Evaluation of the EGM2008 Geopotential Model for Egypt

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1. Introduction

EGM2008 (Pavlis et al., 2008) is the most recent earth geopotential model complete to degree and order 2159, and contains some additional spherical harmonic coefficients up to degree 2190. It represents a spatial resolution of 5 minutes of arc (about 9 km). The scaling parameters GM, a associated with this model have the numerical values:

$$\begin{aligned} GM &= 3986004.415 \times 10^8 \text{ m}^3 \text{ s}^{-2}\,, \\ a &= 6378136.3 \text{ m}\,. \end{aligned}$$

The EGM2008 zero tide model is used within this investigation for the validation process. The coefficient C_{20} of the EGM2008 zero tide model has the numerical value:

$$C_{20} = -0.484169317366974 \times 10^{-03}\,.$$

In this paper, an evaluation of EGM2008 for Egypt is given. The validation data include GPS/levelling, the latest Egyptian geoid model EGGG2008, point gravity anomalies and topographic heights derived by DTM2006.0 model for Egypt. A summary of the results and a conclusion are provided at the end of the paper.

2. Basic Equations

The gravitational potential V can be expressed in spherical harmonic expansion as (Torge, 1989, p. 28; Dragomir et al., 1982, p. 53)

$$V(r,\theta,\lambda) = \frac{GM}{r}\left[1 + \sum_{n=2}^{\infty}\left(\frac{a}{r}\right)^n \sum_{m=0}^{n}\left(\bar{C}_{nm}\cos m\lambda + \bar{S}_{nm}\sin m\lambda\right)\bar{P}_{nm}(\cos\theta)\right]\,, \qquad (1)$$

where GM is the geocentric gravitational constant, r is the geocentric radius, θ is the polar distance, λ is the geodetic longitude, a stands for the equatorial radius of the mean earth's ellipsoid, $\bar{P}_{nm}$ denotes the associated fully normalized Legendre functions and $\bar{C}_{nm}$ and $\bar{S}_{nm}$ are the fully normalized potential coefficients. The polar distance θ can simply be expressed in terms of the geocentric latitude ψ as:

$$\theta = 90^{\circ} - \psi\,, \qquad (2)$$

where ψ is related to the geodetic latitude ϕ through the following expression (Torge, 1980, p. 50):

$$\tan\psi = (1-f)^2\tan\phi\,, \qquad (3)$$

where f is the flattening of the earth's ellipsoid. The geocentric radius r can easily be expressed by

$$r = \sqrt{x^2 + y^2 + z^2}\,, \tag{4}$$

where x, y, z are the geodetic cartesian coordinates given by (Moritz, 1980, p. 8)

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} (\rho + h)\cos\phi\cos\lambda \\ (\rho + h)\cos\phi\sin\lambda \\ [\rho(1-e^2) + h]\sin\phi \end{bmatrix}, \tag{5}$$

where ρ is the radius of curvature in the prime vertical plane, given by

$$\rho = \frac{a}{(1 - e^2\sin^2\phi)^{\frac{1}{2}}}\,. \tag{6}$$

Here h stands for the ellipsoidal height and e is the first eccentricity of the ellipsoid.

The disturbing potential T is defined by

$$T(r, \theta, \lambda) = V(r, \theta, \lambda) - U(r, \theta) \tag{7}$$

where U is the normal gravitational potential of the mean earth's ellipsoid, given by (Torge, 1989, p. 37)

$$U(r, \theta) = \frac{GM}{r}\left[1 + \sum_{n=2}^{\infty}\left(\frac{a}{r}\right)^n \bar{C}^u_{n0}\bar{P}_{n0}(\cos\theta)\right]. \tag{8}$$

Here $\bar{C}^u_{n0}$ denotes the fully normalized harmonic coefficients implied by the reference equipotential ellipsoid. Because of the rotational symmetry of the mean earth's ellipsoid, there will be only zonal terms. And because of the symmetry with respect to the equatorial plane, there will be only even zonal harmonics $\bar{C}^u_{2n,0}$ (Heiskanen and Moritz, 1967, p. 72). The even degree zonal harmonic coefficients $\bar{C}^u_{2n,0}$ converge quickly toward zero, so that (8) may safely be truncated after $n = 6$. The degree zonal harmonics of the equipotential earth's ellipsoid J_n can be given by (Rapp, 1982, p. 7)

$$J_2 = \frac{2}{3}\left[f(1 - \frac{f}{2}) - \frac{m}{2}(1 - \frac{2f}{7} + \frac{11f^2}{49})\right], \tag{9}$$

$$J_4 = -\frac{4f}{35}(1 - \frac{f}{2})\left[7f(1 - \frac{f}{2}) - 5m(1 - \frac{2f}{7})\right], \tag{10}$$

$$J_6 = \frac{4f^2}{21}(6f - 5m)\,, \tag{11}$$

where m is given by (Torge, 1980, p. 58)

$$m = \frac{\omega^2 a^3(1-f)}{GM}, \tag{12}$$

where ω is the angular velocity. The degree zonal harmonic coefficients J_n are related to the fully normalized coefficients of the reference ellipsoid $\bar{C}^u_n$ through the following relationship (Rapp, 1982, P. 7)

$$\bar{C}^u_n = -\frac{J_n}{\sqrt{2n+1}}\,. \tag{13}$$

Thus inserting (1) and (8) into (7), the disturbing potential T can be expressed as (Torge, 1989, p. 43)

$$T(r,\theta,\lambda) = \frac{GM}{r}\sum_{n=2}^{\infty}\left(\frac{a}{r}\right)^n \sum_{m=0}^{n}\left(\bar{C}^*_{nm}\cos m\lambda + \bar{S}_{nm}\sin m\lambda\right)\bar{P}_{nm}(\cos\theta)\,, \qquad (14)$$

where $\bar{C}^*_{nm}$ is the difference between the actual coefficients $\bar{C}_{nm}$ and those implied by the reference equipotential ellipsoid $\bar{C}^u_{nm}$. In view of the above discussion, one may write the following relation for $\bar{C}^*_{nm}$:

$$\begin{aligned} \bar{C}^*_{n0} &= \bar{C}_{n0} - \bar{C}^u_{n0} && \text{if } m = 0\,, \\ \bar{C}^*_{nm} &= \bar{C}_{nm} && \text{if } m \neq 0\,. \end{aligned} \qquad (15)$$

The gravity anomaly Δg can be expressed by (Moritz, 1980, p. 14)

$$\Delta g(r,\theta,\lambda) = -\frac{\partial T}{\partial r} + \frac{1}{\gamma}\frac{\partial \gamma}{\partial r}T(r,\theta,\lambda)\,, \qquad (16)$$

where γ is the normal gravity. The normal gravity γ may be expressed in terms of the normal gravity on the surface of the ellipsoid $\gamma_\circ$, with sufficient accuracy for our purpose, as

$$\gamma = \gamma_\circ - 0.3086\,h\,, \qquad (17)$$

where h stands for the height above the reference ellipsoid. Here γ and $\gamma_\circ$ are in mgal and h is in meter. The normal gravity on the surface of the ellipsoid $\gamma_\circ$ can be expressed as (Heiskanen and Moritz, 1967, p. 76):

$$\gamma_\circ = \gamma_e \frac{1 + k\sin^2\phi}{\sqrt{1 - e^2\sin^2\phi}}\,, \qquad (18)$$

where k is given by

$$k = \frac{(1-f)\,\gamma_p - \gamma_e}{\gamma_e}\,. \qquad (19)$$

Here γ_e and γ_p stand for the normal gravity at the equator and the pole, respectively.

