A comparative study of the geoid-quasigeoid separation term C at two different locations with different topographic distributions

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Abstract:
The present study has been carried out to compare the geoid-quasigeoid separation in two areas of Pakistan with different topographic distributions. The height datum of Pakistan is based on the orthometric height system. Since Pakistan has a diversity of terrain distribution as regards the elevation from mean sea level due to vast expanse comprising both land and hilly areas, the emphasis of this study has been placed on the quantification of the geoid-quasigeoid separation term with respect to elevation distribution for future geoid determination. Bouguer gravity anomalies and digital terrain elevation were used to estimate the minimum/maximum separation between these two reference surfaces. This was also compared with a separation term C computed from EGM96 gravity anomalies. The statistics of the results in the two areas exhibit a one to one correspondence of EGM96 gravity anomalies with observed gravity data and digital elevation data. The area with high mountains (Kalat) has more offset between the two surfaces. The standard deviation of separation term is ~77 mm from observed and 62.5 mm from model data. In contrast with the low elevation area (Dadu), the standard deviation of the separation becomes as small as ~2 mm from observed data and ~7mm from gravity anomalies computed from the EGM96 model. The terrain correction has measurable effect on the standard deviation in Kalat and is very insignificant in Dadu area. The difference of the separation term from observed data and the model can be related to assumption of the topographic density, local mass irregularities and inherent omission error of the EGM96 model. These results also show that the separation term C is significant in Pakistan and may be required to be incorporated in the final geoidal solution.

Keywords: Geoid, Quasigeoid, Bouguer Gravity anomaly, Height Anomaly, C Term

1-Introduction:

The investigation made in this paper is a comparative study of the effect of terrain on the geoid to quasigeoid separation in two selected areas of Pakistan. This study also focuses on the issue of significance of this term with reference to Pakistan and surrounding areas for onward more feasible geodetic boundary values solution.

The geoid is an equipotential surface of the Earth that corresponds to mean sea level, whereas the quasi geoid is a geometrical surface referred to a normal height system. The geoid undulation (N) is the separation between the ellipsoid and the geoid measured along the ellipsoidal normal. The height anomaly (ζ) is the separation between the reference ellipsoid and quasigeoid along the ellipsoid normal. There is similar concept of orthometric heights (H^o) measured along plumb line whereas, normal heights (H^N) are measured along the ellipsoidal normal. These reference surfaces of the geodesy are shown in Figure 1

