Identifying sea-level rates by a wavelet-based multiresolution analysis of altimetry and tide gauge data

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Abstract. Satellite altimetry offers opportunities for studying crustal motion of regional and local origins. This is achieved by the precise determination of the rate of change of the sea level from altimetry data combined with tide gauge records of relative sea level change. In this study, we investigate an application of a wavelet-based multiresolution analysis of TOPEX/Poseidon altimetry data (from 1992 to 2002) and tide gauge records to evaluate trends of sea level change in the Baltic Sea. We compare the sea level trends with those estimated via a least-squares (LS) regression approach. The conducted numerical experiments demonstrate that, in contrast to the wavelet-based approach, LS regression tends to underestimate the sea level trends when short and noisy time series are analyzed. In the presence of an anomalous sea level signal, the wavelet-based approach can detect and model this signal and also accounts for this effect on the sea level trend.

1. Introduction

Today, satellite altimetry over oceans is being considered as complementary to the space positioning techniques over land (SLR, GPS, DORIS, and VLBI) which, in combination with tide gauges, can be employed in vertical crustal motion studies (see, e.g., Cazenave *et al*., 1999; Nerem and Mitchum, 2002). While altimetry determines sea level in a geocentric coordinate system, tide gauges are attached to the deformable solid Earth and measure the sea level with respect to it. Differences of the rate of change of the two signals at the tide gauge location determine the absolute uplift (or subsidence), $\dot{h}$, of the Earth's crust as follows:

$$\dot{h} = \dot{s} - \dot{S} , \tag{1}$$

where $\dot{s}$ is the rate of absolute change of the sea level determined by altimetry and $\dot{S}$ is the rate of change of relative sea level at the tide gauge. Eq. (1) requires that the altimeter instrument drift be precisely known and the periodic variability in the altimetric and tide gauge sea level signals be either removed or cancelled out. Differences between altimetric sea surface heights and tide gauge sea levels have been analyzed for the purpose of postglacial rebound studies by, e.g., Shum *et al*., (2002) and Kuo *et al*., (2004) and for vertical datum monitoring by Jekeli and Dumrongchai (2003).

In semi-enclosed sea regions such as the Baltic Sea, long-term oscillations in the altimetry signal cannot be averaged out when linear trends are sought in contrast to global studies of sea level rise (see Nerem and Mitchum, 2001). In an analysis of the Stockholm tide gauge record, Ekman and Stigebrandt (1990) pointed out that the sea level in the Baltic Sea exhibits low-frequency atmospherically forced oscillations. Such oscillations, and the presence of non-linear trends in the sea level signal, may obscure the estimation of the rate of change of the sea level using least-squares (LS) regression (see also Barbosa *et al*., 2004). Moreover, the absence of accurate tidal corrections for the Baltic Sea can be a critical factor for the estimation of trends from short time span altimetry data since short-periodic oscillations may not average out. If differences between altimetric and tide gauge sea level signals are utilized, some of the periodic components may cancel out. However, to apply this method, only the last decade of the historic tide gauge records can be used. To overcome this shortcoming, Kuo *et al*. (2004) used more than 40-year-long tide gauge records in a Gauss-Markov model constrained by monthly sea level differences from averaged altimetry sea surface heights and tide gauge records.

The aim of this study is to investigate the capabilities of the wavelet-based multiresolution analysis to identify and remove the periodic components from the sea level signal prior to estimating the sea level trend. Since wavelets have good localizing properties both in the time and scale domains, periodic components present in the signal can be detected. This is achieved by imposing restrictions, i.e., threshold values, on the detailed coefficients at different levels of decomposition. The numerical experiment in this study has been conducted in a way that only the smoothing part of Mallat's algorithm (Mallat, 1998) is applied by suppressing the effect of detail coefficients from all levels. Under this condition, wavelets have been used to define the low-pass filter (scaling functions) of Mallat's algorithm, and we expect that identical results should be obtained by means of other low-pass filtering techniques. By filtering out the short-periodic components and using the smoothest time series obtained at the last possible level of decomposition, the wavelet multiresolution analysis can be tested and compared to the traditional LS regression technique in which a trend line is fitted to the time series. Both shorter altimetric and longer tide gauge time series in the Baltic Sea are analyzed herein.