Using the spherical approximation, we may write (ibid, p. 87)

$$\frac{1}{\gamma}\frac{\partial\gamma}{\partial r} = -\frac{2}{r}\,. \qquad (20)$$

Then (16) becomes

$$\Delta g(r,\theta,\lambda) = -\frac{\partial T}{\partial r} - \frac{2}{r}T(r,\theta,\lambda)\,. \qquad (21)$$

Inserting (14) into (21), one may write the following expression for the gravity anomaly Δg

$$\Delta g(r,\theta,\lambda) = \frac{GM}{r^2}\sum_{n=2}^{\infty}(n-1)\left(\frac{a}{r}\right)^n \sum_{m=0}^{n}\left(\bar{C}^*_{nm}\cos m\lambda + \bar{S}_{nm}\sin m\lambda\right)\bar{P}_{nm}(\cos\theta)\,. \qquad (22)$$

The height anomaly ζ can be given by the generalized Bruns formula as (Moritz, 1980, p. 353)

$$\zeta(r,\theta,\lambda) = \frac{T(r,\theta,\lambda)}{\gamma} \,. \tag{23}$$

Inserting (14) into (23) gives

$$\zeta(r,\theta,\lambda) = \frac{GM}{\gamma r} \sum_{n=2}^{\infty} \left(\frac{a}{r}\right)^n \sum_{m=0}^{n} \left(\bar{C}^*_{nm} \cos m\lambda + \bar{S}_{nm} \sin m\lambda\right) \bar{P}_{nm}(\cos\theta) \,. \tag{24}$$

It should be noted that (23) can also be used for the calculation of the geoid undulation N but with the evaluation of T on the surface of the geoid by an appropriate choice of r.

3. Gravity Anomalies Comparison

The available point gravity anomalies data set consists of 13566 points. This data set has been used for the EGGG2002 geoid for Egypt (Abd-Elmotaal, 2003). Figure 1 shows the distribution of the free-air gravity anomalies for Egypt used for the current investigation. The distribution of the free-air gravity anomaly stations on-land is very poor, concentrated mainly along the Nile valley. Many areas are empty.

The point free-air gravity anomalies range between −190.51 mgal and 294.74 mgal with an average of −3.28 mgal and a standard deviation of about 60.36 mgal. Highest values are in sea area. These values are gridded at $5' \times 5'$ grid using krigging interpolation technique. Figure 2 shows the gridded $5' \times 5'$ free-air gravity anomalies for Egypt used for the current investigation.

The program GRVABD (Abd-Elmotaal, 1998) has been used to create $5' \times 5'$ free-air gravity anomalies based on EGM2008 model till degree and order 2159. These anomalies are shown in Figure 3. These free-air anomalies range between −186.60 mgal and 389.59 mgal with an average of 0.86 mgal and a standard deviation of about 38.49 mgal. Figure 3 shows more rough structure for the free-air gravity anomalies than Fig. 2.

It should be noted that the steep structure of the free-air anomalies produced by EGM2008 model at Qena ($\phi = 25.3°$, $\lambda = 32°$) refer to wrong gravity data at that region (as it has been proved in previous investigations (cf. Abd-Elmotaal, 2003)), which is believed that they have been included in producing EGM2008.

The steep structure of the free-air anomalies produced by EGM2008 model in Sinai might come from the lack of data included in producing EGM2008, beside the steep structure of topography there (cf. Fig. 5).

Figure 4 shows the difference between the gridded point free-anomalies and those computed using EGM2008 model till degree and order 2159. These differences range between −185.11 mgal and 135.88 mgal with an average of −2.73 mgal and a standard deviation of about 20.37 mgal. White areas mean differences less than 20 mgal in magnitude. Figure 4 shows very small differences in sea areas, but larger differences in land areas, especially in Sinai, where no data point are available, and also at Qena, where wrong gravity data were included in producing the EGM2008 model.

More interesting is the evaluation of the EGM2008 model at the point gravity stations. Program GRVHRM (Abd-Elmotaal, 1998) has been used to compute the free-air

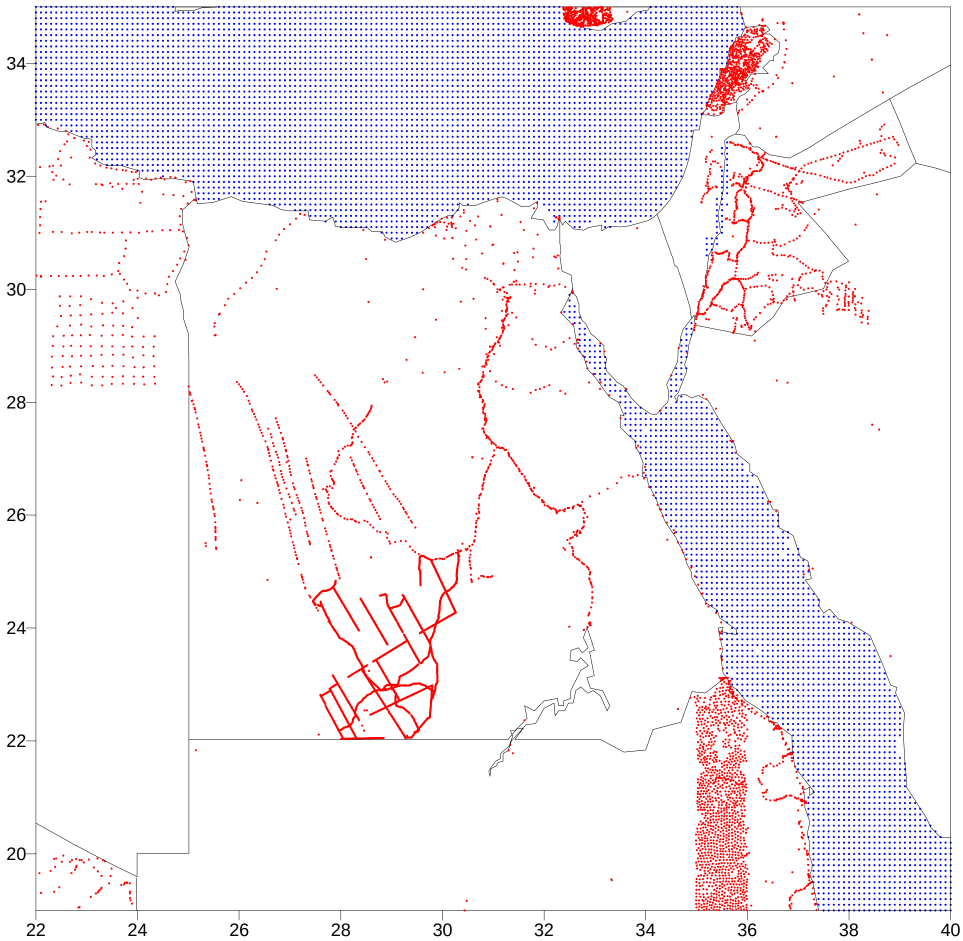

Figure 1: Distribution of the free-air gravity anomalies for Egypt.