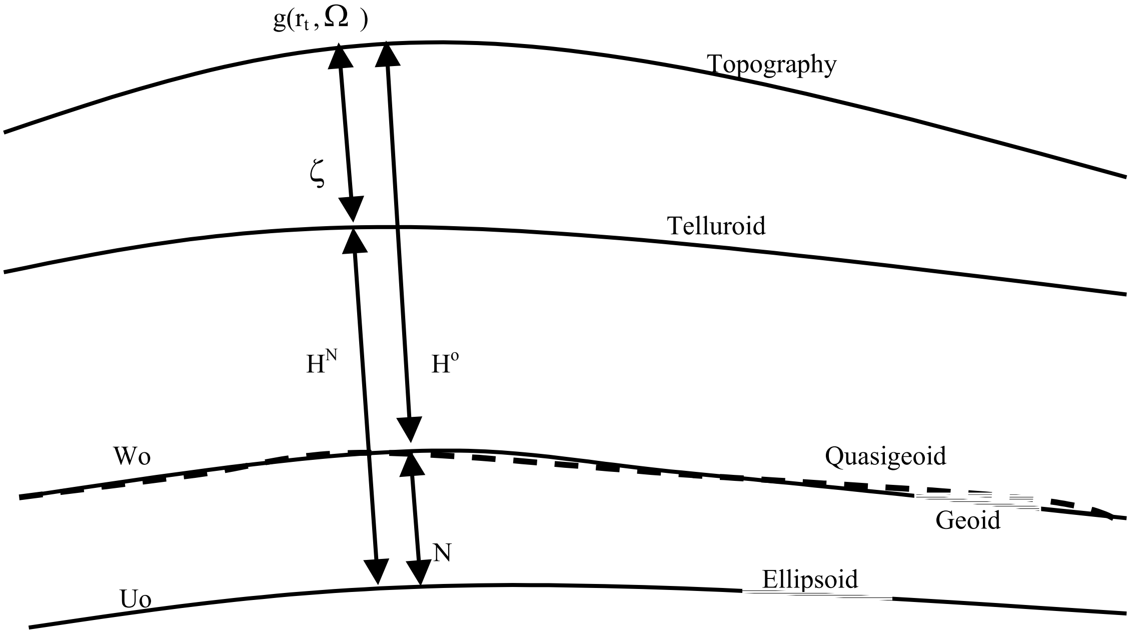

Figure-1: The geoid undulation(*N*), Orthometric heights (*H*), Height Anomaly (ζ) and Normal heights (H^N)

In this study, a comparison is made of the geoid to quasigeoid correction term for two study areas with different topographic distribution. This comparison is made with observed gravity/elevation data, gravity anomalies computed from the EGM96 global model and digital elevation model data extracted from Shuttle Radar Topographic Mission (SRTM) for these areas. The EGM96 global model is a recent estimate of the global gravity and height anomalies [Rapp (1997)]. The global correction term was determined using EGM96 potential coefficients. The Bouguer gravity anomalies and elevation data of the 3495 points in the low land area (Dadu) and 927 points in the high elevation (Kalat) area were determined from observed land gravity data while orthometric heights were determined using spirit leveling. The terrain correction was also calculated in these areas and added to the Bouguer anomaly to quantify the effect of terrain on geoid to quasigeoid separation.

2-Theoretical Background

Molodensky et al. (1962) formulated the geodetic boundary value problem on the earth surface and introduced two new surfaces called the telluroid and the quasigeoid. The quasigeoid is not an equipotential surface of the earth's gravity field and thus has no physical meaning (Heiskanen and Moritz, 1967). However, the quasigeoid can be determined somewhat more directly from surface gravity data without prior knowledge of the topographic bulk density. The separation between the reference ellipsoid and the quasigeoid is called the height anomaly and is defined as;

$$\zeta = h - H^N \tag{1}$$

The telluroid is the surface defined by plotting the points at a distance equal to ζ below the earth surface. The distance between the ellipsoid and the telluroid is called the normal height H^N and can be computed from optical leveling measurements using the integral mean of normal gravity between the reference ellipsoid and the telluroid as,

$$\bar{\gamma} = \frac{1}{H^N}\int_0^{H^N} \gamma \, dH \tag{2}$$

This equation makes it possible to determine the normal heights without prior knowledge of the topographic density distribution along the ellipsoidal normal.

The orthometric height H^o of any point on the surface of the earth is the height of point above the geoid along the geoidal normal. It can be determined by optical leveling measurement along the plumb line between the point on surface and the geoid. It requires the integral mean of the gravity along the geoidal normal and can be determined using the relationship [Heiskanen and Moritz (1967)].

$$\bar{g} = \frac{1}{H^o}\int_0^{H^o} g dH \tag{3}$$

However, this requires the knowledge of the subsurface density along the plumbline and this information is not available in normal routine work. For this purpose, the Poincare-Prey gravity gradient is often used. Assuming that the ellipsoidal normal and plumb line are coincident between the earth surface and the geoid, the geoid separation can be approximated by

$$\mathbf{N \cong h - H^o} \tag{4}$$

Eliminating h from Eq. 1 & 4 gives

$$\mathbf{H^N - H^o = N - \zeta \cong C2} \tag{5}$$

Where, C2 is the geoid-quasigeoid separation term. It depends on the Bouguer anomaly, average theoretical gravity and orthometric height of the point. This term can be derived as follows.

