RESULTS OF EGM08 GEOPOTENTIAL MODEL TESTING AND ITS COMPARISON WITH EGM96

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1. Introduction

The knowledge of geopotential models accuracy and its problematic areas is necessary for all users. In case of World Height System (WHS) development, the geopotential model accuracy limits

- WHS accuracy;
- determination of the geoidal geopotential W_0;
- connection of local vertical datum to WHS;
- computation of the geopotential values W;
- height computations.

What do we need for geopotential model testing?

- Theory of testing and its applications
- value of the geoidal geopotential W_0
- four primary constants defining the level ellipsoid and its gravity field
- geopotential model evaluation and monitoring network (GMEMN), which covers (if possible) the whole Earth´s surface.

The theory for Geopotential Model Testing (GMT), developed by Burke et al. (1995), has applied to the recent EGM08 model, and for comparison purposes, also to the previous EGM96 one. The methodology requires the four primary constants defining the level ellipsoid and its gravity field and accurate geocentric positions as well as normal Molodensky heights of the testing sites on the Earth's surface. Over the oceans, geocentric positions, altimetric heights observed by TOPEX/POSEIDON (TP) or Jason 1 as well as h_{SST}, obtained from a sea surface topography model (POCM4B), are also needed. No hypothetical quantities such as, for example, the orthometric heights and/or geoid heights are used in this GMT. Although a global coverage of the testing sites is preferable, regional testing networks are also useful for GMT

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2. Primary constants used for GMT

Essential progress was made in refining the geoidal potential W_0 (Burša et al., 1999a, 2001/2002) on the basis of TP altimeter data and the sea surface topography (SST) model POCM 4B (360 by 360) of Rapp et al. (1996), based on the global circulation model:

$$W_0 = (62\ 636\ 856.0 \pm 0.5)\ \mathrm{m}^2 \cdot \mathrm{s}^{-2}. \quad (1)$$

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The above value has been adopted for our GMT, the other three adopted constants were: the geocentric gravitational constant

$$GM = (398\ 600\ 441.8 \pm 0.8) \times 10^6\ \mathrm{m}^3 \cdot \mathrm{s}^{-2}, \quad (2)$$

(Ries et al., 1992), the angular velocity of the Earth's rotation

$$\omega = (7\ 292\ 115.8 \pm 0.8) \times 10^{-11}\ \mathrm{rad} \cdot \mathrm{s}^{-1} \quad (3)$$

(IAG SC3 Rep., 1995) and the second zonal Stokes parameter

$$J_2 = (1\ 082\ 635.9 \pm 0.1) \times 10^{-9}, \quad (4)$$

(IAG SC3 Rep., 1995) which is in the zero-frequency tide reference system. Note that the constants (1), (2) and (3) are independent of the tide reference system. Since some GMT data are in the mean tide system (oceans) or in the tide-free system (continents), the constant (4) should be corrected to account for the tidal reference system.

From constants (1) - (4), three derived constants can be computed as follows: the geopotential scale factor

$$R_0 = \frac{GM}{W_0} = (6\ 363\ 672.56 \pm 0.05)\ \mathrm{m} \quad (5)$$

the semimajor axis of the level ellipsoid in the mean and tide-free reference systems, respectively

$$a_{mean} = (6\ 378\ 136.68 \pm 0.05)\ \mathrm{m},\ \ a_{tide\text{-}free} = (6\ 378\ 136.55 \pm 0.05)\ \mathrm{m}, \quad (6)$$

and its flattening

$$\alpha_{\mathrm{mean}} = 1\ /\ (298.252\ 34 \pm 0.000\ 02),\ \alpha_{\mathrm{tide\text{-}free}} = 1\ /\ (298.257\ 69 \pm 0.000\ 02). \quad (7)$$

The geopotential scale factor (5) is also independent of the tide systems. Parameters (6) and (7) define the mean and tide-free level ellipsoids.