gravity anomalies at the gravity data points using the EGM2008 model complete to degree and order 2159. For the sake of comparison, EGM96 and OSU91A models complete to degree and order 360 have also been used to compte free-air gravity anomalies at the gravity data points. No topographic-isostatic reduction has been made at this stage. Table 1 shows the statistics of the point free-air gravity anomalies, free-air anomalies produced using EGM2008, EGM96 and OSU91A models, and their differences. Table 1 shows that EGM2008 model fits best to the point free-air data in view of the smallest standard deviation of the differences between the point free-air and produced model anomalies, however, it produces a larger range difference than that of EGM96. As far as the mean difference is concerned, all three geopotential models give nearly the same good centered differences.

Table 2 shows the differences between the point free-air anomalies and those produced using the EGM2008, EGM96 and OSU91A models after removing the effect of the topographic-isostatic masses employed by the Airy-Heiskanen isostatic model, using the following parameters:

$$T_\circ = 30 \text{ km}, \tag{25}$$

$$\Delta\rho = 0.4 \text{ g/cm}^3. \tag{26}$$

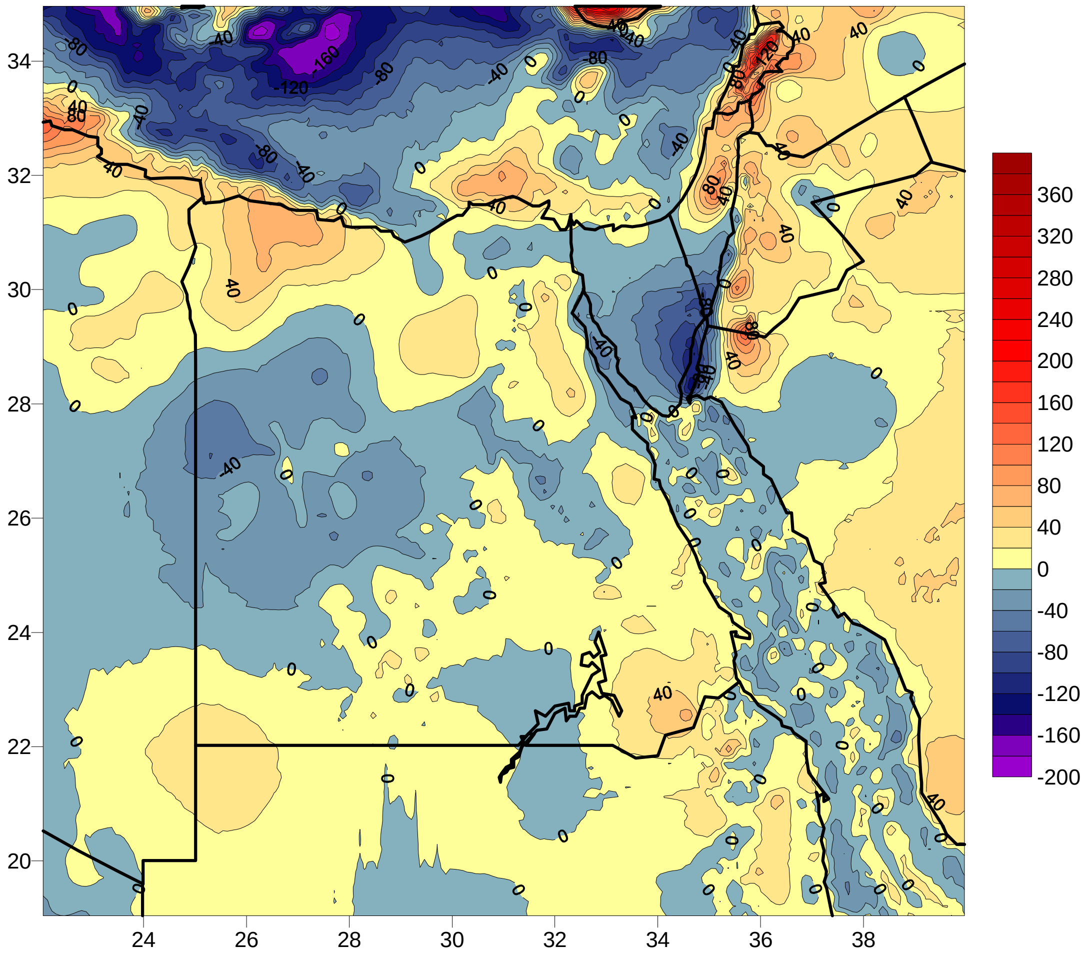

Figure 2: Gridded $5' \times 5'$ free-air gravity anomalies for Egypt using the points gravity anomalies. Contour interval: 20 mgal.

Table 1: Statistics of the gravity anomalies (13566 gravity stations) without topographic reductions

gravity anomalies	statistical parameters			
	min.	max.	mean	st. dev.
	mgal	mgal	mgal	mgal
Point free-air	−190.51	294.74	−3.28	60.36
OSU91A	−165.64	169.37	−3.59	50.36
EGM96	−183.31	211.61	−2.72	56.62
EGM2008	−211.13	290.92	−3.51	61.82
Point free-air − OSU91A	−108.16	137.13	0.30	23.39
Point free-air − EGM96	−96.37	100.20	−0.56	19.12
Point free-air − EGM2008	−83.44	140.39	0.22	11.39