The basic formula for the definition of orthometric height is given by (Heiskanen and Moritz, 1967)

$$H^o = \frac{C[r_t(\Omega)]}{\bar{g}} \qquad \forall \Omega(\varphi,\lambda) \in \Omega_0(-\pi/2 \le \varphi \le \pi/2; 0 \le \lambda \le 2\pi) \tag{6}$$

Where $C[r_t(\Omega)]$ is the geopotential number, and $\bar{g}(\Omega)$ is the mean value of the gravity along the plumb line between geoid and earth surface determined using Poincare-Prey's gravity gradient.

The Molodensky's normal height $H^N(\Omega)$ reads (Molodensky, 1945)

$$H^N = \frac{C[r_t(\Omega)]}{\bar{\gamma}} \qquad \forall \Omega \in \Omega_o \tag{7}$$

Where $\bar{\gamma}(\Omega)$ is the mean value of normal gravity along the ellipsoidal normal between the surface of the geocentric reference ellipsoid and the telluroid.

The difference between the normal height and orthometric height can be determined by the following relation.

$$H^N - H^o = H^o(\Omega)\,\frac{\bar{g}(\Omega) - \bar{\gamma}(\Omega)}{\bar{\gamma}(\Omega)} \qquad \forall\Omega \in \Omega_o \tag{8}$$

The difference between the mean gravity $\bar{g}(\Omega)$ and mean normal gravity $\bar{\gamma}(\Omega)$ can be determined using their mathematical definitions with some assumptions as. [Heiskanen and Moritz, 1967]

$$\bar{g}(\Omega - \bar{\gamma}(\Omega) = g\,[r_t(\Omega)] - \gamma\,[r_o(\Omega) + H^N] - 2\pi G\rho_o H^o(\Omega) \tag{9}$$

Where G is Newton's gravitational constant and ρ_o is the mean topographical density $\rho_o = 2.67$ g.cm^{-3}. The expression on the right side of Eq. 9 is the Simple Bouguer Anomaly. Therefore Eq. 8 can be written for geoid – quasigeoid separation term as,

$$H^N - H^o = \frac{\Delta g_B}{\bar{\gamma}(\Omega)} H(\Omega) \tag{10a}$$

or

$$N_p - \zeta_p = \frac{\Delta g_B}{\bar{\gamma}(\Omega)} H(\Omega) \tag{10b}$$

Eq. 10 is also called the C2 term as mentioned by Sjoberg, 1995, Rapp, 1997 and Featherstone and J.F. Kirby, 1998.

$$N(\varphi,\lambda) = \zeta_o(\varphi,\lambda,r_E) + C1(\varphi,\lambda) + C2(\varphi,\lambda) \tag{11}$$

or

$$N(\varphi,\lambda) - \zeta_o(\varphi,\lambda,r_E) = C1(\varphi,\lambda) + C2(\varphi,\lambda) \tag{12}$$

where

$$C1(\varphi,\lambda) = \frac{\partial\zeta}{\partial r} H + \frac{\partial\zeta}{\partial\gamma}\frac{\partial\gamma}{\partial h} H \tag{13}$$

and

$$\mathbf{C2} = \frac{\Delta g_B}{\bar{\gamma}(\Omega)} H^o(\Omega) \tag{14}$$

However, the Sjoberg (1995) paper includes a term dependent on H^2 as well as H in the N - ζ difference. For this analysis, only the 1st order term in terrain elevation (H^o) is considered.

Here Δg_B is the Bouguer gravity anomaly, $\bar{\gamma}(\Omega)$ is the average normal gravity between the reference and the telluroid. It is calculated between geoid and earth surface in routine work taking orthometric height as height term in its calculations.

