinates of these stations were determined over a time span of several decades with various astronomical instruments and with inhomogeneous accuracy. Astronomical positioning was obtained in ten Laplace stations with a Prin transit instrument between 1923 and 1928 (Jelstrup, 1929) by the Geographical Survey of Norway to orient the geodetic first order network. Further extensions were made with an Askania transit instrument between 1930 and 1950. A Wild T4 was used to establish 27 Laplace stations between 1958 and 1969, and further observations were made in the 1970s with Wild T2 and Zeiss Ni 20 instruments (Blankenburgh, 1987). A total of 96 stations resulted from five decades of efforts. Some were reobserved as new instruments were introduced. Vertical deflections were determined for 22 additional stations in 1981 and 1985 with a zenith camera (Danielsen and Sundsby, 1995) leading to a database of 118 stations in total. Table 1 shows the mean standard deviations obtained with each instrument type.

	Transit instrument	T4	T2	Zenith camera
Std.dev. (ξ)	0.18	0.23	0.44	0.15
Std.dev. (η)	0.24	0.08	0.44	0.18

Tab. 1: Mean standard deviations for different types of astronomical instruments for the north-south (ξ) and east-west (η) components of the deflection of the vertical (Danielsen and Sundsby, 1995). All values are given in arc seconds.

The astronomical observations are unevenly distributed over Norway, see Fig. 1c. The majority of points are located along the coast of southern Norway. After identifying and removing eight outliers from the database (by performing the Grubb's test from the differences between the modeled and the observed deflections of the vertical), the remaining 110 stations were used in the following analysis. The NS component varies from -22.88″ to +16.46″. The EW component varies from -10.36″ to +18.89″. The total deflection of the vertical at the 110 stations is depicted in Fig. 1c.

3. Numerical experiment

3.1 Modeling of height anomalies, free-air gravity anomalies and deflections of the vertical using SEM

A consistent comparison between terrestrial (containing all spectral constituents) and satellite (containing low and middle frequency constituents) observations has to be followed in validation studies. Below we combine a high-resolution GGFM and a digital elevation model (DEM) for gravity field modeling purposes. Such combination is known as the SEM, see (Hirt et al., 2010). We firstly test the performance of SEM by direct comparison of the observed and modeled height anomalies, free-air gravity anomalies, and deflections of the vertical.

Spherical harmonic coefficients of EGM2008 (Pavlis et al., 2012) up to d/o 2190 have been exploited for evaluation of the low, middle, and high frequencies of the modeled quantities. Note that the primary and secondary stations of the Norwegian gravity network were also included in the development of EGM2008 (Omang, 2014, private communication). The synthesis was performed by the GRAFIM software (Janák and Šprlák, 2006), which avoids the numerical problems of Legendre's functions by implementing Horner's scheme (Holmes and Featherstone, 2002). For the free-air gravity anomalies and the NS component of the vertical deflection, harmonic expansions based on the ellipsoidal approximation have been applied (Wenzel, 2005). The deviation between the spherical radius and the ellipsoidal normal is accounted for by the ellipsoidal approximation. It does not affect the height anomalies and the EW component of the vertical deflection. Therefore, these two functionals have been synthesized by the standard spherical harmonic expansions.

The very-high frequencies of the modeled quantities have been calculated from a residual terrain model (RTM, Forsberg, 1984). The RTM has been constructed by subtracting the high resolution DEMs ACE2 (Berry et al., 2010) or ASTER (Tachikawa et al., 2011) with a smooth elevation model DTM2006.0 (Pavlis et al., 2007). The elevations from the DTM2006.0 have been synthesized by the GRAFIM software up to d/o 2160. Zero elevations have been considered over oceans and seas.

Numerical integration over the RTM has been performed point-wise, i.e., at each station of the height anomalies, free-air gravity anomalies and deflections of the vertical, to calculate the very-high frequencies. For simplicity, only planar approximation of the corresponding integral kernels has been considered. We have assumed a constant rock density value of 2670 kg m^{-3} of the masses within the RTM. The numerical integration has been divided into the inner and outer zone. In the inner zone we make use of the ASTER DEM with sampling of 1″ x 1″ of the mass elements and apply this discretization within the integration radius $\psi_0 = 0.1°$.