The four fundamental constants GM, ω, J_2, W_0 or GM, ω, α, a make it possible to compute the actual potential at any site on the physical surface of the Earth with known geocentric position, thus the normal Molodensky height and/or the geopotential number can also be considered known. The GMT is limited by the errors of the normal Molodensky heights and uncertainties of the fundamental constants. The fundamental constant uncertainties contribute about ±0.5 $\mathrm{m}^2 \cdot \mathrm{s}^{-2}$ error to the geopotential and/or ±0.05 m error to the radius of the local equipotential surface, determined by the tested geopotential model.

3. Geopotential model evaluation and monitoring network (GMEMN)

The GMEMN consists of 31,557 testing sites, covering about 82% of the Earth´s surface (Tab. 1, Fig. 1). The altimetry data of TP AVISO Altimetry project was used for determinations of geocentric positions and altimeter heights of repeat cross-over points, forming 20,768 oceanic testing heights. The POCM 4B (360 by 360) SST model (Rapp et al., 1996) has been used for computations of SST heights in order to obtain the normal Molodensky heights over the oceans. No bias or systematic low frequency errors (possibly due to SST) were removed here. This may be attempted in the future work. The testing sites on the continents are identical with the GPS/leveling sites. However, where only orthometric Helmert heights were available, they had to be first transformed into the normal Molodensky heights. In case of Australia, the height system was uncertain, so we have also assumed orthometric Helmert heights as a first approximation.

Table 1 Regions and number of testing sites assembled in the past and used for of GMEMN used for geopotential model testing

Territory/blocs	Number of testing site	Territory/blocs	Number of testing site
U.S.A. and Canada	**6 479**	**France**	**973**
Mexico	**686**	**Spain**	**305**
Australia	**866**	**Belgium**	**42**
Argentina	**32**	**Greece**	**5**
Brazil	**182**	**Baltic Region**	**25**
Chile	**45**	**Portugal**	**122**
Uruguay	**10**	**Czech Republic, Hungary, Slovak Republic, Poland**	**654**
Venezuela	**21**	**Federal Republic of Germany, Netherlands, Scandinavia, Latvia, Lithuania**	**324**
Indonesia	**18**	**Oceans (repeat cross-over points)**	**20 768**

