Evaluation of EGM2008 and PGM2007A over Sweden

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Abstract

An important part of the work performed by the IAG/IGFS Joint Working Group "Evaluation of Global Earth Gravity Models" is to test the new Earth Gravitational Model EGM2008 and its preliminary versions. The purpose of this paper is to present the evaluation of the preliminary PGM2007A and final model EGM2008 over Sweden. The evaluation is done by comparing the model to 195 high quality GPS/leveling observations, to the best regional quasigeoid model presently available and finally to observed gravity anomalies. The regional (gravimetric) quasigeoid model in question has previously been computed in cooperation between Lantmäteriet and the Royal Institute of Technology in Stockholm (KTH) using least squares (stochastic) kernel modification with additive corrections.

The most important result is that EGM2008 agree with the Swedish GPS/leveling data with a RMS of 2.7 cm after a 1-parameter transformation. This is comparable to the corresponding RMS value of 2.2 cm obtained for the regional quasigeoid model. Thus, EGM2008 agrees well with the GPS/levelling data and the regional quasigeoid model in Sweden.

1. Introduction

The purpose of this paper is to present the Swedish evaluation of EGM2008 (Pavlis et al. 2008) and the preliminary model PGM2007A against GPS/leveling observations, the best regional quasigeoid model available over Sweden in 2008 and the gravity anomalies utilised to compute the regional model. This work is done under the umbrella of the IAG/IGFS Joint Working Group "Evaluation of Global Earth Gravity Models"; see the Working Group home page at "http://users.auth.gr/~kotsaki/IAG_JWG/ IAG_JWG.html"

The regional quasigeoid model to which the Earth Gravity Models (EGMs) are compared has been computed in cooperation between Lantmäteriet (Swedish mapping, cadastre and registry authority) and the Royal Institute of Technology in Stockholm (KTH). This work is documented in Ågren et al. (2008). The model is computed using one version of the so-called KTH method, developed by Prof. Lars E Sjöberg and his group at the Royal Institute of Technology in Stockholm. The technique includes least squares (stochastic) kernel modification with additive corrections for the topography, downward continuation, atmosphere and ellipsoidal shape of the Earth. The method is well documented in a long row of publications; see for instance Sjöberg (1991, 2000, 2003a, 2003b, 2004 and 2007), Sjöberg and Nahavandchi (2000), Ågren (2004a, 2004b) and Kiamehr (2006).

The paper is organised in the following way. The GPS/leveling observations are described in Section 2, which also contains the corresponding evaluations of EGM2008, PGM2007A and EGM96. Section 3 contains a short description of the regional quasigeoid model referred above and the corresponding comparisons to EGM2008. After that, the point gravity anomalies that were used to compute the regional model are compared to the predicted gravity anomalies from EGM2008 and PGM2007A. Finally, the paper ends with our conclusions.

2. Comparison with GPS/leveling observations

The quasigeoids computed by the above EGMs were evaluated using 195 high quality GPS/leveling height anomalies in the Swedish reference systems SWEREF 99 and RH 2000. More information about SWEREF 99 can be found in Jivall and Lidberg (2000) while RH 2000 is documented in Ågren et al. (2006) and Ågren and Svensson (2007). The normal heights in RH 2000 have either been determined in the RH 2000 adjustment (Ågren and Svensson 2007) or by utilising high quality leveling connections relative to the RH 2000 benchmarks. The stations are divided into two groups depending on the method employed to determine the GPS ellipsoidal height. The two groups are summarised in Table 1, in which approximate standard errors are also given. The distribution of the stations is illustrated in Figure 1.

Table 1: *The GPS/Leveling observations and their approximate standard errors. The permanent GPS stations (SWEPOS) and the stations (SWEREF) determined relative to SWEPOS are shown in red and blue on Figure 1, respectively.*

Data set	#	Short description	Appr. standard errors (mm)		
			GPS height	Normal height	Height anomaly
SWEPOS (red)	24	Permanent GPS stations whose coordinates define SWEREF 99.	5-10	5-10	7-14
SWEREF (blue)	171	Determined relative to SWEPOS using 48 hours of observations, DM T antennas and the Bernese software	10-20	5-10	11-22

It is clear from Table 1 that the observations are of high quality. Figure 1 also shows that the observations are uniformly distributed over the country. The lack of stations in the north-west corresponds to an area with high mountains where no levelling lines are available.