Beyond that, the outer integration zone exploits coarser ACE2 DEM with regular sampling of 30″ x 30″. We have considered the size of the integration radius in the outer integration zone ψ_0 = 0.5° for free-air gravity anomalies and deflections of the vertical. In case of height anomalies the integration radius in the outer zone was ψ_0 = 3.0°. Numerical experiments showed that the contributions beyond the selected integration radii are negligible. Note that the harmonic correction has to be considered for points located inside the RTM. This issue has been solved implicitly by the approach proposed in (Kadlec, 2011).

Several corrections have been calculated to avoid inconsistencies between the observed and modeled quantities. To get consistent height anomalies we have corrected the GNSS observations by a 5-year correction due to the postglacial rebound based on the model NKG2005LU (Ågren and Svensson, 2007). Due to different tide systems in GNSS and levelling data, a tidal correction (from zero-tide to tide-free system; Mäkinen and Ihde, 2008) has been considered for the leveled heights. Atmospheric correction (Wenzel, 1985) and a tidal correction (from zero-tide to tide-free system) have been applied to the observed free-air gravity anomalies. The NS component of the vertical deflection has been corrected by the topographic reduction (the synthesis is performed on the telluroid rather than on the Earth surface) and for the curvature of the normal plumb line (which links the direction of the normal gravity vector to the direction of the ellipsoidal normal). Only the first correction has also been applied for the WE component. Correction due to postglacial rebound has not been applied to the deflections of the vertical due to missing information of the exact year of observation for several of the stations. We estimate that the neglected postglacial rebound may reach up to 0.2″, see (Gerlach et al., 2013). Note that the tidal effect may reach 0.01″ for the deflections of the vertical and has been neglected in our numerical investigations.

Quantity	RTM	min.	max.	mean	std. dev.
ζ	no	0.102	0.477	0.351	0.051
	yes	0.202	0.608	0.470	0.067
Δg	no	-197.1	100.1	-5.7	20.7
	yes	-51.4	65.5	-0.3	4.3
ξ	no	-5.60	9.27	-0.37	2.06
	yes	-3.44	4.40	-0.38	1.20
η	no	-5.96	8.00	-0.72	2.24
	yes	-4.03	3.56	-0.59	1.47

Tab. 2: Statistics of the differences between the observed and the modeled (using EGM2008 and RTM) quantities with and without the RTM (in meters for height anomalies, in mGal for gravity anomalies, and in arc seconds for the deflections of the vertical).

We summarize the statistics of the differences between the observed and the modeled quantities in Tab. 2. In the case of height anomalies, the standard deviation of the differences increases from 0.051 m (without RTM) to 0.067 m (with RTM). Such a result is in contradiction with Hirt et al. (2010) who reported significant improvements in height anomaly modeling in Switzerland due to the RTM contribution. On the other hand, Kadlec

(2011) showed that the effect of the RTM does not contribute to better performance of SEM in modeling height anomalies in the Czech Republic. For free-air gravity anomalies, we note a decrease of standard deviation from 20.7 mGal (without RTM) to 4.3 mGal (with RTM). Thus the effect of the RTM improves the modeling by 79%. Improvements of SEM due to the RTM effect may also be seen for the deflections of the vertical. For the NS component, the standard deviation decreases from 2.06″ (without RTM) to 1.20″ (with RTM). For the EW component the standard deviation decreases from 2.24″ (without RTM) to 1.47″ (with RTM). Both components are thus improved by approximately 40 % when the RTM effect is included.

The results prove the sensitivity of free-air gravity anomalies and deflections of the vertical to very-high frequencies of the Earth's gravity field. On the other hand, RTM leads to worse performance for the height anomalies. In this case, the unmodified EGM2008 fits better to the observed height anomalies. Such result will be analyzed in more detail in our future research.

3.2 Validation of GOCE-derived GGFMs

We opted for testing seven GGFMs. We consider all five releases based on the time-wise approach (Pail et al., 2010, 2011, Brockmann et al., 2014) and the two most recent models based on the direct approach (Bruinsma et al., 2013, 2014), see Tab. 3.

