Evaluation of EGM08 – Globally and Locally in South Korea

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Abstract
The newest global geopotential model, EGM08, is evaluated locally using GPS/leveling data in South Korea and globally in terms of its degree variances (power spectral density). It is found that the agreement between the EGM08 geoid and the geoid undulation derived from GPS/leveling data over 500 irregularly distributed points has a standard deviation of 18.5 cm. This agreement is better than with EGM96 (27.4 cm). From the mean difference between EGM08 and GPS/leveling it is found that the local geoid (national vertical datum of South Korea) is offset from the global geoid by 43.4 cm, which agrees roughly with the 34 cm offset implied by the EGM96 analysis. The degree variances of the EGM08 geoid show that the EGM96 is significantly underpowered at its high degrees. They also imply that the Earth's high-frequency gravitational field does not have constant fractal dimension, which may provoke additional investigation.

Introduction
We start with the spherical harmonic expansion of the disturbing potential, *T*, in terms of spherical polar coordinates, (r,θ,λ):

$$T(r,\theta,\lambda)=\frac{GM}{a}\sum_{n=2}^{\infty}\sum_{m=-n}^{n}\left(\frac{a}{r}\right)^{n+1}C_{nm}\bar{Y}_{nm}(\theta,\lambda), \tag{1}$$

where *GM* is the product of Newton's gravitational constant and Earth's total mass (including the atmosphere), *a* is the radius of the bounding (Brillouin) sphere, the C_{nm} are the (unit-less) Stokes coefficients, and the $\bar{Y}_{nm}(\theta,\lambda)$ are surface spherical harmonics defined by

$$\bar{Y}_{nm}(\theta,\lambda)=\bar{P}_{n,|m|}(\cos\theta)\begin{cases}\cos m\lambda, & m\ge 0\\ \sin|m|\lambda, & m<0\end{cases} \tag{2}$$

The functions, $\bar{P}_{n,|m|}$, are associated Legendre functions, fully normalized so that

$$\frac{1}{4\pi}\iint_{\sigma}\left(\bar{Y}_{nm}(\theta,\lambda)\right)^2 d\sigma = 1 \quad \text{for all } n, m, \tag{3}$$

where σ represents the unit sphere. Representation (1) assumes that M is also the mass of the normal ellipsoid, whose center, furthermore, coincides with Earth's center of mass, as well as the coordinate origin.

Let W be the gravity potential, U the normal gravity potential, and $\boldsymbol{x}$ a point in space. Then,

$$W(\boldsymbol{x}) = T(\boldsymbol{x}) + U(\boldsymbol{x}). \tag{4}$$

In linear approximation, we have

$$W(\boldsymbol{x}) = T(\boldsymbol{x}) + \left(U(\boldsymbol{x}_0) + \frac{\partial}{\partial h} U(\boldsymbol{x})\Big|_{\boldsymbol{x}=\boldsymbol{x}_0} h \right), \tag{5}$$

where both $\boldsymbol{x}$ and $\boldsymbol{x}_0$ are on a normal to the ellipsoid and h is the vertical distance between them. If $\boldsymbol{x} = \bar{\boldsymbol{x}}$ is a point on the geoid and $\boldsymbol{x}_0 = \bar{\boldsymbol{x}}_0$ is the corresponding point on the ellipsoid (Figure 1), then in this case, $h = N$, the geoid undulation, and we have

$$N(\bar{\boldsymbol{x}}) = \frac{1}{\gamma(\bar{\boldsymbol{x}}_0)} T(\bar{\boldsymbol{x}}) - \frac{1}{\gamma(\bar{\boldsymbol{x}}_0)} (W_0 - U_0), \tag{6}$$

where $\gamma(\bar{\boldsymbol{x}}_0) = -\partial U(\bar{\boldsymbol{x}}) / \partial h \big|_{\bar{\boldsymbol{x}}=\bar{\boldsymbol{x}}_0}$ is normal gravity on the ellipsoid, $W(\bar{\boldsymbol{x}}) = W_0$, and $U(\bar{\boldsymbol{x}}_0) = U_0$. The surface, $W(\bar{\boldsymbol{x}}) = W_0$, represents the global geoid (e.g., the level surface that best fits the mean sea surface on a global scale, although one may adopt other definitions), and N, given by equation (6), is the geoid undulation with respect to the given normal ellipsoid.