All computations were determined in the zero permanent tide system. Consequently, the GPS/leveling height anomalies were consequently corrected so that they refer to the same system. Since RH 2000 is already defined in a zero permanent tide system, the correction amounts to converting the height above the ellipsoid in SWEREF 99 from non-tidal to zero permanent tide system. This was done using (Ekman 1989):

$$h_{zero} = h_{non-tidal} + h\left(0.099 - 0.296 \cdot \sin^2 \phi\right) \text{ meter} \quad (1)$$

where the Love number h was chosen to 0.62.

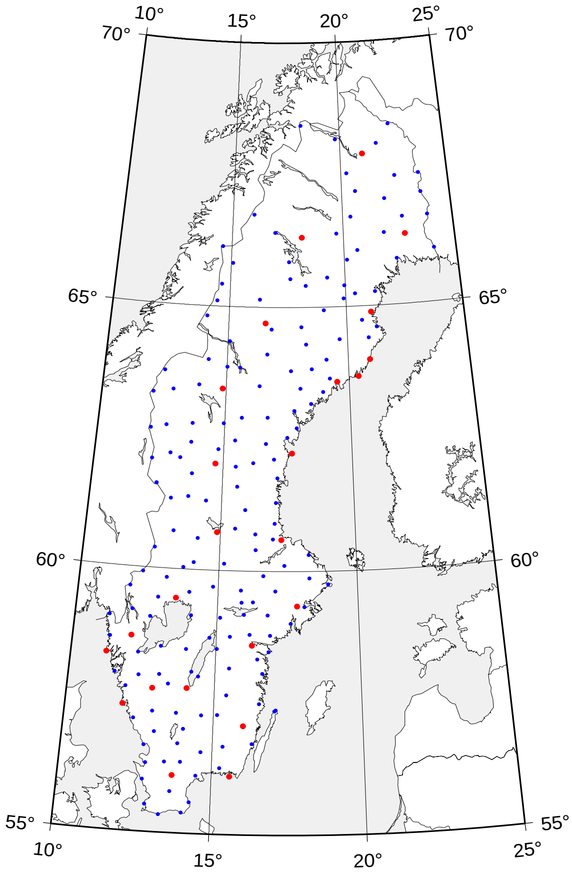

Figure 1: *Locations of the GPS/leveling observations. SWEPOS = Red and SWEREF = Blue.*

The program *Harmonic_synth_v2* is used to compute height anomalies for various spherical harmonic maximum degrees (*M* = 360, 720, 1440 and 2190). The scattered point computation mode (isw=00) is used with the normal height of each station given as input. The statistics of the residuals after a 1-parameter transformation are given in Table 2 for the 4 selected maximum degrees. EGM96 (Lemoine et al. 1998) is here included for comparison. The residuals for both PGM2007A and EGM2008 are illustrated in Figure 2 for the maximum degree 2190.

Table 2: *Statistics for the GPS/leveling residuals of EGM2008, PGM2007A and EGM 96 after a 1-parameter transformation. Unit: m.*

EGM	*M*	# gpslev	Min	Max	Mean	StdDev
EGM2008	2190	195	-0.074	0.095	0.000	0.027
	1440	195	-0.116	0.089	0.000	0.037
	720	195	-0.172	0.124	0.000	0.045
	360	195	-0.266	0.257	0.000	0.099
PGM2007A	2190	195	-0.248	0.085	0.000	0.039
	1440	195	-0.289	0.130	0.000	0.045
	720	195	-0.295	0.118	0.000	0.051
	360	195	-0.428	0.252	0.000	0.103
EGM 96	360	195	-0.376	0.509	0.000	0.172