GGFM	N_{max}	Reference
TIM R1	224	(Pail et al., 2010)
TIM R2	250	(Pail et al., 2011)
TIM R3	250	
TIM R4	250	
TIM R5	280	(Brockmann et al., 2014)
DIR R4	260	(Bruinsma et al., 2013)
DIR R5	300	(Bruinsma et al., 2014)

Tab. 3: Characteristics of the validated GGFMs

We have performed the validation of the GOCE-derived models as follows. The modeled quantities were determined by SEM as described in Section 3.1. Thereby we synthesized the lowest part of the spectrum from each of the GOCE models up to a variable d/o of the spherical harmonic coefficients. The variable d/o ranges from 2 up to the maximum available. Above this spectral range, we have considered the contribution of EGM2008 up to d/o 2190. Then the RTM effect has been calculated above d/o 2190. Note that we have considered the RTM effect also for the height anomalies despite its lacking improvement by SEM. This does not affect any of the conclusions. Finally, the observed and the modeled quantities have been compared and the corresponding statistics have been evaluated.

The standard deviations of differences between the observed and the modeled height anomalies are depicted in Fig. 2. The differences between all models are within 1 cm up to d/o 160. In particular, the differences are negligible for the time-wise and the direct approach for the fourth and fifth release up to d/o 160. Theoretically, we may expect better performance of the models based on the direct approach in the low frequencies. This is due to GRACE measurements involved in the direct approach. We suppose that EGM2008 errors dominate the total error budget and overwhelm the better performance of the direct approach. Above d/o 160 we observe an exponential increase of the standard deviations. It starts at higher d/o for later releases of the GOCE models. We note a 10% increase of the standard deviation for GOCE models with respect to EGM2008 (indicated by the dashed line) at d/o 165 for TIM R1, d/o 185 for TIM R2, d/o 195 for TIM R3, d/o 200 for TIM R4, and d/o 220 for TIM R5. In other words, more gradiometric data leads to better performance of the models, as is expected. The performance of DIR R4 with respect to TIM R4 is slightly worse between degrees 175-220. However, the standard deviation for DIR R4 is lower by more than 1 cm above d/o 220. Only negligible differences are seen between DIR R5 and TIM R5 up to d/o 265. Above d/o 265 the model based on the direct approach performs better. Šprlák et al. (2015) demonstrated an improvement of third release GOCE models with respect to EGM2008 of up to 1.5 cm between d/o 100-200. However, this improvement is not as pronounced in Fig. 2. This discrepancy is due to different datasets of height anomalies. In this study we exploit height anomalies over the whole continental Norway, whereas Šprlák et al. (2015) used height anomalies over southern Norway. We also suspect that the possible improvements relative to EGM2008 are more pronounced in southern Norway. The gain of GOCE models with respect to EGM2008 over Norway will be investigated in a future study.

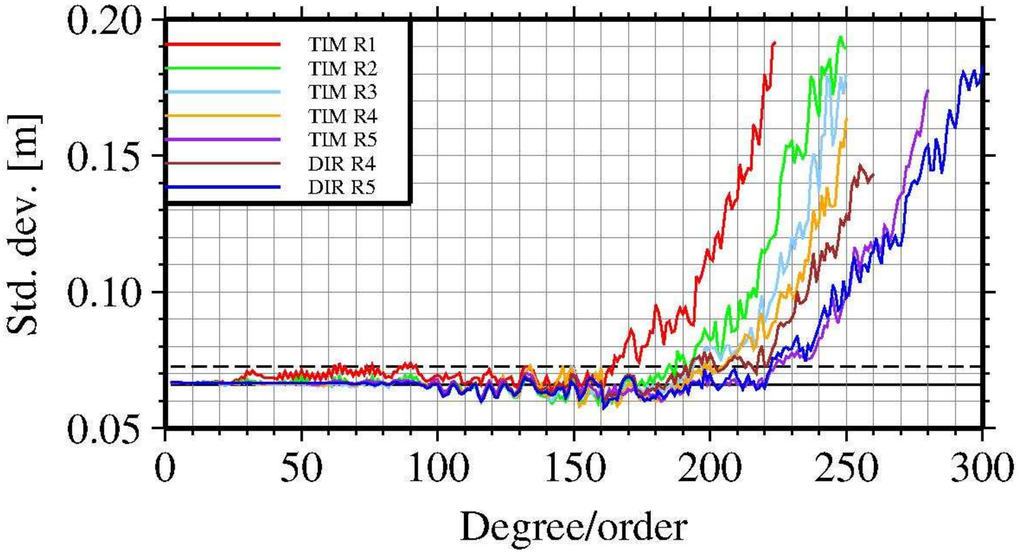

Fig. 2: Standard deviation of differences between the observed and the modeled (using the validated GOCE-derived GGFMs, EGM2008 and RTM) height anomalies as a function of d/o of the GOCE-derived GGFMs (RTM included). The solid black line represents the standard deviation of differences between the observed and the modeled (using EGM2008 and RTM) height anomalies. The dashed black line represents the standard deviation of differences between the observed and the modeled (using EGM2008 and RTM) height anomalies increased by 10%.