Local Vertical Datum

If the adopted normal ellipsoid is the one that also best fits the global mean sea surface, then $U_0 = W_0$ and we obtain the geoid undulation of the *global* geoid with respect to the best-fitting ellipsoid:

$$N(\bar{\boldsymbol{x}}) = \frac{1}{\gamma(\bar{\boldsymbol{x}}_0)} T(\bar{\boldsymbol{x}}). \tag{7}$$

If the ellipsoid is not optimum (such as the WGS84 ellipsoid, by today's accuracy standards), then the geoid undulation of the global geoid with respect to this ellipsoid is

$$N^{(\mathrm{WGS84})}(\bar{\boldsymbol{x}}) = N(\bar{\boldsymbol{x}}) - \frac{1}{\gamma(\bar{\boldsymbol{x}}_0)}\left(W_0 - U_0^{(\mathrm{WGS84})}\right). \tag{8}$$

We may also consider a local geoid, which defines a national vertical datum. The origin point for the vertical datum often does not lie on the global geoid, since it is tied to a local mean sea level point. In that case, the local geoid does not coincide with the global geoid; and, the geoid undulation of the local geoid with respect to the non-optimum ellipsoid is given by (see also Figure 1)

$$\begin{aligned} N_{\mathrm{local}}^{(\mathrm{WGS84})}(\bar{\boldsymbol{x}}) &= \frac{1}{\gamma(\bar{\boldsymbol{x}}_0)} T(\bar{\boldsymbol{x}}) - \frac{1}{\gamma(\bar{\boldsymbol{x}}_0)}\left(W_0^{\mathrm{local}} - U_0^{(\mathrm{WGS84})}\right) \\ &= N(\bar{\boldsymbol{x}}) - \frac{1}{\gamma(\bar{\boldsymbol{x}}_0)}\left(W_0 - U_0^{(\mathrm{WGS84})}\right) + \frac{1}{\gamma(\bar{\boldsymbol{x}}_0)}\left(W_0 - W_0^{\mathrm{local}}\right) \end{aligned} \tag{9}$$

The offset given by the second term on the right side, $-\left(W_0 - U_0^{(\mathrm{WGS84})}\right)\big/\gamma$, is about $-53\ \mathrm{cm}$, based on the current best value, $W_0 = 62636856.33\ \mathrm{m}^2/\mathrm{s}^2$, and the parameter values of the WGS84 normal ellipsoid. The offset given by the third term, $\left(W_0 - W_0^{\mathrm{local}}\right)\big/\gamma$, depends on a local determination of W_0^{local} (see below).

In principle, the infinite spherical harmonic series (1) for the disturbing potential converges to the true value only on or above the Brillouin sphere (neglecting the atmospheric and other extra-terrestrial masses). Therefore, we should not express the geoid undulation in terms of the series by substituting (1) into (6) because this would require the evaluation of the series at a point on the geoid, $\bar{\boldsymbol{x}}$, which is likely inside the bounding sphere. However, in practice, the infinite series must be truncated to some finite degree, $n_{\max}$, since one can determine only a finite number of coefficients, C_{nm}, from a given finite set of data on a bounding sphere. As such, the concern about formal series convergence disappears, although there is still the question of whether the truncated series accurately represents the disturbing potential. On the other hand, we can now use the truncated series anywhere in free space, even below the bounding sphere and on or above the Earth's surface, since the truncated series is harmonic anywhere in free space. Still, in order to respect theory, we cannot use the series *inside* the Earth, specifically on the geoid, which would be the case in equation (6), because the disturbing potential is not harmonic there.

If we apply equation (5) to two points again on a normal to the ellipsoid, but now $\boldsymbol{x} = \bar{\boldsymbol{x}}^e$ is a point on the Earth's surface and $\boldsymbol{x}_0 = \bar{\boldsymbol{x}}_0^e$ is a point on the *telluroid*, which is a surface such that $U\left(\bar{\boldsymbol{x}}_0^e\right) = W\left(\bar{\boldsymbol{x}}^e\right)$, then the distance between these is called the height anomaly ($h = \zeta$) and is given (in linear approximation) by

$$\zeta\left(\bar{\boldsymbol{x}}^e\right) = \frac{1}{\gamma\left(\bar{\boldsymbol{x}}_0^e\right)} T\left(\bar{\boldsymbol{x}}^e\right). \tag{10}$$

Now, we can substitute legitimately the series for T (truncated at $n = n_{\max}$), because $\bar{\boldsymbol{x}}^e$ is not below the Earth's surface, and get

$$\zeta\left(\bar{\boldsymbol{x}}^e\right) = \frac{GM}{a\,\gamma\left(\bar{\boldsymbol{x}}_0^e\right)} \sum_{n=2}^{n_{\max}} \sum_{m=-n}^{n} \left(\frac{a}{r^e}\right)^{n+1} C_{nm} \bar{Y}_{nm}\left(\theta, \lambda\right), \tag{11}$$

where r^e is the radius to the point, $\bar{\boldsymbol{x}}^e$. Without significant error at this stage of the analysis, we may also approximate, $\gamma\left(\bar{\boldsymbol{x}}_0^e\right) \approx GM/a^2$, which simplifies the antecedent factor of the series to a, and which can further be approximated by $a \approx R$, where R is a mean Earth radius.