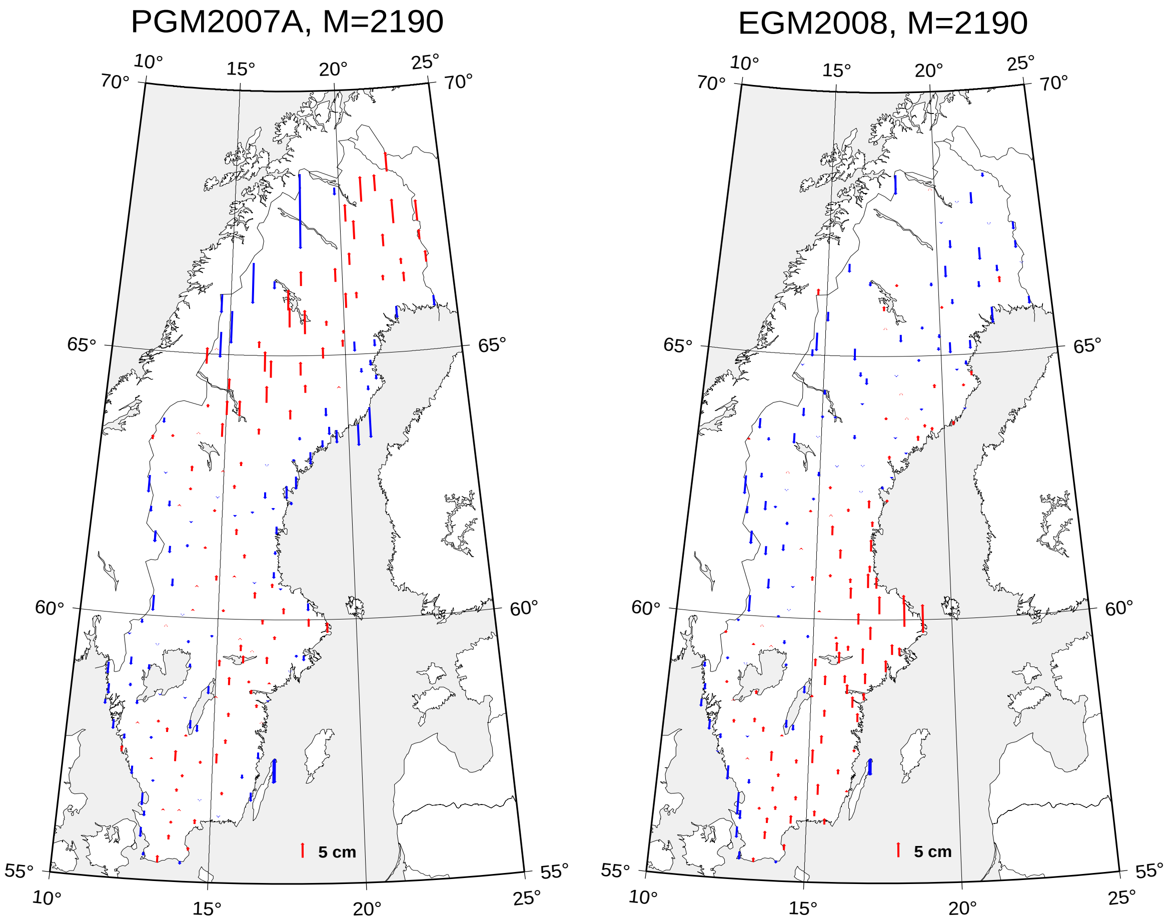

Figure 2: *GPS/leveling residuals of PGM2007A and EGM 2008 for M = 2190 after a 1-parameter transformation. The scale is given by the arrow to the South East.*

The first thing that can be seen in Table 2 and Figure 2 is that EGM2008 fits considerably better with the GPS/leveling data compared to the preliminary PGM2007A. For the latter, the fit is poor in the northern half of the country. For southern Sweden, however, the results are comparable. The fact that EGM2008 is so much better indicates that some improvement must have been made by the processing team based on the preliminary evaluation of PGM2007A. It can further be seen in Table 2 that EGM2008 is a considerable step forward compared to EGM96, also when only the maximum degree M=360 is considered.

It should finally be emphasised that the achieved fit for EGM2008 with M=2190 is impressive. The question now is how this result compares with the best Swedish regional quasigeoid model available in 2008. This is the topic of the next section.