Validation by the free-air gravity anomalies offers a possibility to confirm the above results. The standard deviations of differences are shown in Fig. 3. As for height anomalies, we observe the similar performance of all models up to d/o 150 and then the exponential increase depending on the time span of gradiometric data. The difference of several tenths of mGal above d/o 210 and reaching 1 mGal at d/o 250 shows better performance of DIR R4 relative to TIM R4. The latest release gives identical results for both approaches up to d/o 250. Above this d/o the direct approach performs better. However, in contrast to height anomalies, the 10% increase of the standard deviation for GOCE models with respect to EGM2008 is reached for higher d/o, see Fig. 2. Also we do not observe any deviations with respect to EGM2008 below d/o 150. Both of these observations originate from the sensitivity of free-air gravity anomalies to higher frequencies. The accuracy of SEM in the case of free-air gravity anomalies (on the level of 4.3 mGal, Tab. 2) overwhelms the magnitudes of the differences between EGM2008 and GOCE models (reaching only a few mGal).

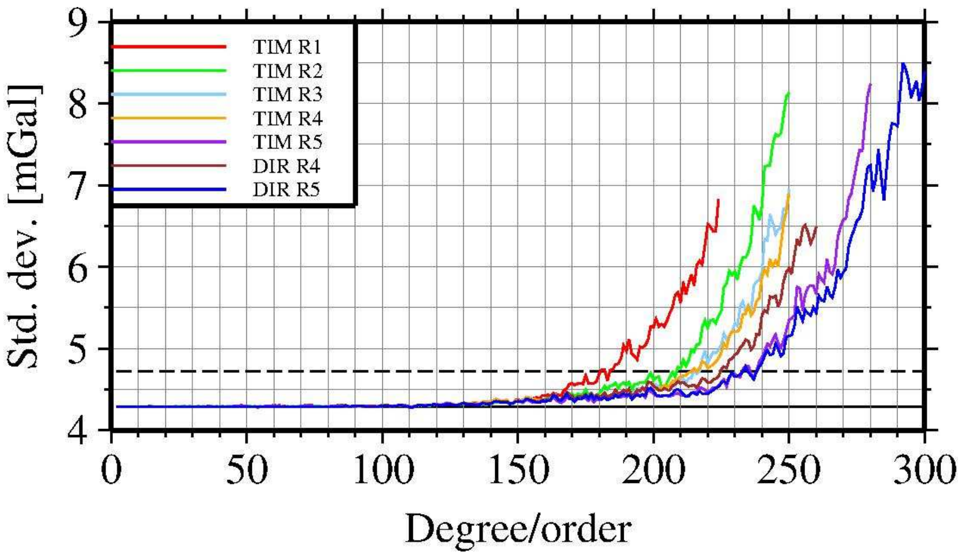

Fig. 3: Standard deviation of differences between the observed and the modeled (using validated GOCE-derived GGFMs, EGM2008 and RTM) free-air gravity anomalies as a function of d/o of the GOCE GGFMs (RTM included). The solid black line represents the standard deviation of differences between the observed and the modeled (using EGM2008 and RTM) free-air gravity anomalies. The dashed black line represents the standard deviation of differences between the observed and the modeled (using EGM2008 and RTM) free-air gravity anomalies increased by 10%.

The deflections of the vertical may also confirm the above results. The standard deviations of differences are shown in Fig. 4. We observe similar performance of the models up to d/o 160. Above this d/o the models differ based on the release. Good agreement between the last two releases of the time-wise and direct approach is confirmed. However, in contrast to height and free-air gravity anomalies, the standard deviations exceed the 10 % threshold even at higher d/o. This may be explained by the different spectral sensitivity of the vertical deflections. Small deviations are visible when comparing the corresponding curves for the NS and EW components. We identify possible improvements, which are less significant compared to height anomalies, relative to EGM2008 for the NS component between d/o 100-200. However, this is not demonstrated by the EW component. Also the improvements with more gradiometric data are not so significant for the EW component. In other words, the spectral range of the crossing for the curves corresponding to the first and the last release with the line indicating the 10 % threshold is wider for the NS component as compared to the EW component. We suppose that this may be related to the different spatial resolution of GOCE observations in both principal directions. At high latitudes, such as it is the case for Norway, the spatial resolution is higher in EW direction than in NS direction for the same d/o numbers. Consequently, the point of divergence (where the curves in Figure 4 start to deviate from the EGM2008 baseline) is shifted towards higher spherical harmonic degrees in case of the EW-component as compared to the NS-component.