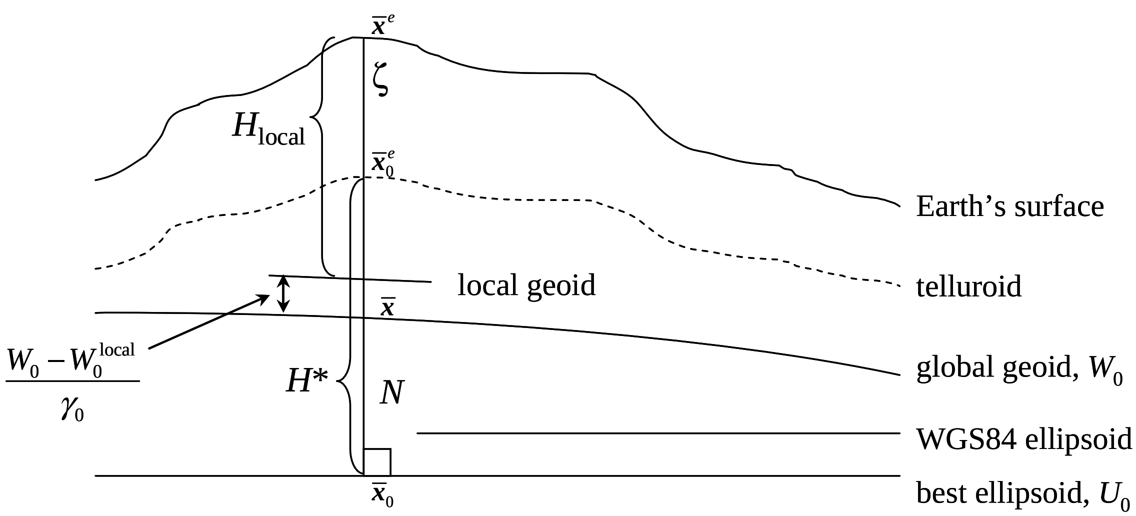

Figure 1: Geometry of geoid, ellipsoid, telluroid, and Earth's surface.

We note that the difference, d, between the height anomaly and the geoid undulation is

$$d = \zeta - N = H - H^* \approx H\,\frac{\Delta g_B}{\gamma_0}, \tag{12}$$

where H is the orthometric height (with respect to the global geoid), Δg_B is the Bouguer gravity anomaly and γ_0 is an average value of normal gravity on the ellipsoid. World-wide, this difference, for the EGM96 model, has a mean value of about -5 cm, with a standard deviation of about 22.3 cm and an extreme absolute value of 311 cm (Lemoine et al., 1998, p.5-14).

Assessment Using GPS/Leveling Data

For a local point-wise assessment of a global model for the geoid undulation, such as EGM96 or EGM08 (Pavlis et al., 2008), one may consider using a set of GPS/leveling data, which yield the geoid undulation of the local geoid with respect to the WGS84 ellipsoid:

$$N_{\text{local}}^{(\text{WGS84})} = h^{(\text{WGS84})} - H_{\text{local}} \,. \tag{13}$$

The points where such data are available for the Republic of South Korea (J. Kwon, personal communication, 2008) are shown in Figure 2. The data set contains 500 points where both the GPS height above the WGS84 ellipsoid, $h^{(\text{WGS84})}$, and the orthometric height with respect to the national vertical datum, H_{local}, have been measured.