3. Comparison to the best regional quasigeoid model available for Sweden in 2008

As mentioned in the introduction, the best (*gravimetric) regional quasigeoid model* available for Sweden in 2008 has been computed in close cooperation between Lantmäteriet (Swedish mapping, cadastre and registry authority) and the Royal Institute of Technology in Stockholm (KTH); see Ågren et al. (2008). The model is derived by the least squares modification of Stokes' formula using additive corrections (the KTH method). The method can also be described as stochastic modification of Stokes' formula using analytical continuation to point level (Moritz 1980) together with improved atmospheric and ellipsoidal corrections (Ågren et al. 2008). Below a short summary is given of the method used to predict height anomalies. The reason for including this description here is that the comparison between the regional model and EGM2008 not only serves as an evaluation of the models themselves, but also of the corresponding processing strategies.

In the least squares modification of Stokes' formula (e.g. Sjöberg 1991), Stokes' kernel is modified in such a way that the expected global mean square error is minimised. This technique can be applied with the standard remove-compute-restore estimator (e.g. Ågren 2004b), but according to KTH practice the so-called *combined estimator* is preferred (Sjöberg 2003b). This means that Stokes' formula (truncated to a cap) is applied to the uncorrected surface gravity anomaly, Δg. After that, the height anomaly ζ is computed by adding a number of so-called *additive corrections*, which are derived in such a way that the same result is ideally obtained as when the remove-compute-restore technique is utilised (except for numerical effects). We thus have

$$\zeta = \frac{R}{4\pi\gamma}\iint_{\sigma_0} S^M(\psi)\Delta g d\sigma + \frac{R}{2\gamma}\sum_{n=2}^{M}\left(s_n + Q_n^M\right)\Delta g_n^{GGM} + \delta\zeta_{TOPO} + \delta\zeta_{DWC} + \delta\zeta_{ATM} + \delta\zeta_{ELL} \tag{2}$$

where

$S^M(\psi)$ is the modified Stokes' function chosen according to Sjöberg (1991).

$\delta\zeta_{TOPO} = 0$ is the combined topographic correction. Vanishes in the height anomaly case.

$\delta\zeta_{DWC}$ includes analytical continuation to point-level of both the gravity anomalies (Moritz 1980) and the spherical harmonic expansion; cf. Sjöberg (2003a) and Ågren (2004a).

$\delta\zeta_{ATM}$ is the atmospheric correction (Sjöberg and Nahavandchi 2000).

$\delta\zeta_{ELL}$ is the ellipsoidal correction (Sjöberg 2004).

The following data is used to compute the regional quasigeoid model:

- Gravity anomalies from the database of the Nordic Geodetic Commission (NKG).
- The combined model GGM02C (to M=200) extended with EGM 96 up to M=360,
- The Swedish photogrammetric Digital Elevation Model (DEM) thinned out to the resolution of 100 m x 100m.

The principles of the weighting of the terrestrial gravity data in relation to the EGM is described in Ågren et al. (2008).

One problem with using the combined quasigeoid estimator in Eq. (2) is that Stokes' quadrature is made on the rough surface gravity anomaly, which results in large *discretisation errors*. However, by taking advantage of the remove-compute-restore philosophy for the gridding of a comparatively dense surface gravity anomaly grid using a smoothing topographic correction, such errors can be counteracted; see Ågren (2004). This makes it possible to take advantage of the high-frequency information available in the DEM. This strategy was adapted in the present case by utilising the Residual Terrain Model (RTM) correction (Forsberg 1997) as implemented in the TC program (Forsberg 2003). The surface gravity anomaly grid was chosen with a resolution of 0.01°x0.02°, which should be sufficiently dense.

The statistics for the fit of the regional quasigeoid model to the GPS/leveling height anomalies are given in Table 3. The residuals are illustrated in Figure 3 (left hand side), which also includes the depiction of EGM2008 residuals for comparison.