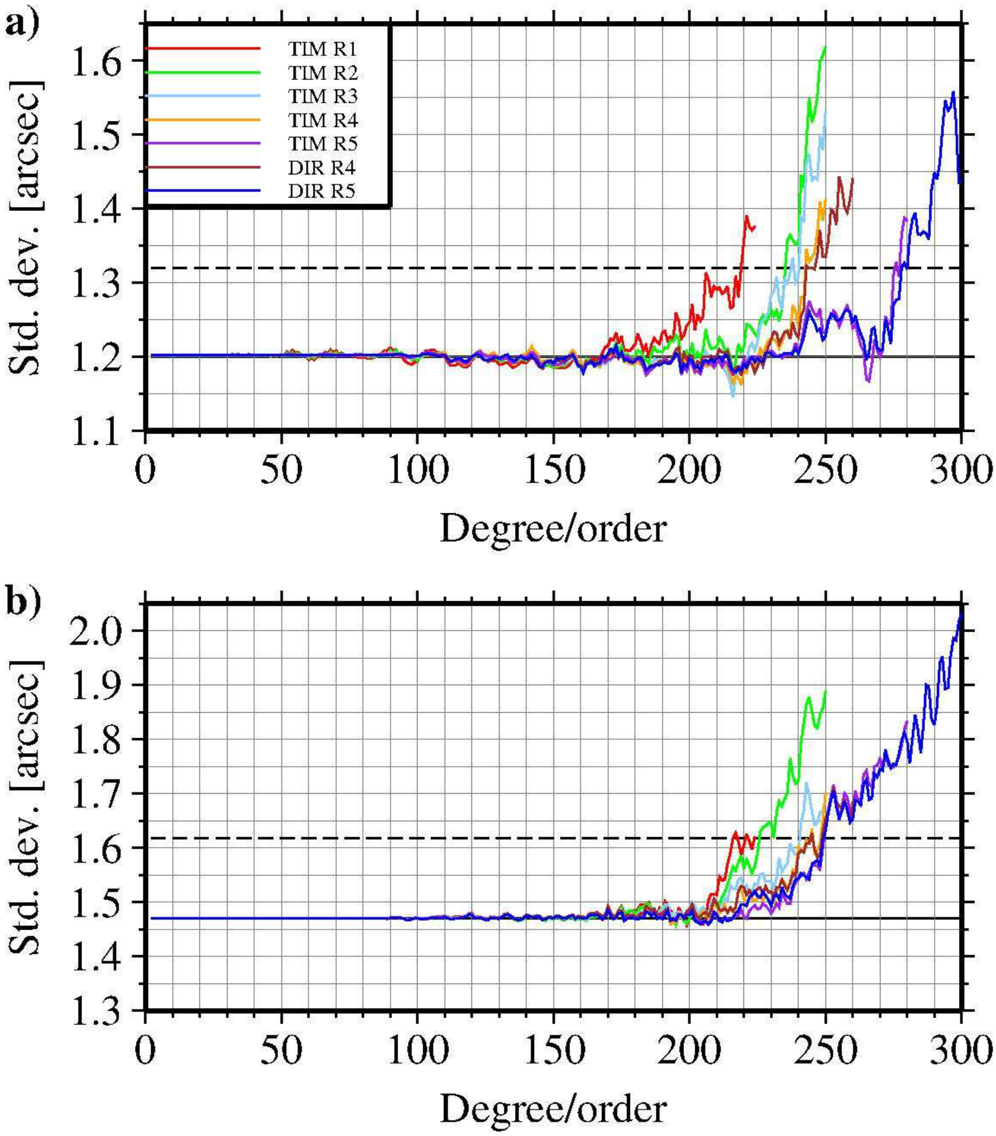

Fig. 4: Standard deviation of differences between the observed and the modeled (using validated GOCE-derived GGFMs, EGM2008 and RTM) deflection of the vertical as a function of d/o of the GOCE GGFMs (RTM included): a) NS component, b) EW component. The solid black line represents the standard deviation of differences between the observed and the modeled (using EGM2008 and RTM) deflections of the vertical. The dashed black line represents the standard deviation of differences between the observed and the modeled (using EGM2008 and RTM) deflections of the vertical increased by 10%.