In view of equation (9), we must consider in our evaluations that the local geoid differs from the global geoid ($W_0^{\text{local}} \neq W_0$). Fell and Tanenbaum (2001) report a bias of about $\left(W_0 - W_0^{\text{local}}\right)\big/\gamma_0 = 90\ \text{cm}$ (i.e., the orthometric heights with respect to the national vertical datum are about 90 cm smaller than with respect to the EGM96 geoid, assumed to be the global geoid). This bias was determined using 27 GPS/leveling stations in South Korea. Although not explicitly stated in their paper, we assume that the EGM96 geoid undulation was computed with respect to the WGS84 ellipsoid, rather than the best-fitting ellipsoid. Hence, the 90 cm bias should be modified by the $-53\ \text{cm}$ difference between these two ellipsoids, so that the bias between vertical datums (global and national) is only 37 cm. In addition, we must assume a mean tide system because the GPS/leveling data presumably were not corrected for tidal effects.

Evaluating the geoid undulation according to

$$N\left(\overline{\boldsymbol{x}}_0\right) = R \sum_{n=2}^{n_{\max}} \sum_{m=-n}^{n} \left(\frac{a}{r^e}\right)^{n+1} C_{nm} \overline{Y}_{nm}\left(\theta, \lambda\right), \tag{14}$$

where r^e is the radial distance to the ellipsoid ($a = 6378137$ m) and according to

$$N\left(\overline{\boldsymbol{x}}_0\right) = R \sum_{n=2}^{n_{\max}} \sum_{m=-n}^{n} \left(\frac{a}{r^e}\right)^{n+1} C_{nm} \overline{Y}_{nm}\left(\theta, \lambda\right) - d\left(\overline{\boldsymbol{x}}_0\right), \tag{15}$$

using software supplied for the evaluation of EGM96 (that includes the correction, $-d$, equation (12)), we found in South Korea only differences with a standard deviation of about 1 cm. Thus, we can safely neglect for present purposes the topographic bias (the mismodeling associated with evaluating a harmonic function inside the masses on the geoid). Table 1 compares the geoid undulation computed using equation (14) for the models EGM96 and EGM08 against the geoid undulation computed from GPS/leveling data, according to equation (13). The standard deviation of the difference includes both commission and omission errors of the spherical harmonic model, and also errors in the GPS/leveling data. The mean difference accounts for different ellipsoids and geoids.

Considering EGM96, the mean difference of 19 cm (Table 1) implies the following difference between the local vertical datum (LD) and the global geoid:

$$
\begin{aligned}
19\text{ cm} = N_{\text{EGM96}} - \left(h_{\text{WGS84}} - H_{\text{LD}}\right) &= N_{\text{EGM96}} - \left(\left(h_{\text{best ellip}} - 53\text{ cm}\right) - H_{\text{LD}}\right) \\
&= \left(N_{\text{EGM96}} - h_{\text{best ellip}}\right) + 53\text{ cm} + H_{\text{LD}} \\
&= -H_{\text{global geoid}} + 53\text{ cm} + H_{\text{LD}} \\
H_{\text{global geoid}} - H_{\text{LD}} &= 34\text{ cm}
\end{aligned}
\tag{16}
$$

which agrees approximately with the modified result by Fell and Tanenbaum (2001). For the EGM08 geoid comparison to the GPS/leveling data, the mean difference of 9.6 cm similarly implies