3-Data Analysis and numerical comparison of the Results:

The data in both study areas comprise the absolute gravity along with orthometric heights. In addition to this Digital terrain model data (3 arc sec.≈ 90 m) from SRTM (USGS, EROS data center) was used to compare its results with orthometric heights to asses its applicability in the terrain correction determination for future geoid computation in Pakistan. The area boundaries and elevation statistics are shown in table 1. The Δg_B was computed using generalized formula [W.E. Featherstone and M.C. Dentith (1997), Eq-23] using point absolute gravity and elevation/DTM data. Theoretical normal gravity γ was computed using Somigliana's formula. For the computation of Δg_B, standard topographical density was assumed, i.e. 2.67 g.cm^{-3}. The Bouguer gravity anomalies and average theoretical gravity in conjunction with elevation data at the corresponding points were used in Eq. 14 to compute the C2 correction term. The results from absolute gravity and terrain/DTM data were compared with the geoid to quasigeoid correction term determined for the EGM96 global geopotential model [Lemoine et al. (1997), National Imagery and Mapping Agency]. The contour plots of C2 term from observed data is shown in figure 6 & 7. The statistics of the C2 term results are shown in table 2.

The Free Air gravity anomalies are correlated (90%) with elevation with inverse correlation (62%) of Bouguer anomalies in Kalat area (Fig-2). The correlation of Bouguer anomalies indicates the considerable Bouguer plate effect demanding the application of terrain correction in this area. This correlation is not significant (Fig-3) in Dadu, as both anomalies are negative and behave approximately similarly with elevation. This small difference of free air and Bouguer anomalies can be related to a small Bouguer plate effect (1.5 to 15.5 mgal only).

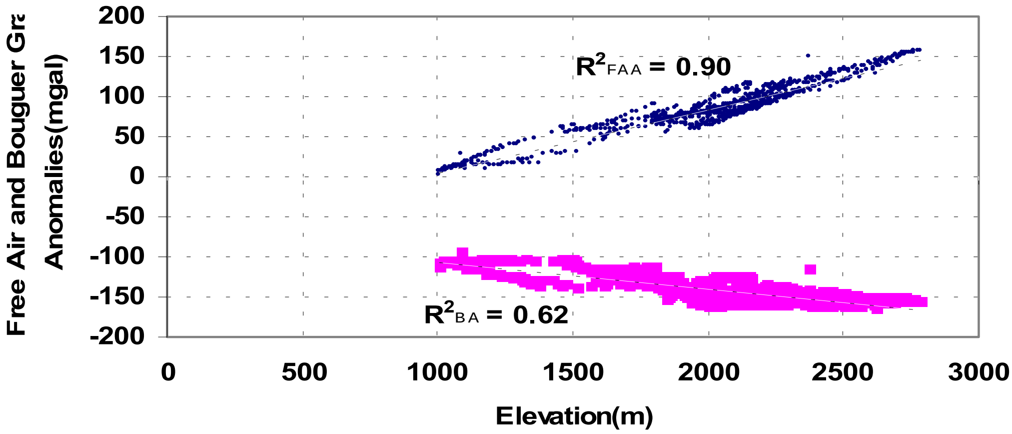

Fig-2. Relationship of Elevation with Free Air and Bouguer gravity Anomalies in Kalat Area