Conclusions

Homogeneous sets of height anomalies, free-air gravity anomalies, and deflections of the vertical have been produced from new terrestrial measurements in Norway. These were exploited for validation of seven GOCE-based models.

Numerical experiments have proven that the gravitational attraction of local masses, computed from DEMs, is important to free-air gravity anomalies and deflections of the vertical. However, height anomalies fit better to the pure EGM2008 model when not taking RTM effects into account.

We found the performance of all models to be very similar up to d/o 160 in the studied local area. We suppose that EGM2008 errors dominate the total error budget and do not allow for seeing better performance of the direct approach in the low frequencies. With more GOCE observations available, higher d/o are determined more accurately. Such finding definitely confirms results of existing studies. Models of the latest (fifth) release compare well to EGM2008 up to d/o ~220. Validation by height anomalies may suggest possible improvements by the GOCE models with respect to EGM2008 in the middle frequencies. This improvement was not proven by free-air gravity anomalies and deflections of the vertical due to their sensitivity to higher frequencies.

Future experiments will be performed to investigate the better fit of pure EGM2008 model to the observed height anomalies. Also possible systematic effects, either due to EGM2008 or the terrestrial datasets, will be investigated. Future models based on GOCE observations will be validated. These tasks are important for improvements of the gravity field over Norway.

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References

Berry PAM, Smith RG, Benveniste J (2010) ACE2: The New Global Digital Elevation Model. In: Mertikas SP (Ed.) Gravity, Geoid and Earth Observation, IAG Symposia, Vol. 135, 23-27 June 2008, Chania, Greece, pp 231-238

Blankenburgh JC (1987) Geodesi med stjerner og satelliter i Norge. Bilag til Kart og Plan nr. 1 – 1987: 91-95 (in Norwegian)

Brockmann JM, Zehentner N, Höck E, Pail R, Loth I, Mayer-Gürr T, Schuh W-D (2014) EGM_TIM_RL05: An independent geoid with centimeter accuracy purely based on the GOCE mission. Geophysical Research Letters, 41: 8089-8099

Bruinsma SL, Förste C, Abrikosov O, Lemoine J-M, Marty J-C, Mulet S, Rio M-H, Bonvalot S (2014) ESA's satellite-only gravity field model via the direct approach based on all GOCE data. Geophysical Research Letters, 41: 7508-7514

Bruinsma SL, Förste C, Abrikosov O, Marty J-C, Rio M-H, Mulet S, Bonvalot S (2013) The new ESA satellite-only gravity field model via direct approach. Geophysical Research Letters, 40: 3607-3612

Danielsen J, Sundsby J (1995) Målinger med senitkamera for loddavvik. Faglig melding, Geodesidivisjonen, Statens kartverk (in Norwegian)

ESA (1999) Gravity field and steady-state ocean circulation mission. Reports for Mission Selection, the Four Candidate Earth Explorer Core Missions, ESA SP-1233(1)

Forsberg R (1984) A study of terrain reductions, density anomalies and geophysical inversion methods in gravity field modeling. Report No. 355, Department of Geodetic Science and Surveying, Ohio State University, Columbus, USA

Gellein J, Olesen O, Reitan M, Sæther PO (1993) Brukerbeskrivelse – database of prosessering av gravimetriske data. Intern NGU Rapport Nr. 93.019, Norges Geologiske Undersøkelse, Trondheim, Norge (in Norwegian)

Gerlach C, Šprlák M, Bentel K, Pettersen BR (2013) Observation, validation, modeling – historical lines and recent results in Norwegian gravity field research. Kart og Plan 73: 128-151

Gruber T, Visser PNAM, Ackermann C, Hosse M (2011) Validation of GOCE gravity field models by means of orbit residuals and geoid comparison. Journal of Geodesy, 85: 845-860

Harsson BG (1978) Beregning av det norske gravimeterbasisnettet. In Proceedings, NKG General Assembly, Oslo (in Norwegian)

Hirt C, Featherstone WE, Marti U (2010) Combining EGM2008 and SRTM/DTM2006.0 residual terrain model data to improve quasigeoid computations in mountainous areas devoid of gravity data. Journal of Geodesy, 84: 557-567