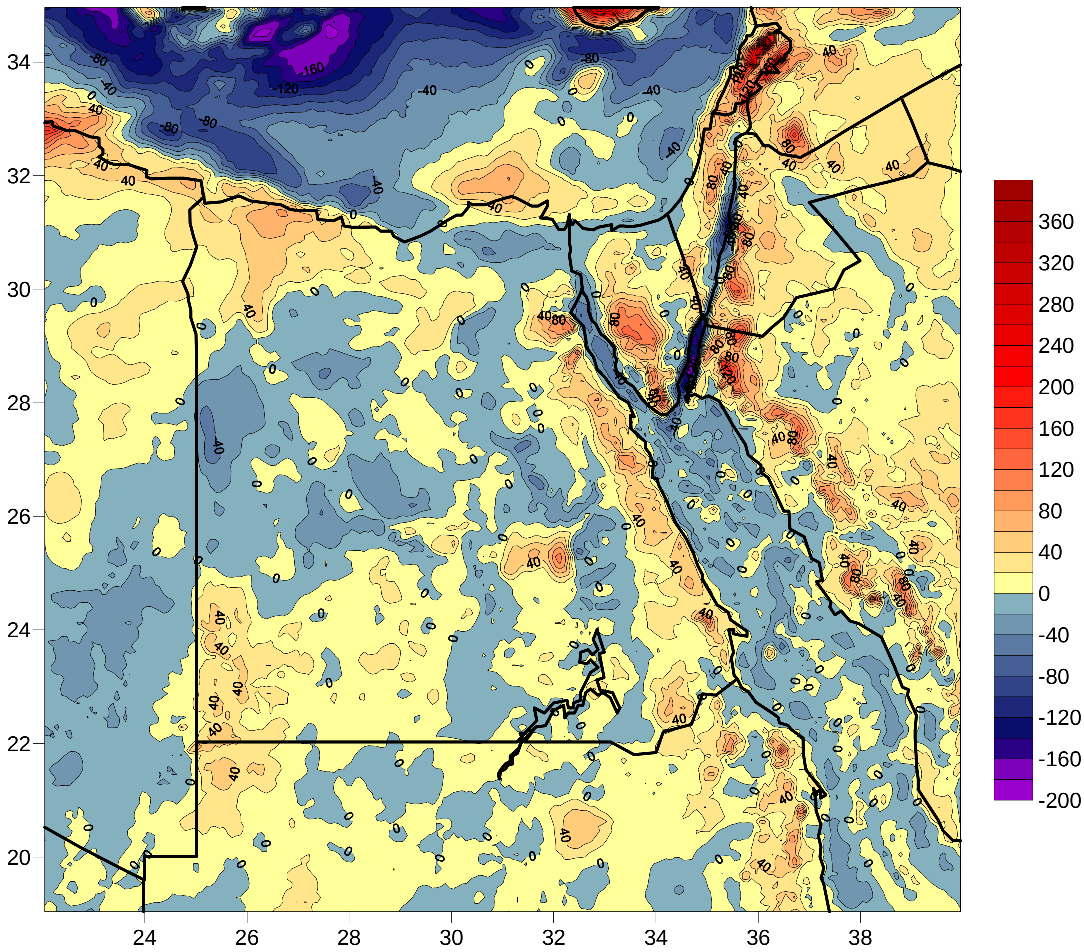

Figure 3: $5' \times 5'$ free-air gravity anomalies for Egypt using EGM2008 model till degree and order 2159. Contour interval: 20 mgal.

Comparing Tables 1 and 2 shows that after removing the effect of the topographic-isostatic masses, the differences became worse. This derives us to study the topographic height, which will be discussed in the following section.

Table 2: Statistics of the gravity anomaly differences (13566 gravity stations) after topographic-isostatic reduction

gravity anomalies	statistical parameters			
	min.	max.	mean	st. dev.
	mgal	mgal	mgal	mgal
Point free-air − OSU91A	−83.47	152.12	5.04	26.65
Point free-air − EGM96	−115.49	122.64	4.17	29.54
Point free-air − EGM2008	−213.58	199.36	4.96	32.47

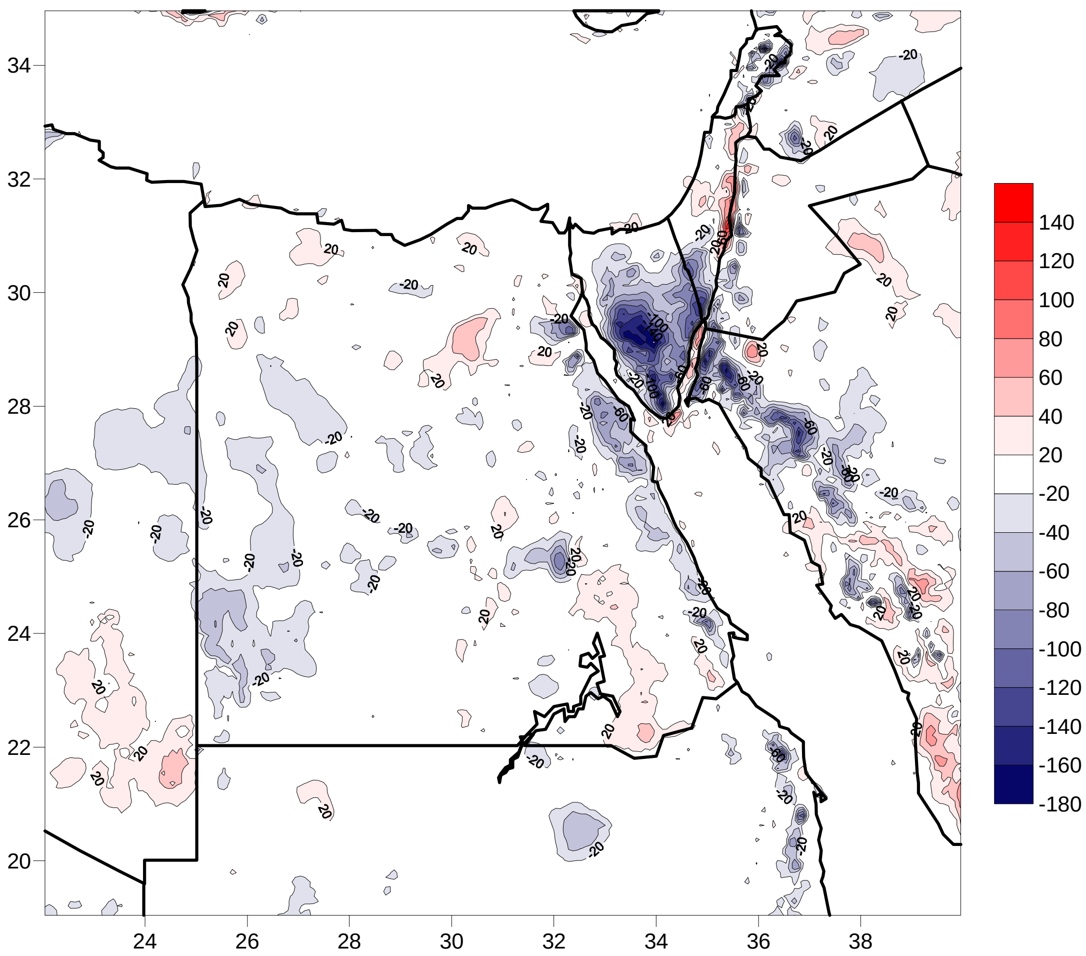

Figure 4: Difference between gridded point free-air anomalies and those computed using EGM2008 model till degree and order 2159. Contour interval: 20 mgal.

4. Digital Height Models Comparison

Abd-Elmotaal (2004 a) has produced Digital Heights Models for Africa with different resolutions, 30″, 3′ and 5′. This represents the most recent DHM for Africa. A window for Egypt out of the African DHM AFH04M5 (5′ × 5′) has been created and illustrated in Fig. 5. The heights within this window range between −4192 m and 2668 m with an average of −19.5 m and a standard deviation of about 993 m.