$$
H_{\text{global geoid}} - H_{\text{LD}} = 43.4\text{ cm}\,. \tag{17}
$$

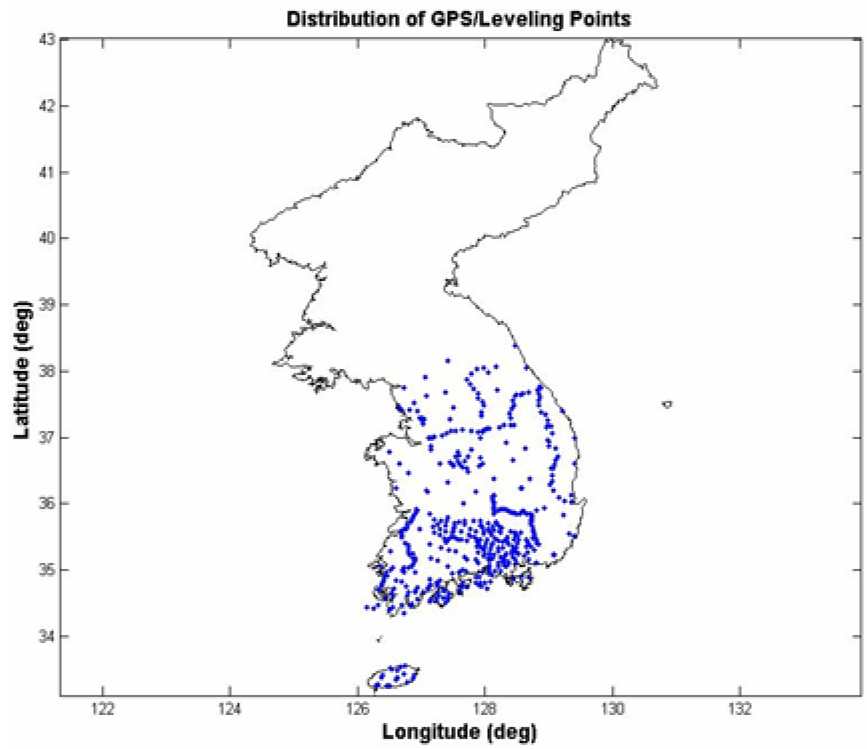

Figure 2: Points where GPS/leveling data are available.

Table 1: Statistics for the differences between geoid models and GPS/leveling data at 500 points shown in Figure 2. Units: [m].

model ($n_{\max}$)	minimum	maximum	mean	st.dev.
EGM96 (360)	-0.52	1.43	0.189	0.274
EGM08 (2160)	-0.54	1.17	0.096	0.185

Degree Variances at High Degrees

As might be expected, the agreement between EGM08 and the GPS/leveling data is better than for EGM96. However, it is also interesting to know the reason for the remaining difference between these two types of geoid undulations. Certainly, errors in the GPS heights and in the leveling data contribute, but the extent of this is not known; and common wisdom would dictate that aside from the occasional bad data neither GPS height errors nor orthometric height errors would contribute at the level of one or two decimeters. Therefore, we assume that the differences are due primarily to errors in the spherical harmonic model.

We consider these model errors in terms of degree variances, which may be separated into commission and omission errors. By putting a bound on the omission error, we will be able to provide some assessment of the commission error, since by assumption Table 1 essentially provides the total error. Quantifying the omission error relies on a statistical model for the harmonic coefficients. That is, the high-degree coefficients are assumed to behave in a stochastic sense with a particular standard deviation (or, variance) for each degree (and zero average). Scaled to the geoid undulation, the degree variance is given by

$$\left(\sigma_N^2\right)_n = R^2\left(\frac{a}{r^e}\right)^{2n+2}\sum_{m=-n}^{n} C_{nm}^2\,, \tag{18}$$

where r^e is taken as constant (e.g., $r^e = a$). W.M. Kaula promoted and developed this idea; and an often-used rule-of-thumb (Kaula, 1966, p.98), bearing his name, provides a rough, global estimate of the standard deviation of the omission error:

$$\sigma_{\text{omission}}^{(n_{\max})} = \sqrt{\sum_{n=n_{\max}+1}^{\infty}\left(\sigma_N^2\right)_n} \approx \frac{64}{n_{\max}}\ [\text{m}]\,. \tag{19}$$

Hence, if $n_{\max} = 360$, then $\sigma_{\text{omission}}^{(180)} = 18\ \text{cm}$. Many other researchers, notably Rapp (1979), have devised refinements of this model. Figure 3 shows the degree variances of the geoid according to the EGM96 and EGM08 models, as well as Kaula's rule.

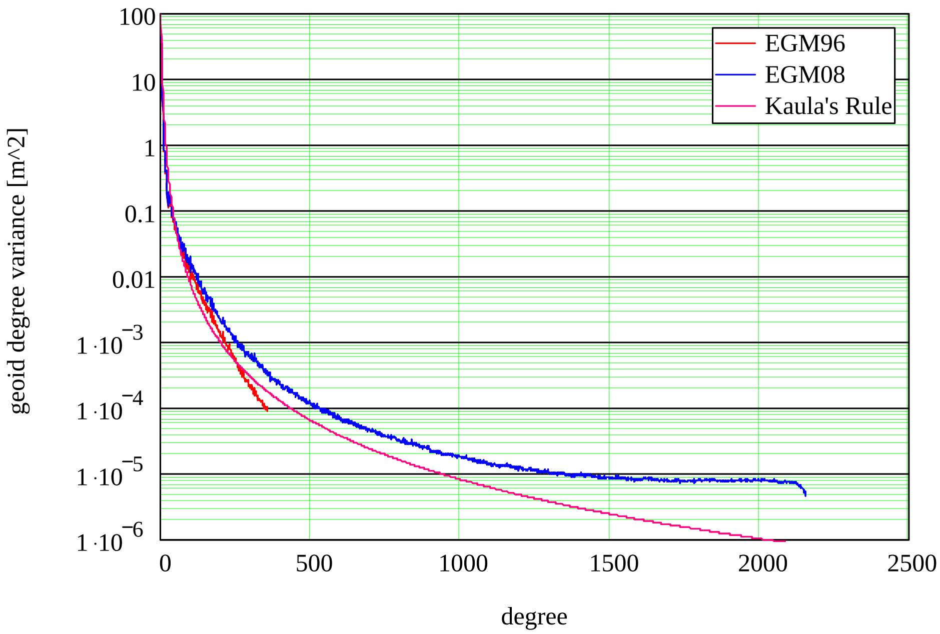

Figure 3: Degree variances of the geoid according to various models.