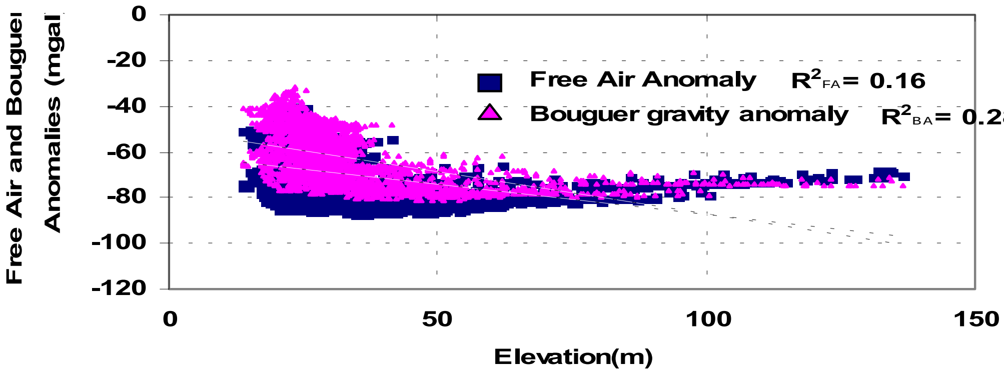

Fig-3. Relationship of Elevation with Free Air and Bouguer gravity Anomalies in Dadu Area

The comparison of geoid to quasigeoid separation terms from observed gravity data and gravity anomalies derived from the EGM96 global geopotential model in Kalat area shows close correlation with each other. The use of the digital terrain model in Kalat has created insignificant difference to the computed C2 term. Although DTM data differs much (of the order of 10-40 m), the effect on the resulting separation term is insignificant due to the reason that the coefficient of H in Eq-14 is of the order of 10^{-3}. The maximum magnitude of the C2 term determined from observed gravity data is 440 mm while that of the EGM96 gravity anomalies is 484mm. However, the standard deviation differs only by 14.3 mm. This term from gravity measurements in Dadu area comes out to be 12 mm while 19.6 mm from EGM96 gravity anomalies whereas, standard deviation differs by only 5mm. The results of DTM data confirm the observed data. This indicates similarity of the elevation data from DTM with leveling data. The difference of values from observed gravity data and EGM96 model may be related to the omission error within EGM96 model. This shows that the overall effect of omission error in Dadu is not significant, whereas it is considerable in Kalat.

The terrain correction was calculated using the software developed by Yecai Li, and Michael G. Sideris (1993) on the grids of orthometric heights to study its effect on this term. The terrain correction is not very large however considerable in Kalat and it has practically insignificant effect on this term in Dadu area. The standard deviation of geoid-quasigeoid separation differs by only 5.4 mm in the Kalat area and really no effect in Dadu area.

Measured Parameter(Kalat area)	**Max**	**Min**	**Mean**	**Measured Parameter (Dadu Area)**	**Max**	**Min**	**Mean**
Longitude(degrees)	67.093	66.496	66.776	**Longitude(degrees)**	67.78	67.35	67.566
Latitude(degrees)	29.426	28.57	29.005	**Latitude(degrees)**	27.29	26.53	26.909
Altitude above MSL(m)	2782.8	1006.2	2024.7	**Altitude above MSL(m)**	136.5	13.7	34.008
Altitude (from DTM)	2782	1021	2029	**Altitude (from DTM)**	148	32	51
Terr. Correction (mgal)	7.663	0.034	0.819	**Terr. Correction(mgal)**	0.155	0.0	0.005

Table-1:Statistics of altitude (in meters) of 927 land gravity points in Kalat(high land) area and 3495 points in Dadu (low land) area along with Terrain correction and latitudinal/longitudinal boundaries.

Description of Model(Kalat Area)	Max	Min	Mean	Std. Dev.	Description of Model(Dadu Area)	Max	Min	Mean	Std. Dev.
C (Obs. Gravity) Before Terrain Correction applied	-103	-440	-297	76.8	**C2(Abs. Gravity) Before Terrain Correction applied**	-0.7	-12	-2.67	1.6
C (Obs. Gravity) After Terrain Correction applied	-106.9	-427.4	-260	71.4	**C2(Obs. Gravity) After Terrain Correction applied**	-0.66	-10.1	-2.3	1.46
C2 (DTM)	-103	-437.4	-295.7	76.6	**C2 (DTM)**	-1.1	- 10	- 3.2	1.5
C2(EGM96)	-210	-484	-411.5	62.5	**C2(EGM96)**	18	-19.6	0.7	7

Table-2: Statistics of the geoid to quasigeoid correction term C2 (in mm) at 927 land gravity points in Kalat (high land) area and 3494 points in Dadu(low land) area from the Bouguer gravity anomaly and elevation with and without terrain correction applied. Also included are the C2 results from DTM and EGM96 global geopotential model gravity anomalies.