The DTM2006.0 model (Pavlis et al., 2007) contains fully-normalized spherical harmonic coefficients of the elevation ($\bar{C}_{nm}, \bar{S}_{nm}$) in units of meters, complete to degree and order 2190. Heights H above Mean Sea Level (MSL) can be computed by:

$$H(\theta, \lambda) = \sum_{n=0}^{N_{max}} \sum_{m=0}^{n} \left(\bar{C}_{nm} \cos m\lambda + \bar{S}_{nm} \sin m\lambda \right) \bar{P}_{nm}(\cos\theta) \,. \tag{27}$$

The DTM2006.0 model has been used to compute the heights for the Egyptian window by using FIELDCRE program (Abd-Elmotaal, 2004 b). The used upper maximum degree N_{max} has been set to 2160.

Figure 6 illustrates a 5′ × 5′ DHM for Egypt derived by using DTM2006.0 model

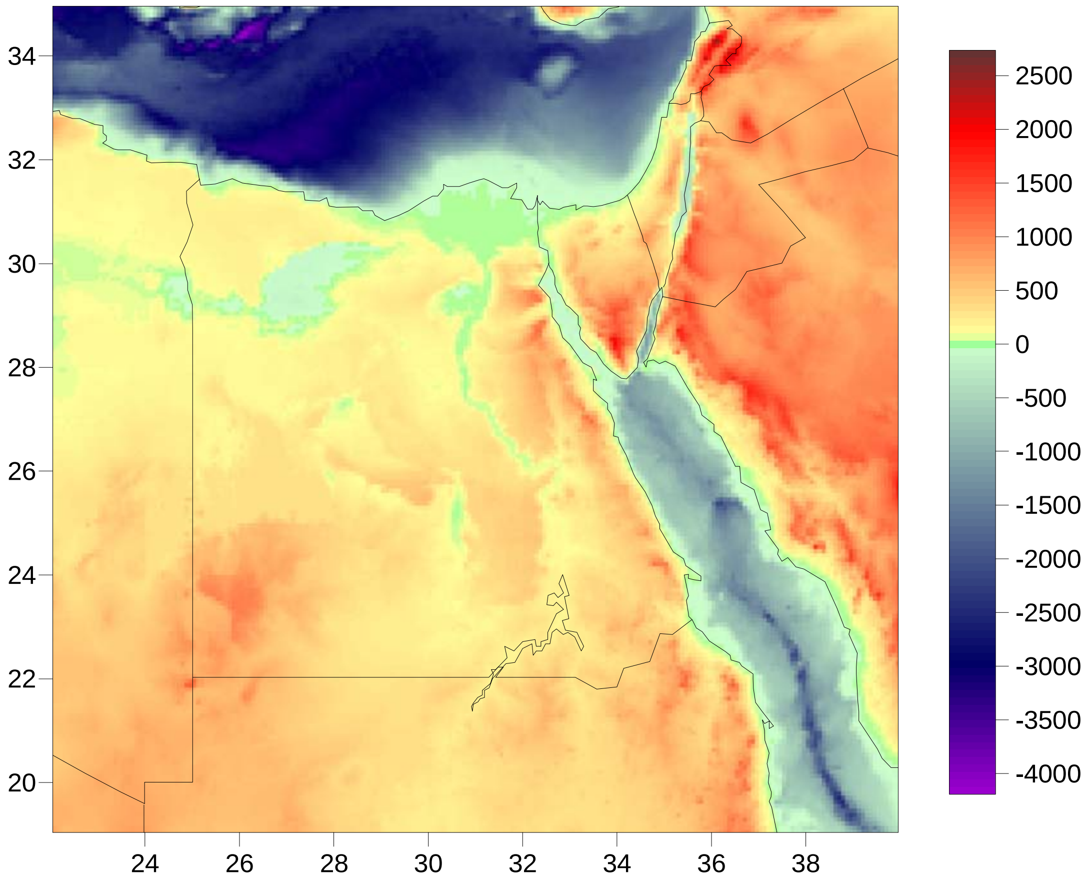

Figure 5: $5' \times 5'$ DHM for Egypt derived from the African DHM AFH04M5 (after Abd-Elmotaal, 2004 a).

complete to degree and order 2160. The heights within the Egyptian window range between -4136 m and 2734 m with an average of -29 m and a standard deviation of about 998 m.

Comparing Figs. 5 and 6 shows a general agreement of the topography. Figure 7 illustrates the difference between AFH04M0 DHM and heights computed by using DTM2006.0 model complete to degree and order 2160. These difference range between -2904 m and 2284 m with an average of 9.5 m and a standard deviation of about 214 m. White areas mean differences less than 100 m in magnitude. Figure 7 shows very small differences in land areas, but larger differences in sea areas, especially in the Mediterranean sea. Some few hundred meters differences in topographic heights in Sinai are remarkable. This might be responsible for the large free-air anomaly differences in Sinai (cf. Fig 4).

5. Geoid Comparison

Figure 8 shows the most recent geoid solution for Egypt EGGG2008 (Abd-Elmotaal, 2008). It is a gravimetric geoid computed using a tailored global geopotential model for Egypt (created basically using the EGM96 global geopotential model) within the window remove-restore technique described by Abd-Elmotaal and Kühtreiber (2003). The geoid