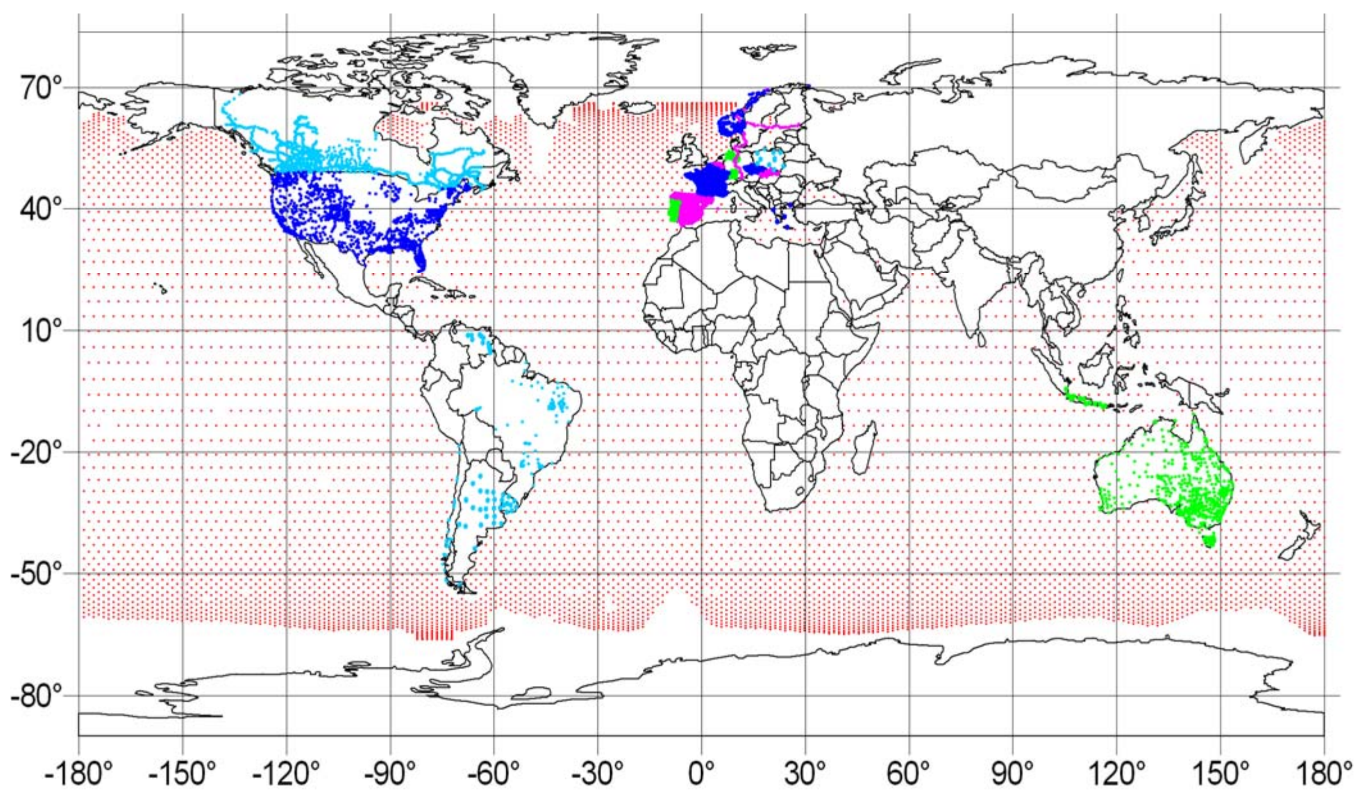

Fig. 1 Geopotential Model evaluation and monitoring network

4. The first results

The GMT methodology is based on the difference δW between the geopotential value at the testing site, computed as a function of positions x_j, normal Molodensky heights H_q and the four fundamental constants

$$W = W(GM,\ \omega,\ J_2^{(0)},\ W_0,\ x_j,\ H_q) \tag{8}$$

(for an explicit functional expression, see Burša et al., 1999b), and the geopotential value, computed from the tested geopotential model:

$$\delta W = W - W(\text{model}). \tag{9}$$

Instead of δW, the corresponding radial distortions δR ($\delta R = -GM\ \delta W / W^2$) of the equipotential surface, passing through the testing site, can be used in GMT. The distortions δW and/or δR are due to the tested geopotential model, if the errors in fundamental constants (1) - (4), as well as, the errors in geocentric coordinates x_j and in normal Molodensky heights H_q of the testing sites are small enough. The uncertainty in the adopted fundamental constants contributes to δR as follows: GM (±13 mm), ω (±l mm), $J_2^{(0)}$ (±0.6 mm), W_0 (±50 mm). The errors in x_j and H_q are believed to amount to a few centimetres only in most cases. However, there are regions, such as those outside Europe and North America, where leveling height errors are likely much larger. Furthermore, there are differences in heights due to different Local Vertical Datums used. That is why, the non-zero biases in δW and/or in δR, i.e. the mean values, computed over GMEMN regions connected to the same tide gauge station, are interpreted as being due to Local Vertical Datum shifts. Therefore it is necessary to exclude mean values $\delta \overline{W}$ and/or $\delta \overline{R}$, i.e., to use standard deviations (Std) rather than RMS. Then, the $\delta \overline{R}$ and the corresponding Std of tested geopotential model obtained over an area covered by testing sites, can be evaluated as follows:

$$\delta \overline{R} = \frac{\sum_1^n \delta R}{n} \quad \text{and} \quad Std = \sqrt{\frac{\sum_1^n \left(\delta \overline{R} - \delta R\right)^2}{n-1}}\ , \tag{10}$$

where n is the number of testing sites within a region. Note that the mean values of (10) can also be used for connecting Local Vertical Datums to a World Height System.

4.1. Numeral results of testing

The first results of the application of the above theory are summarised in **Table 2**. Here, the geopotential models EGM96 and EGM08 were evaluated in selected regions only.