In the following, the methodology and the data sets used are described. Next, the investigated method is tested using a synthetic sea level signal. This is followed by an analysis of the altimetric and tide gauges time series. A discussion on the capabilities of the proposed method based on the wavelet multiresolution analysis to model linear sea level trends is presented in the last section.

2. Description of the Methodology and Data Used

Wavelet-based multiresolution analysis (MRA) is the tool employed to numerically decompose and reconstruct a signal. MRA helps in constructing filter banks based on dyadic wavelets (see Mallat, 1998), and it consists of an application of low- and high-pass filters together with down-sampling and up-sampling. The MRA smoothing step (scale functions) can be defined using different types of wavelets. The general relation

$$\hat{\psi}(f)=\frac{1}{\sqrt{2}}\hat{h}\left(\frac{f}{2}\right)\hat{\varphi}\left(\frac{f}{2}\right) \tag{2}$$

between Fourier transforms of scale functions and the corresponding dyadic wavelets is used. Once the dyadic wavelet is chosen, the corresponding scale function can be defined based on:

$$\hat{\varphi}(f)=\frac{1}{\sqrt{2}}\hat{l}\left(\frac{f}{2}\right)\hat{\varphi}\left(\frac{f}{2}\right), \tag{3}$$

where $\hat{\psi}(f),\hat{\varphi}(f)$ are the Fourier transforms of the wavelet and scale functions; $\hat{h}\left(\frac{f}{2}\right),\hat{l}\left(\frac{f}{2}\right)$ are the Fourier transforms of the high-pass and low-pass filters at twice finer resolution; and f is the frequency.

For the numerical experiment Daubechies 8 wavelets have been used and, following Keller (2000), the corresponding scaling coefficients can be determined in the subsequent steps:

Step 1: Represent the scaling function in the form $\varphi(x)=\sqrt{2}\sum_{k\in Z} l_k\varphi(2x-k)$, where scaling coefficients l_k need to be determined.

Step 2: Define a trigonometric polynomial

$$q_N(f)=\frac{1}{2}+\sum_{k=1}^{N}a_k\cos(2(2k-1)\pi f),\ \ \sum_{k=1}^{N}a_k=\frac{1}{2}\ \text{ assuming that } N=8\,.$$

Step 3: Find the spectrum of the scaling function $\hat{\varphi}(f)$ based on $\left|\hat{\varphi}(f)\right|^2=q_N(f)$.

Step 4: Represent the spectrum of scaling function in terms of scaling coefficients

$$\hat{\varphi}(f)=\frac{1}{\sqrt{2}}\sum_{k=0}^{N}l_k e^{-jk2\pi f}\ .$$

Step 5: Determine the scaling coefficients l_k based on the assumed trigonometric polynomial and the relation in step 3. For the reconstruction step, wavelet coefficients are assumed to be equal to zero.

The scaling coefficients depend on the choice of the trigonometric polynomial, which has a specific form for Daubechies 8 wavelets (Mallat, 1998). Once the scaling coefficients are determined, the smoothing part of Mallat's algorithm can be applied.

MRA can be represented by the diagram in Fig.1 as decomposition and reconstruction steps of a signal. The detail coefficients d_{j+1} are results of the high-pass filter while the approximated coefficients a_{j+1}

represent the smoothing part of the MRA at the $(j+1)^{th}$ level of decomposition. For the numerical experiment in this study, only the smoothing part of the MRA is used due to the fact that all detail coefficients are suppressed.

To exploit the good localization properties of the wavelets in the time and scale domains, a further investigation of the evaluated different periodic components and the proper threshold values for every level of decomposition is necessary. A complete application of wavelets for analyzing the periodic signals present in altimeter and tide gauge time series will be discussed in a future paper.

The sea level trend is parameterized in the form of a second order polynomial with respect to the time t as follows:

$$S = a + bt + ct^2\,, \tag{4}$$

where S is the time series and a, b, and c are the polynomial coefficients determined via a LS fit to the time series. From eq. (4), a linear trend is derived via differentiation with respect to time. This parameterization is applied both to the last approximation level in the wavelet-based analysis and the original signal in the LS regression method.