Hirt C, Gruber T, Featherstone WE (2011) Evaluation of the first GOCE static gravity field models using terrestrial gravity, vertical deflections and EGM2008 quasigeoid heights. Journal of Geodesy, 85: 723-740

Holmes SA, Featherstone WE (2002) A unified approach to the Clenshaw summation and the recursive computation of very high degree and order normalized associated Legendre functions. Journal of Geodesy, 76: 279-299

Janák J, Pitoňák M (2011) Comparison and testing of GOCE global gravity models in central Europe. Journal of Geodetic Science, 1: 333-347

Janák J, Šprlák M (2006) A new software for gravity field modelling. Geodetic and Cartographic Horizon, 52: 1-8 (in Slovak)

Jelstrup HJ (1929) Determinations of astronomical longitudes, latitudes and azimuths. Geodetiske arbeider, hefte II (1929) og III (1931), Norges geografiske opmåling (in Norwegian)

Kadlec M (2011) Refining gravity field parameters by residual terrain modeling. Doctoral Thesis, Department of Mathematics, Faculty of Applied Sciences, University of West Bohemia, Pilsen, Czech Republic, 150 p.

Lysaker DI, Vestøl O (2012) Norwegian vertical reference system NN2000. Norwegian Mapping Authority, Hønefoss, Norway (in preparation)

Mäkinen J, Ihde J (2008) The permanent tides in height systems. In: Sideris MG (Ed.) Observing our Changing Earth, IAG Symposia, Vol. 133, 2-13 July, 2007, Perugia, Italy, pp 81-87

Pail R, Bruinsma S, Migliaccio F, Förste C, Goiginger H, Schuh WD, Höck E, Reguzzoni M, Brockmann JM, Abrikosov O, Veicherts M, Fecher T, Mayrhofer R, Krasbutter I, Sansó F, Tscherning CC (2011) First GOCE gravity field models derived by three different approaches. Journal of Geodesy, 85: 819-843

Pail R, Goiginger H, Mayrhofer R, Schuh WD, Brockmann JM, Krasbutter I, Höck E, Fecher T (2010) GOCE gravity field model derived from orbit and gradiometry data applying the time-wise method. In: Proceedings of the ESA Living Planet Symposium, 28 June - 2 July 2010, Bergen, Norway

Pavlis NK, Factor JK, Holmes SA (2007) Terrain-related gravimetric quantities computed for the next EGM. In: Gravity Field of the Earth, Proceedings of the 1st International Symposium of the International Gravity Field Service, Harita Dergisi, Special Issue 18, Istanbul, Turkey, pp 318-323

Pavlis NK, Holmes SA, Kenyon SC, Factor JK (2012) The development and evaluation of the Earth Gravitational Model 2008 (EGM2008). Journal of Geophysical Research, Vol. 117, B04406, 38 pp, doi:10.1029/2011JB008916

Šprlák M, Gerlach C, Pettersen BR (2012) Validation of GOCE global gravity field models using terrestrial gravity data in Norway. Journal of Geodetic Science, 2: 134-143

Šprlák M, Pettersen BR, Omang OCD, Lysaker DI, Sekowski M, Dykowski P (2015) Comparison of GOCE global gravity field models to test fields in southern Norway. In: Marti U (Ed.) Gravity, Geoid and Height Systems, IAG Symposia, Vol. 141, 9-12 October, 2012, Venice, Italy, pp. 59-65

Tachikawa T, Hato M, Kaku M, Iwasaki A (2011) The characteristic of ASTER GDEM version 2. In: IEEE International Geoscience and Remote Sensing Symposium (IGARSS), July 24-29, Vancouver, Canada, pp 3657-3660

Wenzel HG (1985) Hochauflösende Kugelfunktionsmodelle für das Gravitationspotential der Erde, Wissenschaftliche Arbeiten der Fachrichtung Vermessungswesen der Universität Hannover, Nr. 137, Geodätisches Institut der Universität Hannover (in German)

Wenzel HG (2005) Global models of the gravity field of high and ultra-high resolution. In: International School for the Determination and Use of the Geoid, 31 January - 5 February 2005, Budapest, Hungary, International Geoid Service, 36 p.

Ågren J, Svensson R (2007) Postglacial land uplift model and system definition for the new Swedish height system RH 2000. Reports in Geodesy and Geographical Information Systems, LMV-Rapport 2007:4, National Land Survey of Sweden, Gävle, Sweden, 124 p.