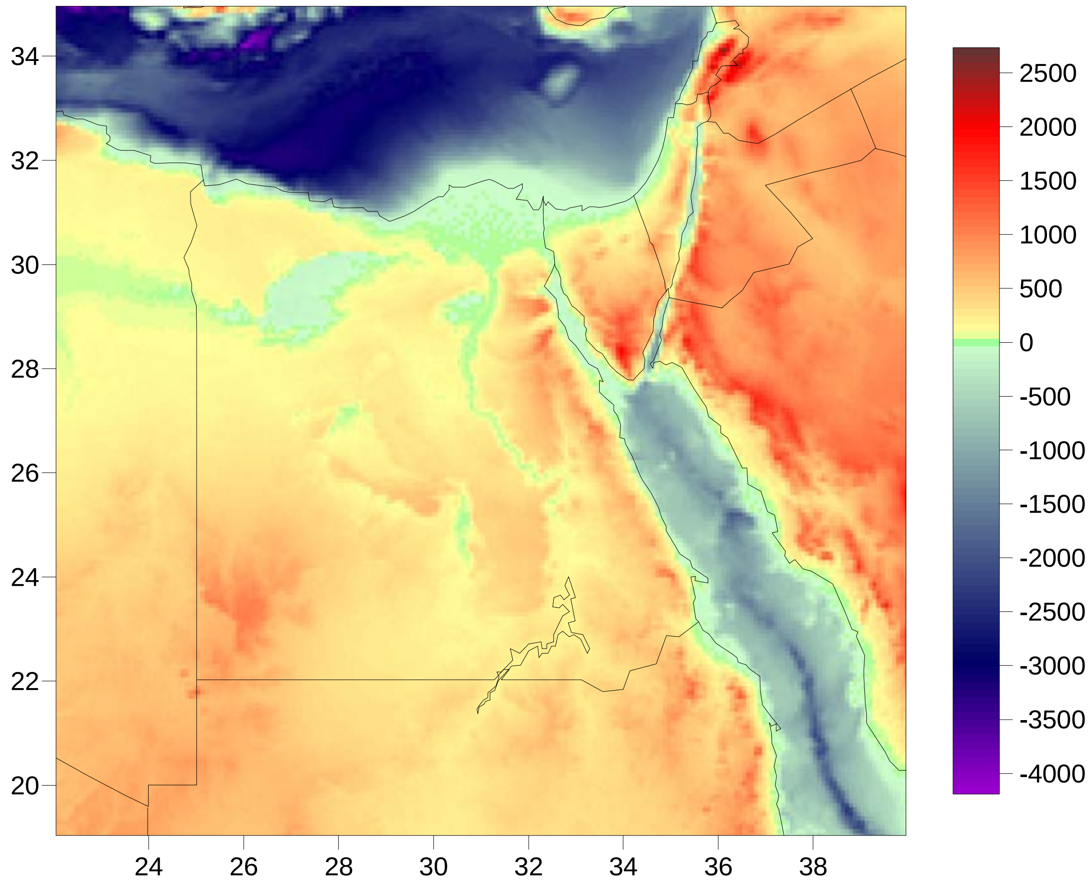

Figure 6: $5' \times 5'$ DHM for Egypt derived by using DTM2006.0 model complete to degree and order 2160.

heights range between 5.95 m and 23.89 m with an average of 14.52 m and a standard deviation of about 3.65 m.

The EGM2008 model has been used to create a geoid model for Egypt using GRVABD program (Abd-Elmotaal, 1998). Figure 9 shows the geoid undulations for Egypt computed by using EGM2008 model complete to degree and order 2159. The geoid heights range between 6.37 m and 22.89 m with an average of 14.80 m and a standard deviation of about 3.16 m.

Figure 10 shows the difference between EGGG2008 geoid model and geoid undulations computed by using EGM2008 model complete to degree and order 2159. These geoid undulation differences range between -7.83 m and 3.40 m with an average of -0.28 m and a standard deviation of about 1.92 m. White areas mean differences less than 0.5 m in magnitude. Figure 10 shows very large differences especially in Sinai and North-Western desert.

6. GPS Comparison

Figure 11 shows the difference of geoid undulations at GPS stations between GPS/levelling undulations and those computed using EGM2008 model complete to degree and order

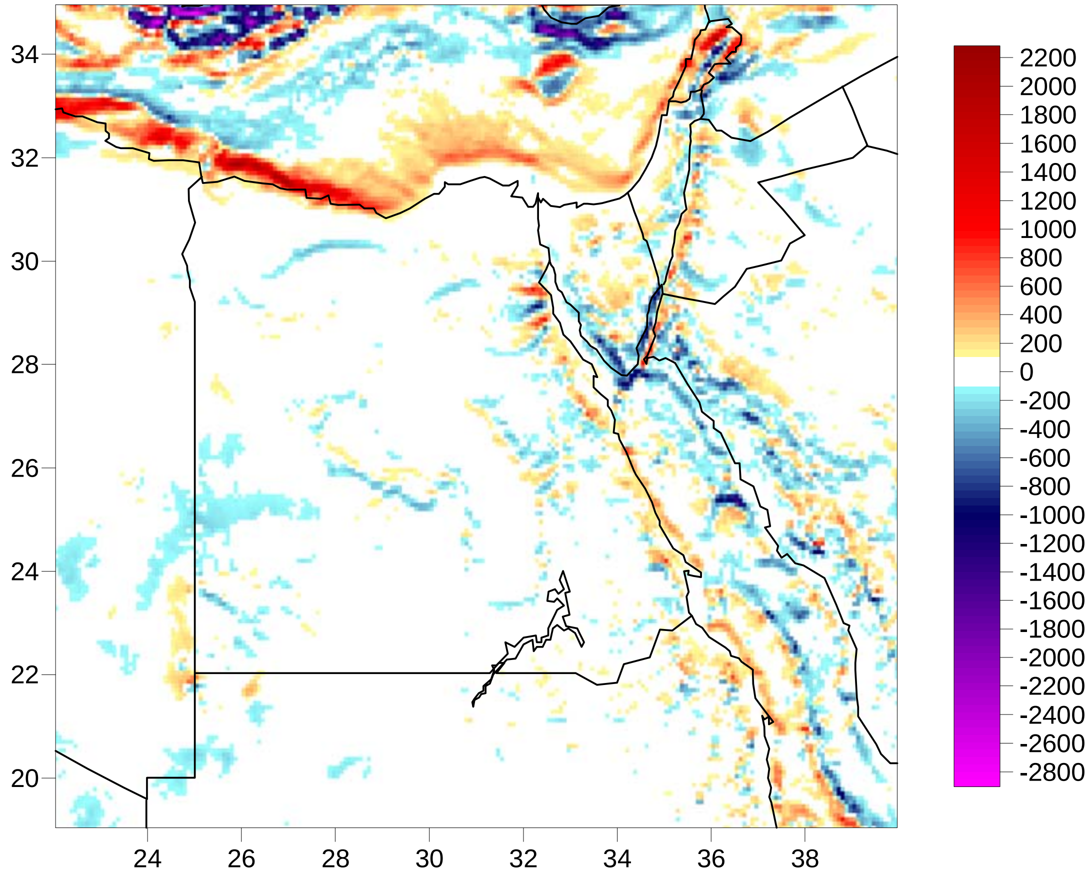

Figure 7: Difference between AFH04M5 DHM and heights computed by using DTM2006.0 model complete to degree and order 2160.

2159. These differences range between −5.01 m and 3.32 m with an average of −0.04 m and a standard deviation of about 2.11 m. Figure 11 also shows the differences of the undulations at the GPS stations (30 stations). Figure 11 confirms the conclusion drawn in the previous section that there are large difference at both Sinai and North-Western desert.

Table 3 illustrates the statistics of the geoid undulations at GPS stations of both GPS/levelling and those computed using EGM2008 model complete to degree and order 2159, as well as their differences. It also shows, for comparison purposes, same statistics for the case of the EGM96 global geopotential model as well as the EGGG2008 local geoid model. Table 3 confirms the already stated conclusion that EGM2008 doesn't fit the geoid undulations of Egypt.