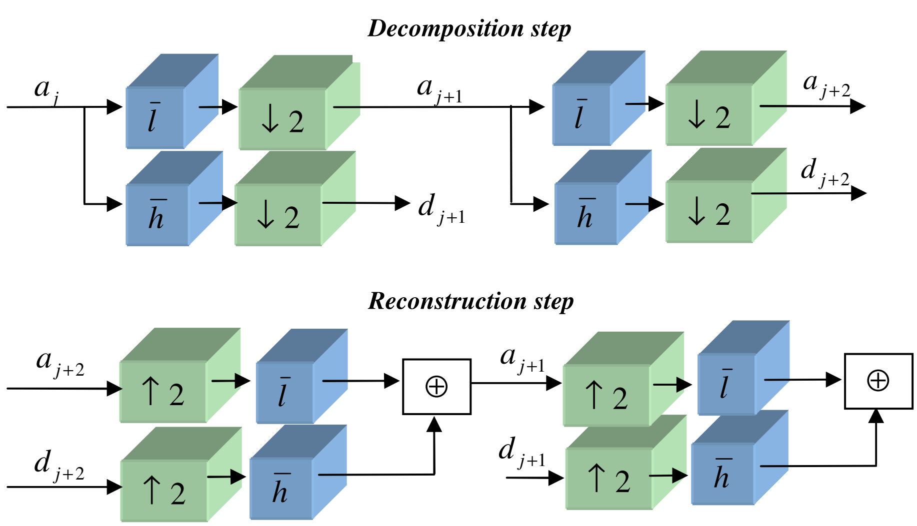

Fig.1 Decomposition and reconstruction steps in the wavelet-based multiresolution analysis (MRA).

We use TOPEX/Poseidon (T/P) time series (from 1992 to 2002 with approximately a 10-days time resolution) at the crossover points in the Baltic Sea available from the AVISO GDR-M products (AVISO, 1996). The high coherence of the sea level signal at frequencies lower than one month has been used to reconstruct the series at missing epochs (Lambeck *et al.*, 1998). Due to the ice coverage at latitudes higher than 63 degrees some of the crossover time series are discontinuous and no reliable interpolation for the missing epochs can be done. In the conducted numerical experiments, we have used crossover time series which, because of the reduced noise, are more appropriate for testing the wavelet-based analysis than the time series obtained from along-track sea surface heights. The crossover adjustment aims to minimize the radial orbit errors, unmodeled tides and other changes in the sea surface due to dynamic ocean factors. Therefore, it is likely that the crossover adjustment reduces long-periodic variations and trends, and we can expect that the sea level trend determined at the crossovers will be underestimated.

For the purpose of this study, tide gauge records relative to the Revised Local Reference (RLR) datum at each site have been obtained from the Permanent Service for Mean Sea Level (PSMSL). To estimate the secular trends, usually time series of annual means are used (see, e.g., Ekman and Stigebrandt, 1990). However, they do not provide a sufficient resolution for the wavelet analysis, especially if shorter time series are analyzed. Therefore, monthly mean values are used herein. No attempt to fill gaps larger than 3 months has been made. This fact and the space coverage of the altimetry data ($54° < \varphi < 63°$ and $14° < \lambda < 26°$) limit the number of the analyzed tide gauge series to sixteen.

3. Internal Validation of the Wavelet Multiresolution Analysis

First, the procedure has been validated by analyzing a synthetic signal. This signal contains periodic variations estimated from the Stockholm tide gauge time series chosen to represent the sea level variability of the entire Baltic Sea. The Stockholm tide gauge record exhibits small high frequency signal combined with large long-term sea level variations (Ekman, 1999). Monthly and annual mean sea levels are commonly used in studies of the seasonal variation in the sea level as well as the pole tide for the region (Ekman and Stigebrandt, 1990; Wróblewski, 2001).

The synthetic signal includes: (i) a small trend of 0.3 mm/yr; (ii) a component due to the nodal tide using estimates at the Stockholm tide gauge; (iii) a 6.5-year tidal component due to the overlapping of the pole and annual tides; (iv) annual, semi-annual, approximately bi-, and tri-monthly periodic components observed in the spectrum of the sea level signal; and (v) stationary random noise with variance of 1 mm^2. Since approximately a half period of the nodal tide component (with a period of 18.6 years) is present in the altimetry signal, the trend can be parameterized as in eq. (4) from which a linear trend of 0.43 mm/yr should be estimated (0.13 mm/yr is due to the nodal tide).