Table 3: *Statistics for the GPS/leveling residuals of the regional quasigeoid model (KTH_080326) after a 1-parameter transformation. Unit: m.*

Model	# gpslev	Min	Max	Mean	StdDev
KTH_080326	195	-0.064	0.061	0.000	0.022

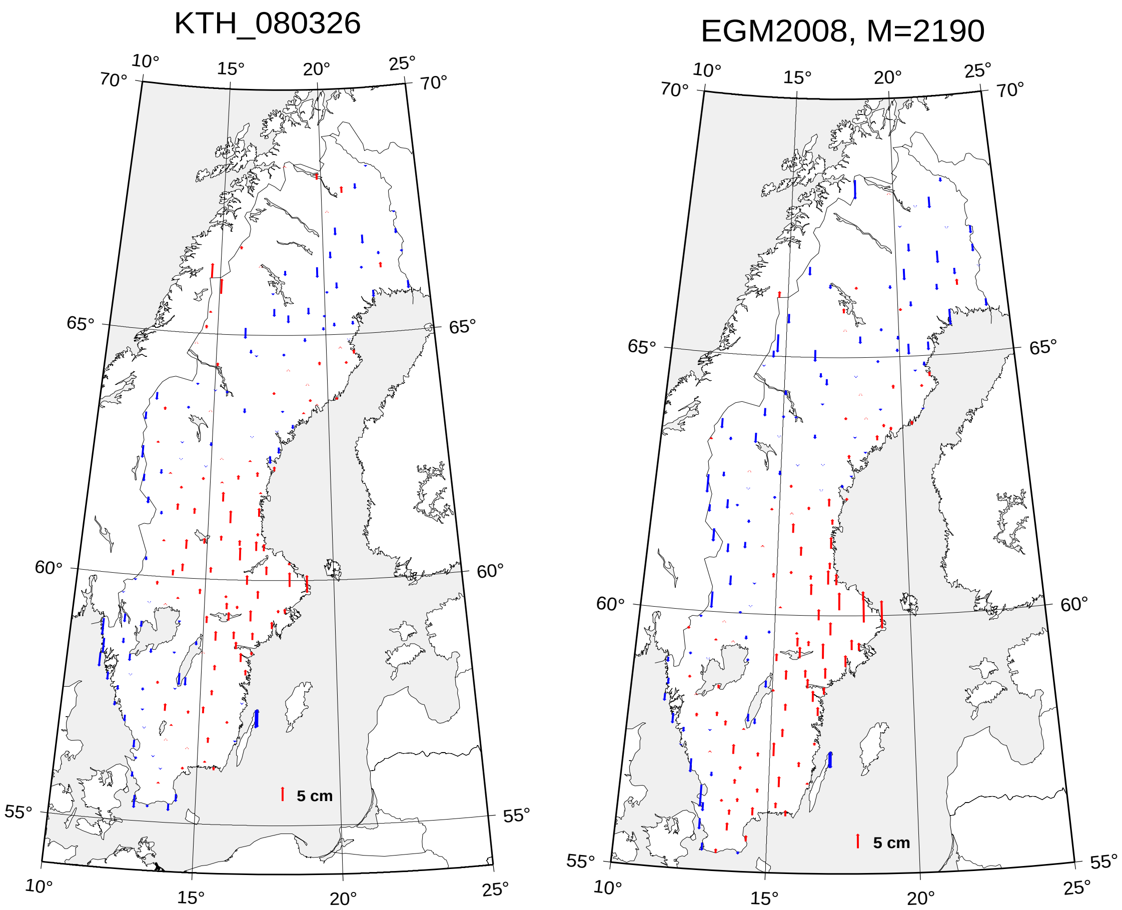

Figure 3: *GPS/leveling residuals of the regional quasigeoid model (KTH_080326) and EGM 2008 after a 1-parameter transformation. The scale is given by the arrow to the South East.*

By comparing the results in Tables 2 and 3, it can be observed that the regional quasigeoid model fits slightly better than EGM2008 to the Swedish GPS/leveling data. As is clear from Figure 3, the agreement between the regional model and EGM2008 is surprisingly good. It is difficult to say, though, how much of the errors in Figure 3 are gravimetric and how much are GPS/leveling. Let us now study the difference in question a little more carefully. For this purpose, the regional height anomaly grid was compared to the same grid computed by EGM2008 using *Harmonic_synth_v2* in the scattered point mode with M = 2190. The comparison was made without transforming the models in any way and without special zero degree corrections. The differences are illustrated in Figure 4 and statistics for the whole grid and for the mainland of Sweden are presented in Table 4.