The separation term C2 is positive in low land area, though very small (quasigeoid is relatively higher than geoidal surface) and these surfaces reverse position with negative increase of C2 term as elevation increases. It is negative in low land (Dadu area) in general, and become more negative in high land (Kalat area). The correlation with elevation is more with observed data (94%) in Kalat area than EGM96 model (49.8 %) values (Fig-4). However, this correlation is reversed in low land (Dadu area) as C2 from observed data is less correlated (35.5 %) than EGM96 model (97%) separation term. This also indicates an important aspect of inherent data deficiency in EGM96 model in these areas. In low land area (Dadu) the EGM96 model with dominant low frequency components has close correlation with low elevation and gravity gradient. In high land area (Kalat), the EGM96 model values are ~50% less correlated than observed data indicating the deficient information regarding the high frequency component of gravity and elevation.

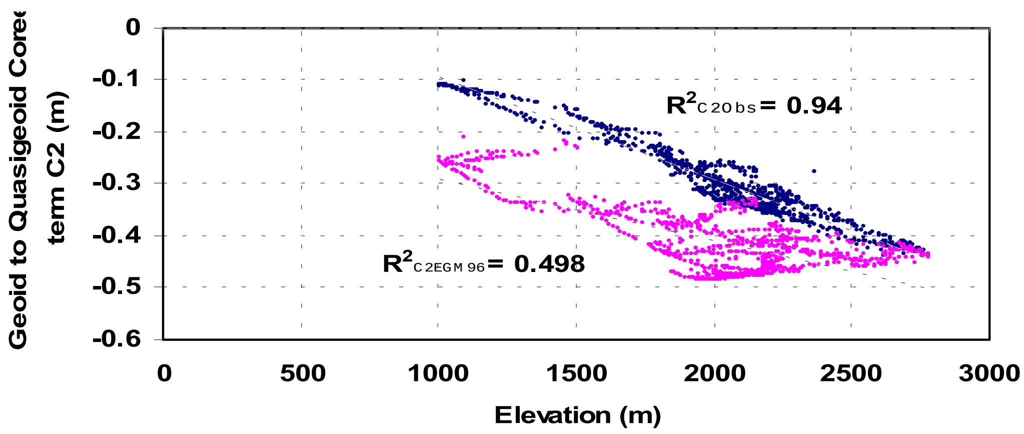

Fig-4. Relationship of Elevation with geoid-quasigeoid correction C2 term from observed gravity data and EGM96 model gravity in the Kalat area