The signal has been analyzed using the Daubechies-8 wavelet (see Fig.2) and the trend has been computed from the second order polynomial fitted to the approximation at the 8^{th} level of decomposition. Next, the trend has been estimated by LS regression. Table 1 compares the results obtained from 1000 runs of both tests at the assumed noise level. From the comparison between the mean, maximum, and minimum trend given in the table, it can be seen that LS regression underestimates the trend while, after filtering out the higher frequency components, the MRA correctly estimates the required trend to be 0.43 mm/yr.

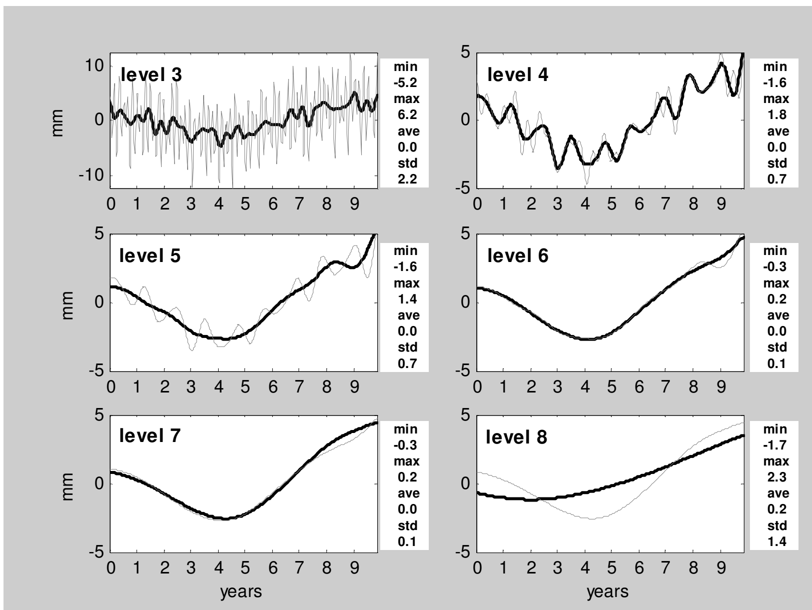

Fig.2 Successive approximations of the synthetic signal and the residuals between them. The trend is estimated at the last level eight of decomposition. Note the different scale of the Y axis for level 3.

Table1 Comparison of the trend (in mm/yr) estimated via the wavelet-based analysis and LS regression for the synthetic time series.

	mean	st dev	min	max
Wavelets	0.43	± 0.03	0.31	0.51
LS Regression	0.37	± 0.02	0.30	0.42

4. Analysis of the Altimetric and Tide Gauge Time Series

The altimetric time series have been analyzed using the same Daubechies-8 wavelet. Fig.3a shows the decomposition and successive approximations of one crossover time series. Fig.3b represents the spectra of the residuals between the successive approximations. Oscillations with a frequency of approximately two months are completely eliminated at level 3, while the semi-annual signal is removed at level 4. At level 5, most of the annual signal is detected and eliminated. Finally, at the last level (8), the oscillations due to the overlapping of the nodal and pole tides are removed, and the approximation represents the small amplitude nodal tide, possibly, plus a trend component. The trend estimated from the last-level approximation is 0.4 mm/yr while the LS regression trend is 0.1 ± 0.5 mm/yr. Table 2 compares the estimated trend for all crossover time series. Apparently, the sea level trend estimated after removing the short periodic variations is larger than the LS estimated trend. This agrees with the result obtained for the synthetic time series. The average trend in the altimetric time series is 0.6±0.1 mm/yr and is less than the mean sea level trend determined from satellite altimetry data (see, e.g., Cabanes *et al.*, 2001). The crossover adjustment is the most likely reason for that since it can reduce the amplitudes of long-term variations and trends.

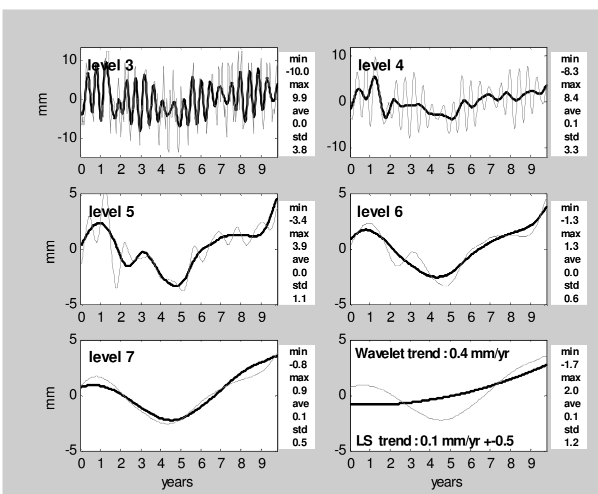

Fig.3a Example of the successive approximations of altimetry crossover time series and statistics of the residuals.