Table 4: *Statistics for the height anomaly difference between the regional quasigeoid model (KTH_080326) and EGM2008. Unit: m.*

Area	# grid points	Min	Max	Mean	StdDev	RMS
Whole grid	1202351	-0.221	0.242	0.006	0.038	0.038
Mainland of Sweden	390816	-0.187	0.093	-0.002	0.023	0.023

It can be seen that the agreement between the models is within ± 2 cm for most parts of the country. In some areas the discrepancies are larger, but in most cases just a little. The differences are considerably larger in the rough mountains to the North-West of the country, which mainly depends on that the regional model contains more high-frequency information than EGM2008. In these areas the frequencies above $M=2190$ are definitely significant. It can also be seen that the models agree reasonably well in the Baltic Sea east of Sweden. We do not comment here on the large deviations outside the coast of Norway. This is not a Swedish matter

Overall, Table 4 and Figure 4 support the conclusion that the two models agree well over Sweden. This is encouraging and shows that the processing strategies for the regional model and for EGM2008 are compatible to a high degree and that very similar gravity data must have been utilised in the processing. Since the regional model has been computed using updated gravity from the NKG database (see above), this shows that good, updated data must have been used for EGM2008 too.

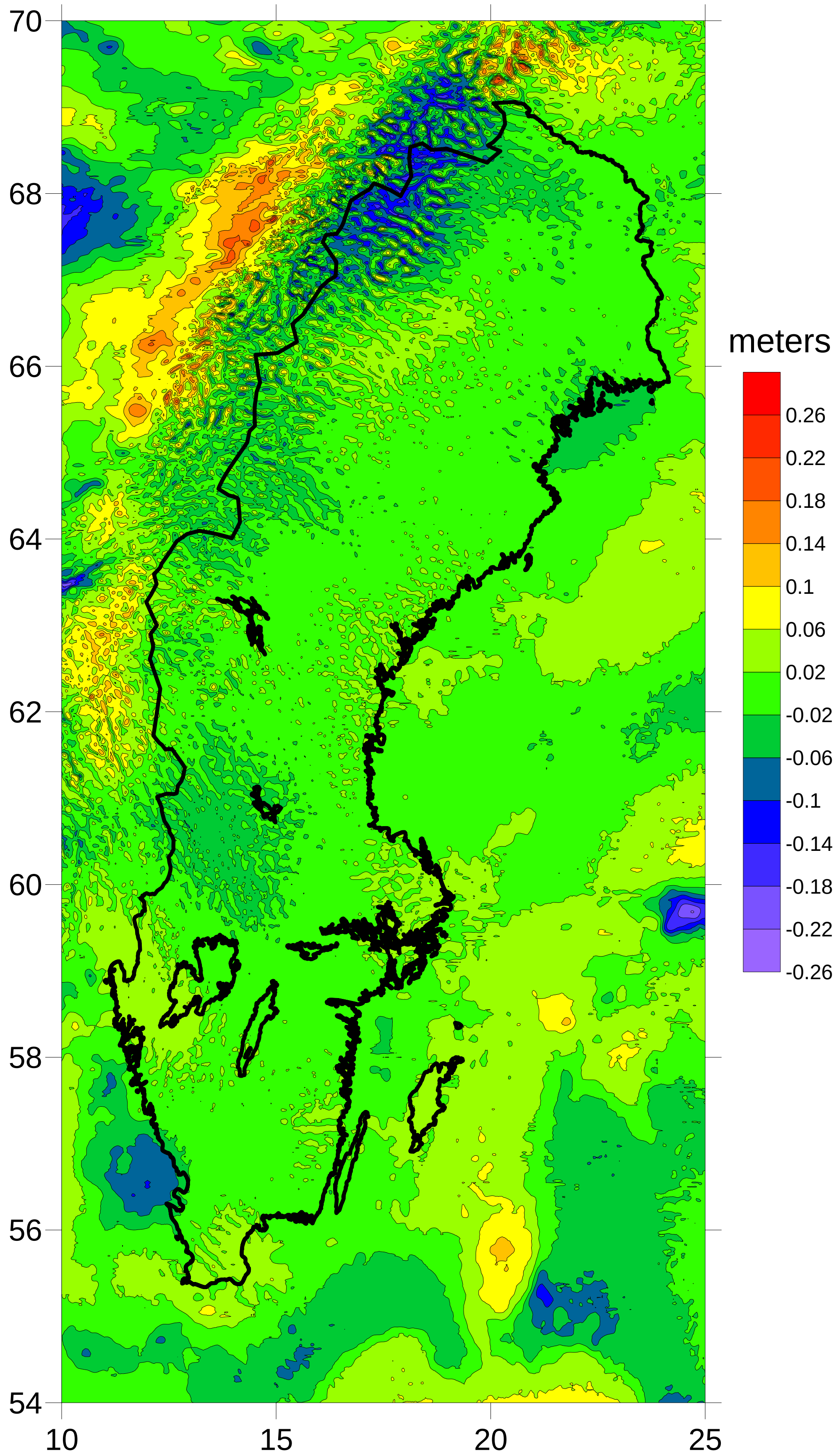

Figure 4: *Difference between the height anomalies from the regional quasigeoid model (KTH_080326) and EGM 2008. Unit: m.*

4. Comparison with gravity anomalies

As mentioned in Section 3, the gravity anomalies used to compute the regional quasigeoid model (KTH_080326) were taken from the NKG (Nordic Geodetic Commission) database, managed by René Forsberg and Gabriel Strykowsky at DTU Space in Denmark. Before estimating the height anomaly using Eq. (2), however, the gravity anomalies were cleaned by

- computing the weighted mean of observations at the same location,
- making a cross validation using the RTM and EGM reduced gravity anomalies and
- finally choosing only the observation with lowest apriori standard error in each compartment of a 0.02°x0.04° grid.

Statistics for the difference between EGM2008 and PGM2007A (using *Harmonic_synth_v2* in the scattered point computation mode with M = 2190) and the cleaned gravity anomalies are given in Table 5.

Table 5: Statistics for the difference between the gravity anomalies selected to compute the regional quasigeoid model (KTH_080326) and PGM2007A/EGM2008. Unit: mGal.