A significant feature of the new EGM08 model is the demonstration that the EGM96 model is under-powered at the high degrees, as already predicted earlier by some investigators (e.g., Jekeli, 1999). We can transform from degree variance to power spectral density (psd), Φ, using the relationship

$$\Phi_N\left(f_n\right)=\frac{2\pi R^2}{n}\left(\sigma_N^2\right)_n, \tag{20}$$

where $f_n = n/(2\pi R)$ is cyclical frequency. Plotting the psd's, now with frequency also on a logarithmic scale, we find a second striking feature (Figure 4). While Kaula's rule significantly overestimates the psd of the geoid undulation at the lower frequencies, it underestimates the psd at the high frequencies, although the power attenuation is essentially correct, at least up to wavelengths of about 30 km (frequency, 3×10^{-5} cy/m). It has been suggested that the Earth's gravitational potential, like its topography and that of other planets, behaves like a fractal (e.g., Turcotte, 1987). That is, its psd obeys a power law, essentially of the form:

$$\Phi\left(f\right)=bf^{-\alpha},\quad f\geq f_0, \tag{21}$$

with constants, α, b, where α is related to the fractal dimension of the fractal. Clearly, the EGM08 model indicates that for frequencies, 2×10^{-6} cy/m $\leq f \leq 3\times10^{-5}$ cy/m, the psd is a power law. If one postulates that the gravitational field is a fractal (with constant fractal dimension) at high frequencies (as often modeled, e.g., by Jekeli (2003), on the basis of local gravity anomaly and topographic data sets), one might question the significant upturn in the

EGM08 psd at frequencies, $f \geq 3\times10^{-5}$ cy/m, corresponding to harmonic degrees greater than about 1200. On the other hand, if accurate at these higher frequencies, EGM08 shows that the Earth's gravitational potential does not possess the same fractal dimension at all (higher) frequencies.

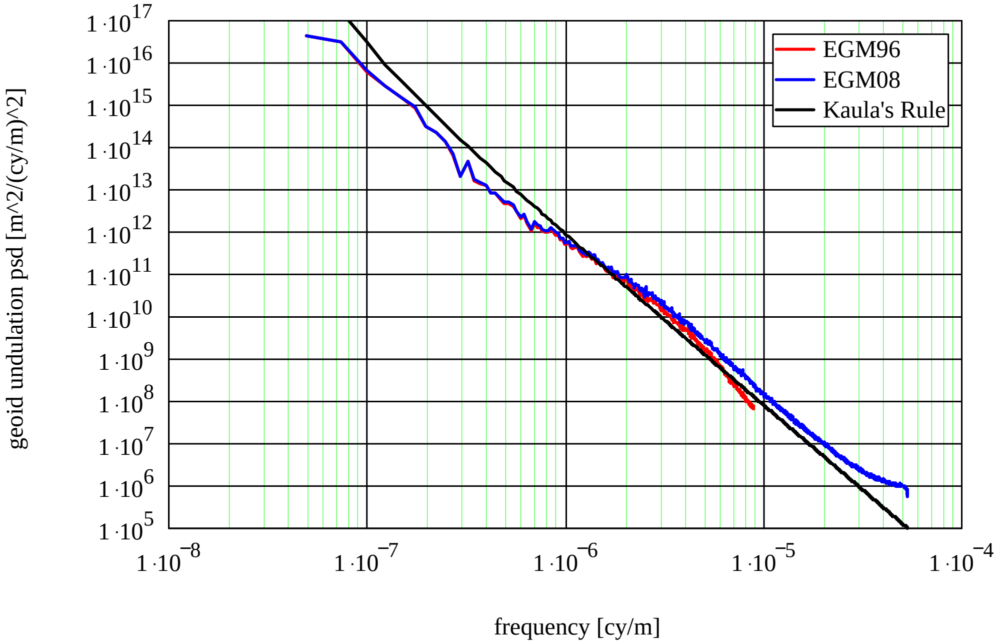

Figure 4: PSDs of the geoid undulation according to various models.