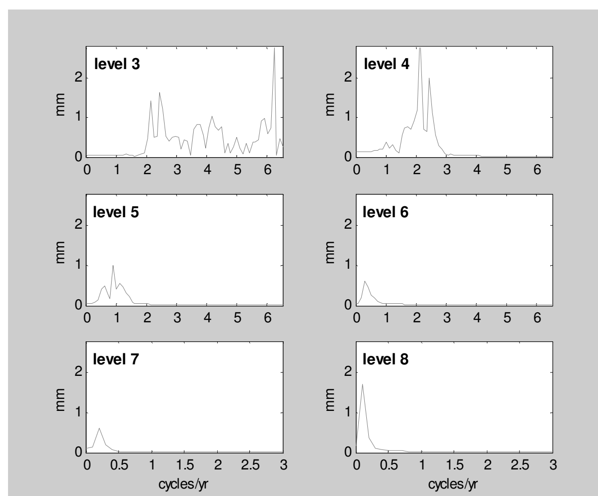

Fig.3b Spectra of the residuals between the successive approximations of the altimetry time series.

Table 2 Comparison of the linear trends (in mm/yr) estimated via wavelets and LS regression for the crossovers.

	1	2	3	4	5	6	7
Wavelets	0.36±0.00	0.50±0.01	0.22±0.02	0.50±0.00	0.68±0.01	0.44±0.00	0.67±0.01
LS Regression	0.12±0.55	0.15±0.64	0.06±0.80	0.17±0.50	0.34±1.0	0.15±0.50	0.26±0.90

Table 3 Secular trend identified in the tide gauge time series. According to the sign convention, the uplift (positive values) and subsidence (negative values) of the Earth's crust are shown.

	Ekman	1892-1991		1892 -2001		1950-2001	
	(1996)	Wavelets	LS Fitting	Wavelets	LS Fitting	Wavelets	LS Fitting
Klagshamn	0.04	-	-	-	-	-0.5 ± 0.2	-0.5 ± 1.8
Kungsholmsfort	0.20	0.2 ± 0.1	0.2 ± 0.9	0.1 ± 0.1	0.1 ± 0.7	-0.9 ± 0.2	-0.7 ± 2.1
Ölands Norra Udde	1.29	1.2 ± 0.1	1.2 ± 0.9	1.1 ± 0.1	1.2 ± 0.8	0.7 ± 0.2	0.9 ± 2.4
Landsort	3.06	3.0 ± 0.1	3.0 ± 0.9	2.9 ± 0.1	3.0 ± 0.8	2.1 ± 0.2	2.2 ± 2.5
Stockholm	4.00	4.0 ± 0.1	4.0 ± 0.9	3.9 ± 0.1	4.0 ± 0.8	3.4 ± 0.2	3.5 ± 2.5
Ratan	8.16	7.8 ± 0.1	7.9 ± 1.0	7.8 ± 0.1	7.9 ± 0.9	7.3 ± 0.2	7.4 ± 2.9
Furuögrund	8.75	-	-	-	-	7.3 ± 0.2	7.5 ± 2.9
Pietarsaari/Jakobsta	8.04	-	-	-	-	6.4 ± 0.2	6.6 ± 2.9
Kaskinen/Kasko	7.11	-	-	-	-	6.0 ± 0.2	6.1 ± 2.8
Mantyluoto	6.31	-	-	-	-	5.3 ± 0.2	5.4 ± 2.7
Rauma/Raumo	5.22	-	-	-	-	4.4 ± 0.2	4.5 ± 2.7
Turku/Abo		-	-	-	-	3.0 ± 0.2	3.1 ± 2.8
Helsinki	2.28	2.4 ± 0.1	2.4 ± 1.0	2.2 ± 0.1	2.3 ± 0.9	1.0 ± 0.2	1.2 ± 2.9
Gdansk	-	-	-	-	-	-3.4 ± 0.2	-3.0 ± 2.5
Ustka	-	-	-	-	-	-2.1 ± 0.2	-1.8 ± 2.4
Warnemünde 2	-1.06	-1.2 ±0.1	-1.3 ± 0.5	-1.3 ±0.1	-1.2 ± 0.4	-1.1 ± 0.2	-1.4 ± 1.4