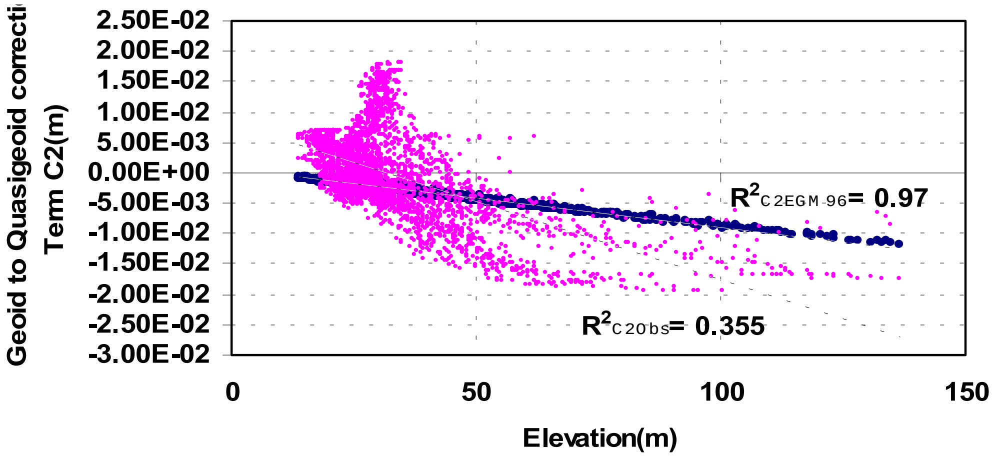

Fig-5. Relationship of Elevation with geoid-quasigeoid correction C2 term from observed gravity data and EGM96 model gravity in the Dadu area

The contour maps of the geoid to quasigeoid separation term from observed gravity data (Fig. 6 & 7 below) show an increase in absolute value in the north-west direction. The trend of the elevation is also increasing towards north-west direction. Most of the western half and complete eastern half of Dadu area is low elevation with 20-40 m above sea level, with the C2 term in the range of 2 -3 mm. The Kalat area has highest elevation in central west part with corresponding maximum separation term of 440 mm. This shows high correlation of C2 term with elevation. This also validates the data quality of observed gravity and elevation in both study areas.

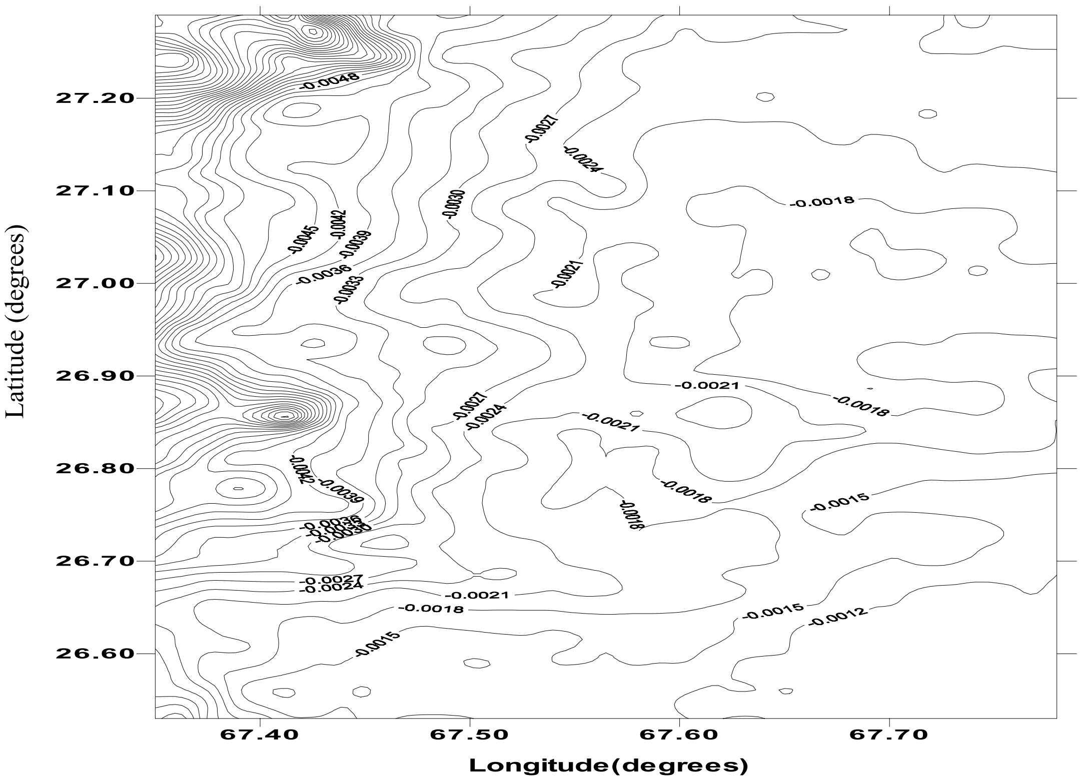

Fig-6. Geoid to Quasigeoid separation term C2 from observed gravity data in Dadu Area. Contour Interval=0.3mm