A power law model for the psd that more accurately fits the frequency band, 3×10^{-6} cy/m $\leq f \leq 3\times10^{-5}$ cy/m (harmonic degrees, 120 – 1200), is given by

$$\Phi_N(f) = 4.044\times10^{-12} f^{-3.898} \text{ m}^2/(\text{cy/m})^2 . \tag{22}$$

Assuming that this new model (shown in Figure 5) yields a reasonable approximation to the standard deviation of the omission error, we find

$$\sigma_{\text{omission}}^{(n_{\max})} = \sqrt{\frac{1}{2\pi}\int_{f_{\max}}^{\infty} \Phi_N(f)\, f\, df} = 0.041 \text{ m}, \quad \text{for } n_{\max} = 2160 . \tag{23}$$

Since the omission and commission errors are independent, we have

$$\left(\sigma_{\text{commission}}^{(n_{\max})}\right)^2 = \sigma_{\text{total}}^2 - \left(\sigma_{\text{omission}}^{(n_{\max})}\right)^2 . \tag{24}$$

Using the standard deviation in Table 1 as the total (i.e., disregarding errors in the GPS/leveling data) and assuming the omission error model , equation (23) is representative of EGM08 in South Korea, the commission error in EGM08 for South Korea is about 18.0 cm. If the new omission error model, equation (23), underestimates the high-degree degree variances (high-frequency psd), as indicated in Figure 5, then it is more difficult to estimate the omission error (and hence, the commission error). Furthermore, it should be emphasized that this model, or Kaula's rule, or any other model extrapolated from the degree variances, is a global model for the standard deviation of the omission error. For a local computation of the geoid undulation from a particular model (like EGM08), the standard deviation of the omission error may be significantly lower or higher.

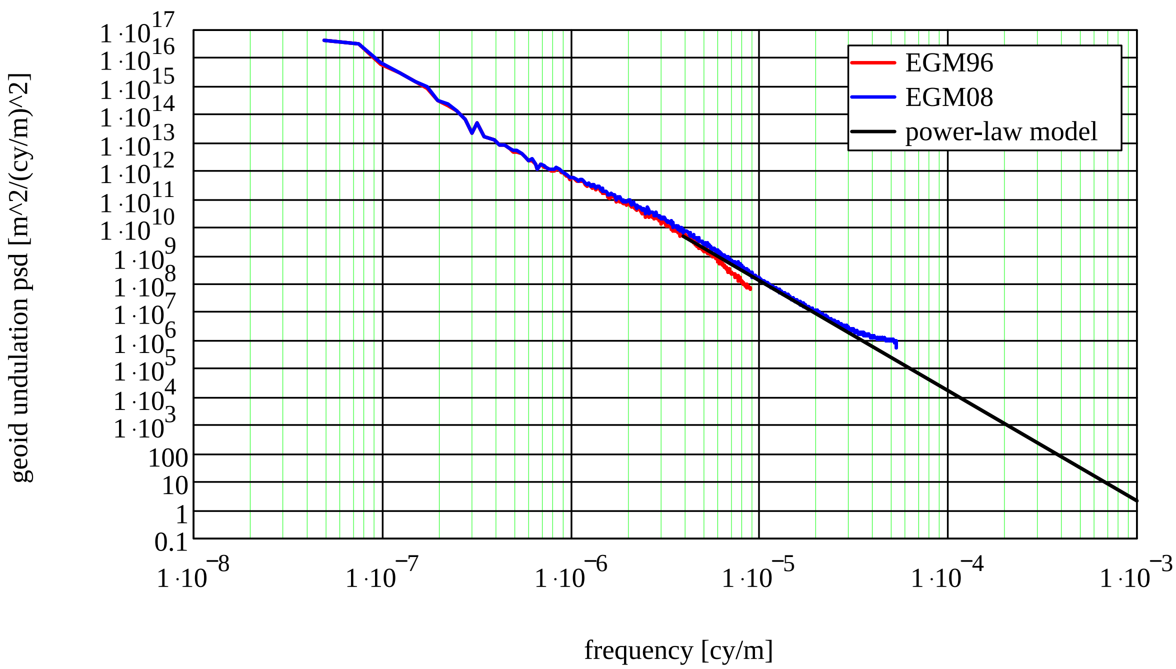

Figure 5: Global power spectral density of the geoid according to the EGM96 and EGM08 models, and their power-law extension.