The tide gauge records have been first analyzed for the time interval from 1892 to 1991. The last decade of the 20th century, which coincides with the time span of the altimetric measurements, is characterized by low-water years and usually is excluded from analyses of the sea level rates for the region (Lambeck *et al.,* 1998). In Table 3, our estimates and those published by Ekman (1996) for the same time interval but using annual mean values are compared. According to the adopted sign convention, the table represents the relative uplift/subsidence of the Earth's surface with respect to the sea level. The differences between Ekman's trend estimates and the ones estimated in this study (using monthly means) is within the formal error of the LS regression. Furthermore, both wavelets and LS regression give very close estimates, which demonstrates that for long time series and when no anomalous sea level signal is present wavelets and LS regression give identical results. Latter in this section, we demonstrate that wavelets are capable of modeling the anomalous signal present in the sea level time series. In the case of shorter time series, this leads to larger estimated trends.

Figures 4a and 4b show the decomposition and the spectra of the residuals between the approximations of the sea level signal at the Stockholm tide gauge. The annual and semi-annual signals are removed at level 3, while long-term oscillations are still present in the approximation at level 8. Similarly to the altimetric time series, a second order trend curve is fitted to the last-level approximation. From Table 3, it is evident that both the wavelet-based analysis and LS regression estimate a trend of 4.0 mm/yr. The much smaller error for wavelets is due to the smaller residuals after filtering out high frequency oscillations from the signal.

The addition of data from the last decade of the 20th century to the longest continuous records reduces the secular trend by 0.1 mm/yr at most of the sites (see Table 3), but this difference is still within the estimation error. If the time series from 1950 to 2001 (from 1951 to 1999 for the tide gauge sites Gdansk and Ustka) are analyzed, the estimated trend is lowered by 0.8 mm/yr on average. For the tide gauges on the south coast of the Baltic Sea, where smaller rates are observed, the differences between the wavelet-based analysis and the LS regression become larger than the formal errors. For the tide gauge Gdansk, LS regression predicts a smaller rate of sea level rise compared to the wavelet analysis (see Fig. 5). Because of the good localizing properties of wavelets in time domain, the low-water signal in the last decade of the record (see the box in Fig.5) is detected and modeled. The polynomial curve fitted to the last-level approximation accounts for this local effect and the estimated trend is larger. This is observed also for Ustka and Kungsholmsfort, but not for Klagshamn and Warnemünde. Because the first two tide gauge sites have larger estimation errors (see Table 3), it is not clear whether the larger trend estimated by the wavelet-based analysis is realistic. Additional numerical experiments and analysis are necessary in order to establish the advantages of using wavelets in the presence of an anomalous sea level signal especially in the case of short and noisy time series.

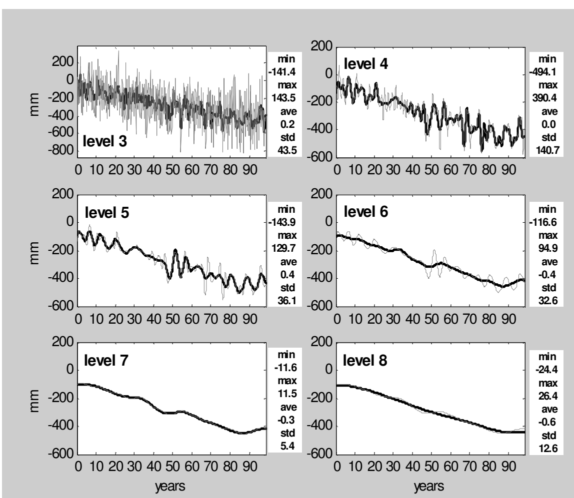

Fig.4a Successive approximations of the Stockholm tide gauge time series and statistics of the residuals. Note the different scales of the Y axis.