EGM	Area	# gravity anomalies	Min	Max	Mean	StdDev	RMS
PGM2007A	Whole grid	270204	-107.2	197.3	1.3	10.2	10.3
	Mainland of Sweden	24570	-60.2	86.9	2.4	7.6	7.9
EGM2008	Whole grid	270204	-99.1	197.8	1.3	10.0	10.1
	Mainland of Sweden	24570	-58.6	80.3	2.6	7.3	7.8

It should be pointed out that no kind of filtering is used in the computation of Table 5. Consequently the differences in question also contain the omission error above the maximum degree 2160, which is definitely not negligible. For instance, the global omission RMS error from the Tscherning and Rapp (1974) degree variance model is 11.2 mGal. With this in mind, it is clear that the observed point gravity data and EGM2008 fit reasonably well over Sweden. No comparisons have been made using filtered gravity anomaly data.

5 Summary

The main purpose of this paper is to present the Swedish evaluation of EGM2008 and its preliminary version PGM2007A. The main conclusions are the following:

- The height anomalies from EGM2008 are almost as accurate as those of the best gravimetric quasigeoid model available for Sweden in 2008. The RMS for the fit to GPS/leveling are 2.7 cm and 2.2 cm, respectively. Considering the standard errors of the GPS/leveling height anomalies, which lies somewhere around 10 – 20 mm, it is clear that both the regional model and EGM2008 are very good.
- The regional quasigeoid model and EGM2008 agree well with each other inside Sweden. This indicates that the respective computation strategies are compatible. Since this is an important result for both techniques, a rather detailed summary has been given of the regional geoid determination method. The good agreement also shows that similar gravity anomaly data is utilized for the two models. Since the regional quasigeoid model has been derived using good, updated data (NKG database), this need to be the case for EGM2008 too.

- EGM2008 is a considerable improvement with respect to PGM2007A as far as Sweden is concerned. The standard deviation improves from 3.9 cm to 2.7 cm when compared to the GPS/leveling-derived height anomalies. This is after correcting for a 1-parameter transformation. The improvements occur in northern half of Sweden, above the 64 degree parallel. Whatever the EGM2008 processing team did to improve PGM2007A, they did the right thing.
- EGM2008 agrees well with the Swedish point gravity anomalies. For the mainland of Sweden, the RMS of the discrepancies is 7.8 mGal. If one considers the magnitude of the omission error for EGM2008, it is clear that this agreement is as good as can be expected.

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