Summary Discussion

A brief local assessment of EGM08 has been performed using GPS/leveling data in South Korea. These data are available at 500 points somewhat unevenly distributed over the South Korean peninsula. However, they cover many types of terrain. The accuracy of these data is not available (at present), but in terms of standard deviation the EGM08 geoid agrees better (18.5 cm) with these geometrically derived undulations than does the EGM96 geoid (27.4 cm). The mean differences (gravimetric model minus geometric measurement) also provide an indication of the offset of the local vertical datum from the global geoid. EGM08 implies a 43.3 cm offset, compared to the EGM96 model, which implies a 34 cm offset.

We note that EGM96 was published as a *non-tidal geoid* model; whereas EGM08 is offered both as a zero-tide and a non-tide geoid model. We performed our computations of EGM08 using its zero-tide version. The difference between the two is (in terms of geoid undulation; see Jekeli (2000, ch.6); Lemoine et al. (1998, ch.11))

$$N_{\text{zero-tide}} - N_{\text{non-tide}} = -0.099 k_2 \left(3\sin^2\psi - 1\right) \text{ [m]}, \tag{25}$$

where $k_2 = 0.29$ is a Love's number and ψ is the geocentric latitude. In South Korea, this latitude is $\psi \approx 36°$, and, hence, $N_{\text{zero-tide}} - N_{\text{non-tide}} = -0.001\text{ m}$. We see that the mean difference, $\text{mean}\left(N_{\text{EGM96}} - N_{\text{EGM08}}\right) = 9.3\text{ cm}$, in South Korea is not due to the difference in tidal correction. The reason for this mean difference is not known to the authors at this time.

The degree variances of the EGM08 geoid show that the EGM96 spectrum is significantly underpowered at its high degrees. Furthermore, it is shown that Kaula's rule is remarkably close in describing the fractal dimension of the field in the frequency band, 2×10^{-6} cy/m $\le f \le 3\times10^{-5}$ cy/m (corresponding to harmonic degrees, 80 – 1200). At higher frequencies, EGM08 departs from this fractal dimension. However, if the power-law attenuation were an accurate representation of the high-degree variances, then the omission error for EGM08 would be approximately 4.1 cm (global standard deviation), which if it further holds for South Korea would imply a commission error in EGM08 for South Korea of about 18.0 cm (st.dev.), assuming relatively insignificant errors in the GPS/leveling data.

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References

Fell, P. and M. Tanenbaum (2001): Preliminary comparisons of the WGS84(EGM96) geoid with national vertical datums. Proceedings of Oceans, 2001, MTS/IEEE Conference and Exhibition, vol.1, pp.571-574, doi: 10.1109/OCEANS.2001.968784.

Jekeli, C. (1999): A further evaluation of the EGM96 geopotential model based on deflections of the vertical. *Bolletino di Geofisica Teorica ed Applicata*, **40**(3-4), 239-244.

Jekeli, C. (2000): Heights, the geopotential, and vertical datums, Report no.459, Geodetic Science, Ohio State University, Columbus, Ohio.

Jekeli, C. (2003): Statistical analysis of moving-base gravimetry and gravity gradiometry. Report no.466, Geodetic Science, Ohio State University, Columbus, Ohio.

Kaula, W.M. (1966): *Theory of Satellite Geodesy*. Blaisdell Publ. Co., Waltham, Mass.

Lemoine F.G., Kenyon S.C., Factor J.K., Trimmer R.G., Pavlis N.K., Chinn D.S., Cox C.M., Klosko S.M., Luthcke S.B., Torrence M.H., Wang Y.M., Williamson, R.G., Pavlis E.C., Rapp R.H., Olson T.R. (1998): The development of the joint NASA GSFC and the National Imagery and Mapping Agency (NIMA) geopotential model EGM96. NASA Technical Paper NASA/TP-1998-206861, Goddard Space Flight Center, Greenbelt, Maryland.

Pavlis, N.K., S.A. Holmes, S.C. Kenyon, and J.K. Factor (2008): An Earth Gravitational Model to Degree 2160: EGM2008. Presented at the General Assembly of the European Geosciences Union, Vienna, Austria, April 13-18, 2008.

Rapp, R.H. (1979): Potential coefficient and anomaly degree variance modeling revisited. Report no.293, Geodetic Science, Ohio State University, Columbus, Ohio.

Turcotte, D.L. (1987): A fractal interpretation of topography and geoid spectra on the Earth, Moon, Venus, and Mars. *Journal of Geophysical Research*, **92**(B4), E597-E601.