7. Conclusion

A validation scheme of the recently generated EGM2008 geopotential model over Egypt has been carried out in this investigation. The validation includes the followings:

- Comparison of gridded $5' \times 5'$ gravity anomalies for Egypt
- Comparison of point gravity anomalies for Egypt

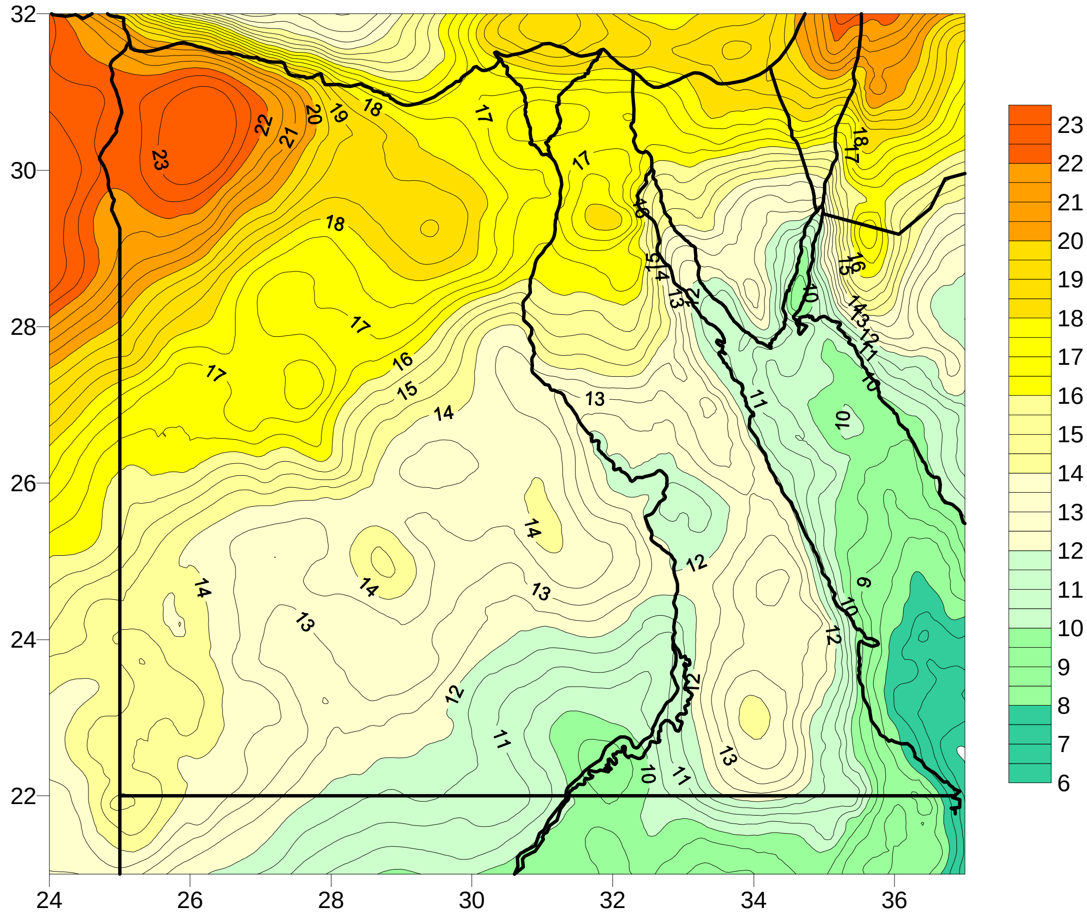

Figure 8: EGGG2008 geoid model for Egypt (after Abd-Elmotaal, 2008). Contour interval: 0.5 m.

Table 3: Statistics of the geoid undulations at GPS stations (30 stations)

geoid undulation	statistical parameters			
	min.	max.	mean	st. dev.
	m	m	m	m
GPS/levelling	7.29	21.51	14.85	3.73
EGM2008	7.39	20.96	14.89	3.06
EGM96	7.53	21.25	14.74	3.06
EGGG2008	7.28	21.56	14.78	3.75
GPS/levelling – EGM2008	−5.01	3.32	−0.04	2.11
GPS/levelling – EGM96	−2.42	2.84	0.10	1.37
GPS/levelling – EGM96	−0.05	1.75	0.07	0.32

- Comparison of geoid model for Egypt
- Comparison of geoid undulations and GPS stations for Egypt

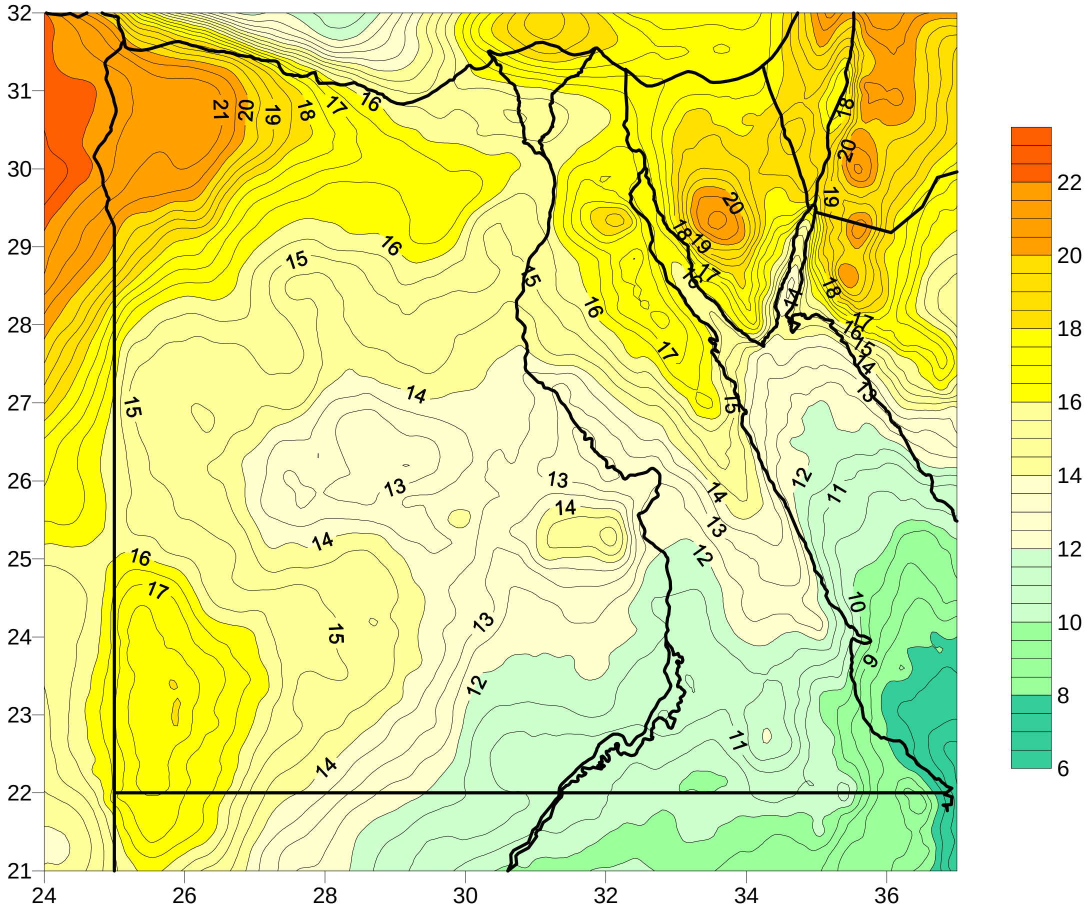

Figure 9: Geoid model for Egypt computed by using EGM2008 model complete to degree and order 2159. Contour interval: 0.5 m.

The validation also includes the comparison of the most updated DHM for Egypt and the DTM2006.0 DHM.

The results proved that the EGM2008 model gives generally good agreement with gravity anomalies in Egypt without applying topographic-isostatic reduction. After performing topographic-isostatic reduction, worse results were obtained.

As far as geoid undulations are concerned, EGM2008 proved to produce incompatible geoid undulations for Egypt. Large difference were obtained, especially in Sinai and North-Western desert.

