Calibration of a 3-accelerometer inertial gravimetry system for moving gravimetry

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Abstract. A moving inertial gravimetric system is being developed, consisting of three high precision accelerometers measuring accelerations along three non parallel axes. The signal delivered by each accelerometer is an electric current, the intensity of which is proportional to the acceleration experienced by the test mass of the accelerometric sensor. This sensor is also very sensitive to temperature variations which are continuously monitored by an internal temperature sensor. The current given by each accelerometer is transformed into a voltage sampled at 31.25 Hz, that is one sample every 32 ms, while the temperature is sampled at a rate of one sample every 4.096 s.

Our aim is to carry out the calibration of this system in order to derive the relationship between each digitalized value given by the accelerometers and the actual acceleration, taking into account temperature variations. Our calibration system permits to tilt simultaneously the three accelerometers above a point where gravity has been precisely determined. Thus, the accelerometers can sense any acceleration value between 0 and the value of gravity at the measuring point (accelerometer axis is then vertical).

We discuss the results of the calibration by looking at the residuals between observed values and those coming from different theoretical calibration functions. We particularly focus on the perturbing phenomena such as temperature or misalignment of the sensitive axis.

Keywords. Vector gravimetry, calibration, accelerometers, least squares.

1 Introduction

Since about a decade, gravimetry has significantly evolved and several techniques can now be used with different precisions and resolutions (Bruton, 2000). There is however a gap affecting medium resolutions, ranging from 5 to 100 km, which cannot be filled by ground-based gravimetry or space gravimetry (Boedecker, 1994, Verdun et al., 2003). The miniaturization of both accelerometers and GPS receivers made possible the design of small size apparatus for moving gravimetry. Such apparatus are used to cover hard access continental regions like mountains, margins, deserts or rivers like Amazon, with a resolution within the range 5 to 100 km.

Three accelerometers, coupled with a 4-antennae GPS system can be used as a cheap and handy instrument to measure the three spatial components of the gravity vector (vector gravimetry).

Such a gravimetry system is now being developed in our laboratory consisting of three high precision accelerometers (type QA3000-20, Honeywell) supported by a triad. Each accelerometer delivers an electric current, the intensity of which is proportional to the acceleration sensed onto its sensitive axis. The current is then transformed into a voltage which is digitalized at a frequency up to 250 Hz by means of a 24 bits A/D converter. The internal accelerometer temperature is also measured by a sensor, and digitalized with a sampling rate of 0.244 Hz (1 sample every 4.096 s). Our approach consists of finding a calibration procedure in order to estimate the relationship between the actual acceleration value and its digitalized value. The effects of temperature have also to be carefully investigated since the electronic system is likely to be very sensitive to temperature variations.

We develop and discuss in this paper the calibration procedure including the materials and the processing of calibration function. A first calibration carried out in February 2005 is analysed, and a first calibration function is derived. We also propose some recommendations in order to improve the reliability of the calibration procedure.

2 Calibration principle

Deriving the calibration function requires the acquisition of a set of accelerometer data at points where the acceleration has been already measured by means of another method. Our calibration system permits to tilt simultaneously the three accelerometers above a point where gravity has been measured beforehand using an absolute gravimeter. By so doing, each accelerometer can sense any acceleration value between 0 (accelerometer sensitive axis is horizontal) and the gravity value at the point (980 855.8 mGal).

Indeed, if the accelerometer is tilted by an angle *α* from the vertical (fig. 1), the accelerometer measurement F consists of the projection of gravity vector $\vec{g}_{\text{abs}}$ onto its sensitive axis, that is

$$F = g_{\text{abs}} * \cos(\alpha - \beta) \quad (1)$$

where *β* is the projection of *θ* (the angle between the geometric axis and the sensitive axis of the accelerometer) on the plan of *α* formed between the geometric axis of the mirror and the sensitive axis of the accelerometer (fig. 2).