Conclusions for geopotential model evaluation:

a) geopotential model EGM96
This model gave Std ranging from **±0.128 m (oceans) up to ±1.601 m (Venezuela)**. The mean Std value was **±0.500 m**.

b) geopotential model EGM08
This model gave the best Std’s, ranging from **±0.071 m (oceans) up to ±0.935 m (Venezuela)**. The mean Std value was **±0.334 m**.

Table 2 First results of geopotential models EGM96 and EGM08 testing.

Territory	Standard deviation [m]		Territory	Standard deviation [m]	
	EGM96	EGM08		EGM96	EGM08
Oceans	**± 0.128**	**±0.071**	**Portugal**	**± 0.347**	**±0.235**
U.S.A.	**± 0.398**	**± 0.283**	**Slovakia**	**±0.451**	**±0.292**
Canada	**± 0.363**	**±0.225**	**Spain**	**± 0.312**	**±0.164**
Australia	**± 0.445**	**±0.323**	**Indonesia**	**±0.650**	**±0.442**
France	**± 0.368**	**±0.116**	**Mexico**	**± 0.613**	**±0.400**
Greece	**± 0.324**	**±0.154**	**Argentina**	**± 0.783**	**± 0.656**
Czech republic	**±0.185**	**±0.113**	**Brazil**	**± 0.884**	**± 0.762**
Hungary	**±0.137**	**±0.064**	**Chile**	**± 0.946**	**±0.696**
Baltic region	**± 0.232**	**±0.123**	**Uruguay**	**± 0.614**	**±0.564**
Poland	**±0.226**	**±0.075**	**Venezuela**	**± 1.601**	**±0.935**

Table 3 Relative **comparison of geopotential models EGM96 and EGM08.**

EGM08 Std decrease (%) with respect to EGM08			
Territory	**EGM96 → EGM08**	**Territory**	**EGM96 → EGM08**
Oceans	**-44.5 %**	**Portugal**	**-32.3 %**
U.S.A.	**-28.9 %**	**Slovakia**	**-35.3 %**
Canada	**-38.0 %**	**Spain**	**-47.4 %**
Australia	**-27.4 %**	**Indonesia**	**-32.0 %**
France	**-68.5 %**	**Mexico**	**-34.7 %**
Greece	**-52.5 %**	**Argentina**	**-16.2 %**
Czech republic	**-38.9 %**	**Brazil**	**-13.8 %**
Hungary	**-53.2 %**	**Chile**	**-26.4 %**
Baltic region	**-47.0 %**	**Uruguay**	**-8.1 %**
Poland	**-66.8 %**	**Venezuela**	**-41.6 %**

4.2. Numeral results of comparisons of geopotential models EGM96 and EGM08

The first results of Std comparisons of the geopotential models EGM96 and EGM08 are shown in **Table 3**.

Conclusions of geopotential models EGM96 and EGM08 evaluation (Std comparison):

The EGM08 Std's are 33.9 % smaller than EGM96 ones. The highest Std decrease was seen in France (**-68.5%),** Poland **(-66.8 %),** Hungary **(-53.2 %)** and Greece **(-52.5 %).**

5. Conclusions

- Our GMT technology is ready for testing of a geopotential model with harmonic expansions up to degree/order n=2190
- It is necessary to enlarge the GMEMN (e.g. in Asia, Africa,…)
- **amsignificant global decrease of EGM08 Std's is evident : the mean standard deviation value is ±0.339 m, (a decrease of about -39% with respect to EGM96!)**
- The highest precision (in terms of Std): oceans (±0.071 m)
  Hungary (±0.064 m)
  Poland (±0.075 m)
- The lowest precision (in terms of Std): Venezuela (±0.935 m)
- The highest Std decrease of EGM08 (wrt EGM96): France (-68.5 %)
  Poland (-66.8 %)
- The lowest Std decrease of EGM08 precision (wrt EGM96): Uruguay (-8.1 %)
  Brazil (-12.2%)
- The observed GMT technology distortions can be used for improvements of the EGM08 geopotential model.

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