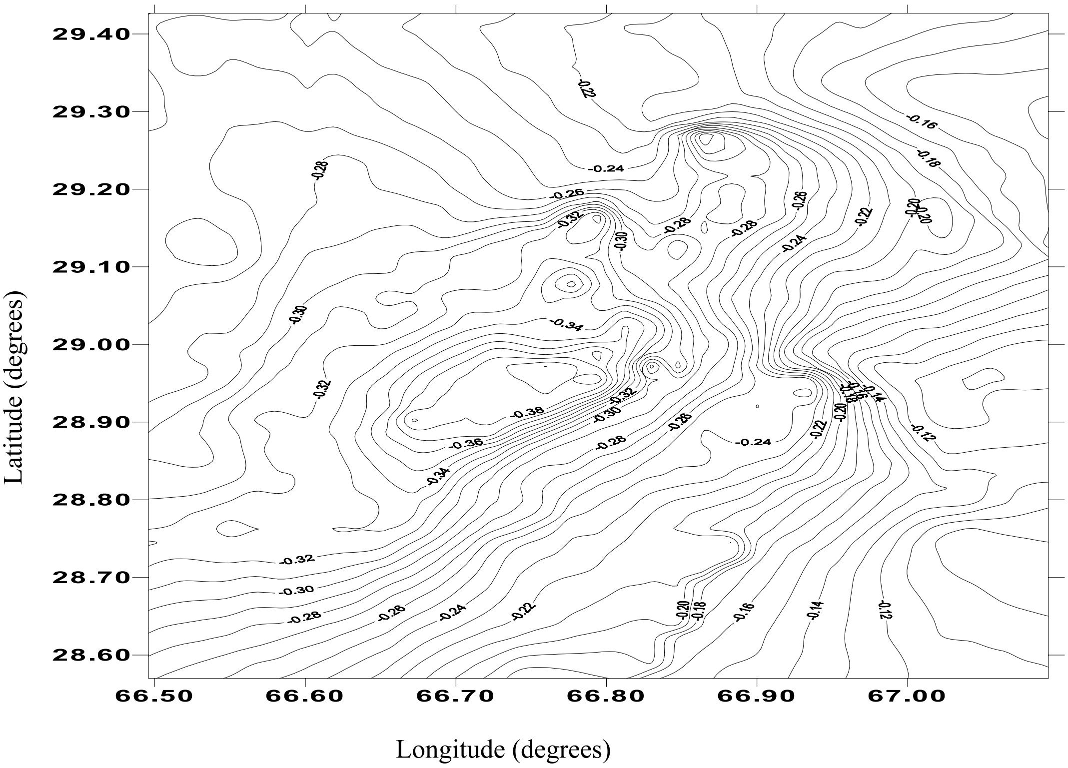

Fig-7. Geoid to Quasigeoid separation term C2 from observed gravity data in Kalat Area. Contour Interval=10 mm

4-Conclusions and Recommendations:

The geoid to quasigeoid separation term C2 was estimated in both study areas and a comparative study with elevation was established. There is a high correlation of the C2 term with elevation, though the correlation of Bouguer anomalies and free air anomalies in the low land area is comparable due to the very small Bouguer plate effect. The terrain correction effect in Kalat is considerable and it is practically insignificant in Dadu area. Keeping in view of the general terrain of Pakistan and the height datum of Pakistan, it is emphasized that the terrain correction be applied in the calculation of geoid to quasigeoid separation term. Also, the magnitude of the geoid to quasigeoid separation term C2 suggests its incorporation in the final geoidal solution, whichever course is followed in solving the geodetic boundary value problem of the gravity field. This also necessitates the estimation of C2 term over the whole area of Pakistan and its validation with other data sources e.g. GPS/Leveling data.

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