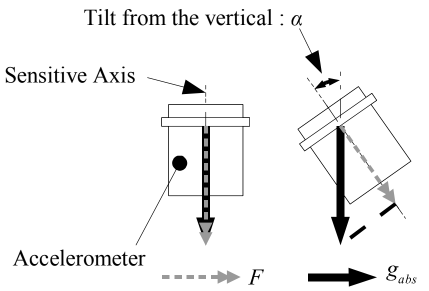


Fig. 1 Tilt of accelerometer sensitive axis

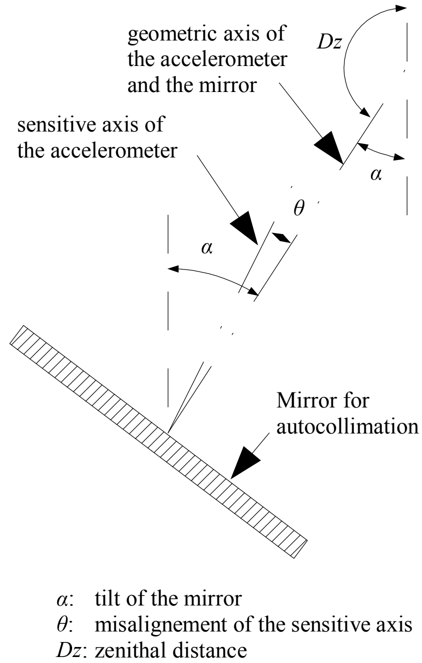
The angle *θ* is designed by the manufacturer to be less than 1 mrad. As a first approximation, we chose to neglect this angle because so far measurements for calibration do not permit to define this angle. So, the sensitive axis is assumed to be aligned with the geometric axis of each accelerometer. Let *Dz* be the zenithal distance of the accelerometer axis (fig. 2), then we obtain

$$F = -g_{\text{abs}} * \cos(Dz) \quad . \quad (2)$$

Equation (2) allows to determine the acceleration *F* sensed by each accelerometer for any zenithal distance *Dz*. In the meantime, the A/D converter provides the digitalized value *Ng* of the acceleration *F*. Our aim is now to obtain, for each accelerometer, a set of N observations (Ng_i , F_i), $i=1,...,N$, in order to estimate the calibration function

$$Am = f_T(Ng) \quad (3)$$

of each accelerometer, for a given temperature.



α: tilt of the mirror
θ: misalignement of the sensitive axis
Dz: zenithal distance

Fig. 2 Measurement of *Dz*

3 Practical determination of acceleration *F*

The three accelerometers are mounted on a platform which can rotate around a horizontal axis, and enclosed in a thermally isolated box where temperature is maintained constant. The platform's rotation is controlled by a screw which can be overtightened in order to maintain the platform in a fixed direction. The platform is also equipped with a mirror placed outside the isolated box by means of an axis. The axes of respectively the mirror and the three accelerometers have been previously yielded parallel by autocollimation with an optical plummet. By so doing, the zenithal distance of the mirror axis corresponds exactly to the geometrical axes of the three accelerometers. The zenithal distance can then be precisely measured by means of autocollimation using a theodolite.

4 Result of the calibration

The calibration was carried out in the laboratory of ESGT (Ecole Supérieure des Géomètres et Topographes) during two days.

4.1 Acceleration measurements and precision

Accelerations were measured for about 20 zenithal distances *Dz* ranging from 109° to 140°, and for two different temperatures. We systematically acquired about 15 measurements for each zenithal distance, so as to estimate the standard deviation σ_{Dz}. Then, assuming that the errors on *Dz* and g_{abs} are uncorrelated, and the error on g_{abs} is negligible, the standard deviation σ_{Am} of *F* can be calculated as:

$$\sigma_{Am} = g_{\text{abs}} |\sin(Dz)| \sigma_{Dz} \quad . \tag{4}$$

The resulting standard deviations have proved to range between 0.8 mGal and 4.5 mGal with a median at 2.1 mGal.

4.2 Digitalizing of accelerations and precision

For a given tilt, acceleration was continuously digitalized with a sampling rate of 31.25 Hz, i.e. one sample every 32 ms. By so doing, we acquired on average 80 digitalized values *Ng* of *F* for each measurement at zenithal distance *Dz*. As a result, using 15 zenithal distance measurements, we obtained 15 * 80 = 1200 digitalized values *Ng* of the same acceleration. The standard deviations of the digitalized acceleration mean values range between 0.5 and 2.5 bits with a median at 1.0 bit. Given the fact that accelerometers range extends over 4 *g* (±*2 g*), coded on 24 bits, the resulting resolution is given by:

$$\frac{4 \times 10 \times 10^5}{2^{24}} = 0.24 \quad mGal/bit$$

These findings indicate that the errors on digitalized acceleration values cause an error on acceleration ranging between 0.12 mGal and 0.60 mGal.

The synchronisation of both the theodolite and the A/D converter was ensured by means of GPS time delivered by a dedicated GPS receiver.

5 Estimation of the calibration function

Following the manufacturer recommendation, the calibration function for a given temperature has to be chosen as a polynomial function. Since the temperature does not vary very much and the range of accelerations is not wide, we chose a first order polynomial function as a model:

$$f_T(Ng) = b + k\, Ng \tag{5}$$

where *b* and *k* are the bias and the scale factor of the calibration function, respectively.