It should also be pointed out that it is believed that some wrong gravity data at Qena ($\phi = 25.3°$, $\lambda = 32°$) have been included in producing EGM2008. It is, therefore, recommended to remove them in producing any upcoming geopotential model.

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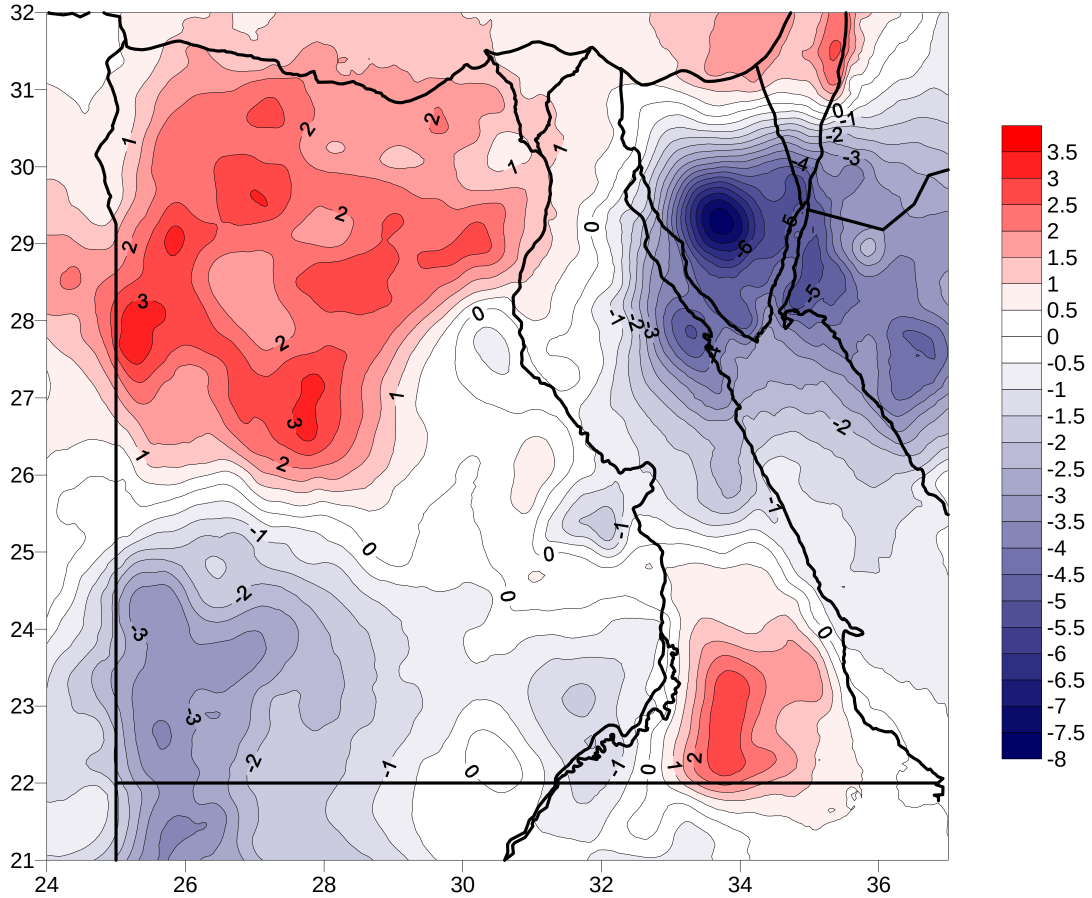

Figure 10: Geoid difference for Egypt between EGGG2008 model and geoid undulations computed by using EGM2008 model complete to degree and order 2159. Contour interval: 0.5 m.

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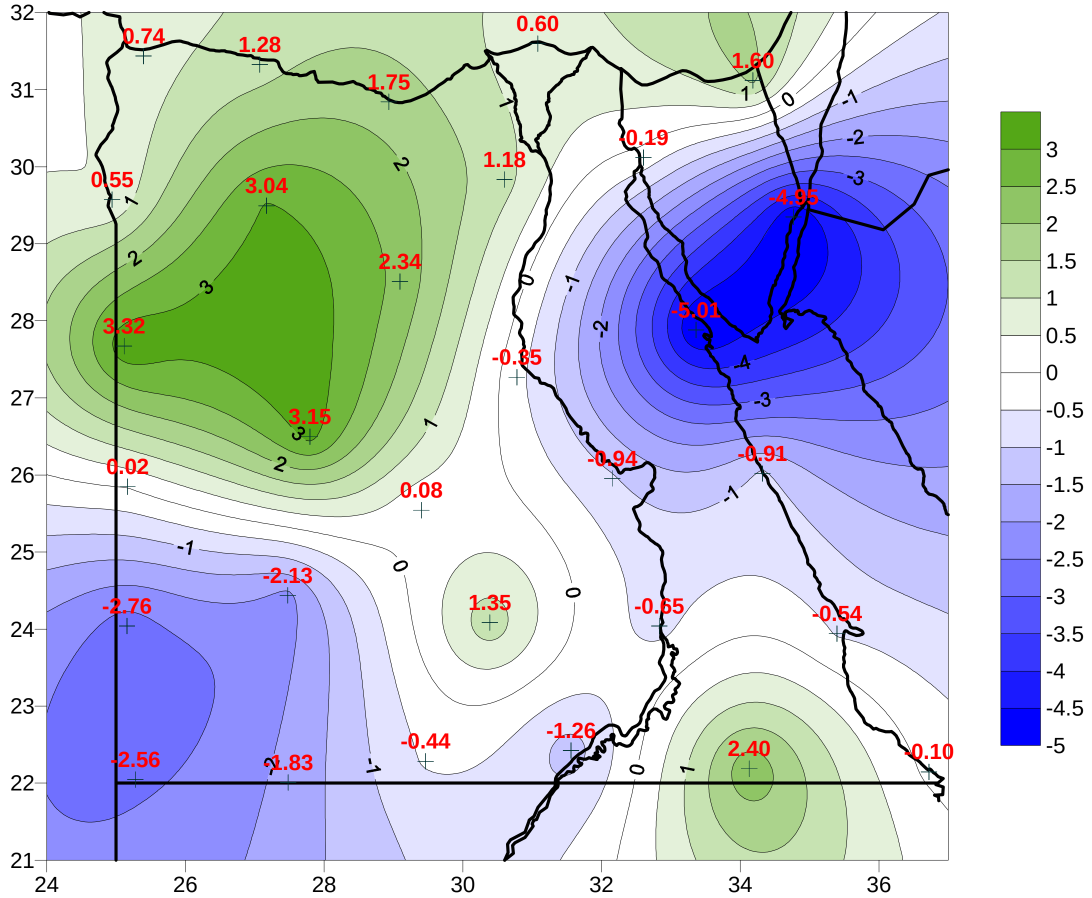

Figure 11: Difference of geoid undulations at GPS stations between GPS/levelling undulations and those computed using EGM2008 model complete to degree and order 2159. Contour interval: 0.5 m.

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