Let b^0 and k^0 be approximate values of parameters b and k, then the calibration function for each observation (Dz_i , Ng_i) may be expressed as

$$g\cos(Dz_i + v_{Dz_i}) + (b^0 + \Delta b) + (k^0 + \Delta k)(Ng_i + v_{Ng_i}) = 0 \tag{6}$$

where v_{Dzi} and v_{Ngi} correspond to the residuals of the observations Dz_i and Ng_i, respectively, and Δb, Δk are the correction to be applied to the parameters *b* and *k*. By keeping only first order terms in the previous equation, we obtain

$$\begin{aligned} &g\cos(Dz_i) - g\sin(Dz_i)\, v_{Dz_i} \\ &\quad + b^0 + \Delta b + k^0\, Ng_i + Ng_i \Delta k + k^0\, v_{Ng_i} = 0 \end{aligned} \tag{7}$$

By denoting

$$A = \begin{bmatrix} \vdots & \vdots \\ 1 & Ng_i \\ \vdots & \vdots \end{bmatrix}, \; X = \begin{bmatrix} \Delta b \\ \Delta k \end{bmatrix}, \; W = \begin{bmatrix} \vdots \\ g\cos Dz_i + b^0 + k^0\, Ng_i \\ \vdots \end{bmatrix}$$

$$V = \begin{bmatrix} \vdots \\ -g\sin Dz_i\, v_{Dz_i} + k^0 \; v_{Ng_i} \\ \vdots \end{bmatrix} \tag{8}$$

equation 7 can be rewritten in matrix form as

$$W + V + AX = 0 \tag{9}$$

Estimates for *b* and *k* parameters can be found by means of the least squares method which consists in minimizing the quadratic form

$$\Phi(V, \Lambda, X) = V^T P_V V - 2\, \Lambda^T (W + V + AX) \tag{10}$$

where P_V is the weight matrix of residual vector V and Λ is the vector of Lagrange's multipliers. The solution can be easily determined using the following relations (Leick, 1990)

$$
\begin{aligned}
X &= -(A^T P_V A)^{-1} A^T P_V W \\
\Lambda &= -P_V (W + A X) \\
V &= P_V^{-1} \Lambda
\end{aligned} \qquad (11)
$$

The variance-covariance matrix of the parameters is the given by

$$C_X = (A^T P_V A)^{-1} \qquad (12)$$

The weight matrix P_V can be deduced from the Cv covariance matrix of vector $V = [v_{Dz}^T, v_{Ng}^T]^T$ by (Fotopoulos, 2005)

$$P_V = (E\, Cv\, E^T)^{-1} \qquad with$$

$$Cv = \begin{bmatrix} \sigma_{v_{Dz_1}} & & & & & \\ & \ddots & & & (0) & \\ & & \sigma_{v_{Dz_N}} & & & \\ & & & \sigma_{v_{Ng_1}} & & \\ & (0) & & & \ddots & \\ & & & & & \sigma_{v_{Ng_1}} \end{bmatrix} \qquad (13)$$

$$E = \begin{bmatrix} -g\sin(Dz_1) & & -k^0 & \\ \ddots & & \ddots & \\ & -g\sin(Dz_N) & & -k^0 \end{bmatrix}$$

σ_{Dz_i} and σ_{Ng_i} for $i=1,..,N$ can be estimated from the measurements described in § 4.1 and 4.2. The above-mentioned calculation can be performed by iteration starting from an arbitrary solution (b_0 , k_0) and testing the sum of squared normalized residuals $V^T P_V V$.

The results obtained for our two-days calibration are tabulated in table 1 for two different temperatures (24.8°C for the first day and 31.0°C for the second).

Table 1 Results of least squares estimation ; the second accelerometer was disconnected during the 2nd day

		b mgal	k mgal/kbit	σb mgal	σk mgal/Mbit	So mgal
Day 1	acc. 1	-29497.03	243.05	4.58	1.83	0.03
222 values	acc. 2	-34476.42	241.77	4.62	1.82	0.03
T°=24.8°C	acc. 3	-23802.24	242.20	4.55	1.83	0.07
Day 2	acc. 1	-29410.07	242.85	2.31	0.95	0.03
413 values	acc. 2	---	---	---	---	---
T°= 31.0°C	acc. 3	-24061.45	242.18	2.29	0.95	0.24

The relative uncertainties on the bias are very small, which gives confidence in its determination. The scale factor has proved to be nearly constant (variation less than 2 mGal/kbits) for all accelerometers and all temperatures which confirms of the reliability of the accelerometer for tracking acceleration variations. The results suggest that the bias might increase with the temperature, which has to be validated by exploring a wider range of temperatures.

6 Conclusions

The calibration procedure proved to be very efficient to estimate the calibration functions of each individual accelerometer. But, the misalignment θ and its direction cannot be defined with this calibration process. Further experiments have to be held in order to determine this angle.

The bias is likely to be significantly influenced by the temperature variations ; thus some additional experiments have to be carried out in order to explore a wider temperature range.

Clearly more work needs to be done particularly in testing higher order calibration functions.

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