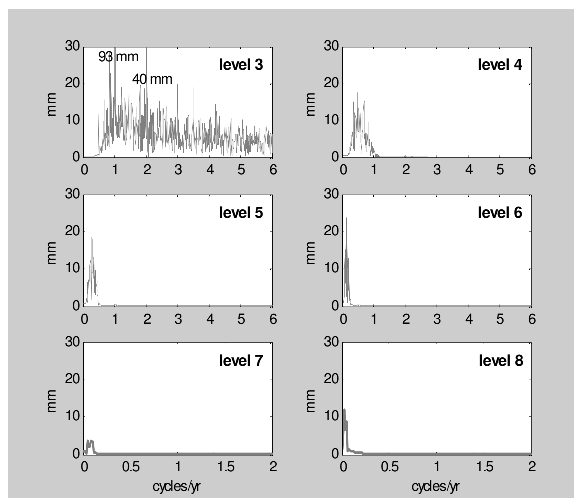

Fig.4b Spectra of the residuals between the successive approximations of the time series.

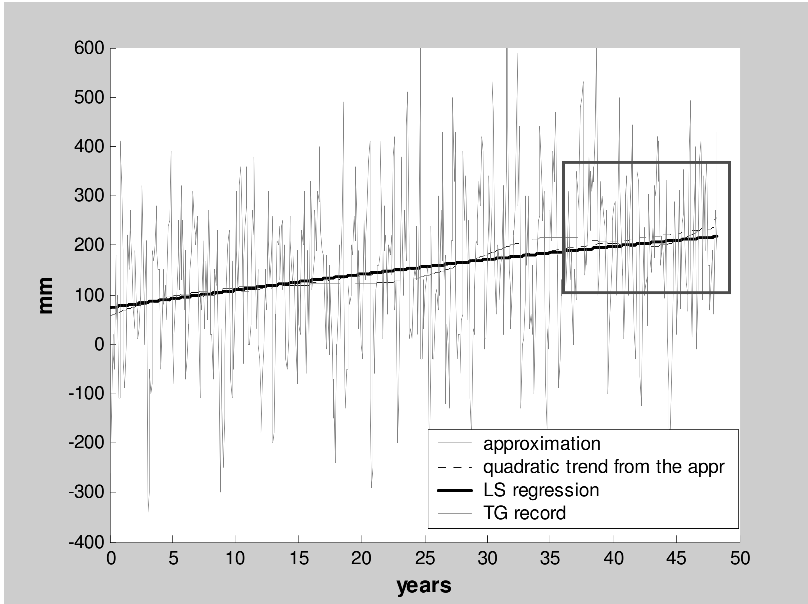

Fig. 5 Sea level trend determined by the wavelet-based analysis and LS regression from the tide gauge time series at Gdansk (1951-1999).

6. Discussion and Conclusions

To compute the absolute rates of the uplift/subsidence, the sea level trend estimated at the altimetry crossovers must be interpolated and added to the sea level rates relative to the Earth's surface according to the sign convention in Table 3. No such attempts have been made in this study. No conclusions about the absolute crustal uplift/subsidence can be drawn based on an analysis performed at a very limited number of crossover points across the Baltic Sea. On the other hand, time series of sea surface heights averaged around tide gauge sites can be created and used in the wavelet-based analysis. Nevertheless, this numerical experiment has demonstrated that LS regression tends to underestimate the sea level trend when short and/or noisy time series are analyzed. In the presence of an anomalous sea level signal, the wavelet-based approach can detect and model this signal and accounts for this effect on the trend.

In order to draw more definite conclusions about the application of the wavelet analysis for estimating sea level trends, a comparison with other smoothing methods is necessary. One such method is the robust non-parametric smoothing based on locally weighted regression; see Barbosa *et al*. (2004). As pointed out by the authors, this non-parametric method offers more flexibility in describing non-linear trends than LS regression, and can be very effective in analyzing short sea level time series.

As discussed herein, the results obtained must be considered as a preliminary step in our attempts to apply the wavelet-based multiresolution analysis in order to determine sea level trends from altimetry data for the region of the Baltic Sea. Based on these results, there exist indications that filtering out high frequency oscillations (partly oceanographic signal) from the short-term sea level records can improve the estimated trend. If proven to provide reliable estimates for sea level trends, the suggested procedure can be incorporated in the determination of the absolute rates of vertical crustal motion associated with postglacial rebound in the region.

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References

AVISO (1996), "AVISO User Handbook for Merged TOPEX/POSEIDON products", AVI-NT-02-101, Edition 3.0.

Barbosa, S.M., M.J. Fernandes, and M.E. Silva (2004), Nonlinear sea level trends from European tide gauge records, *Annales Geophysicae*, 22, pp. 1465 - 1472.

Cabanes, C., A. Cazenave, and C. Le Provost (2001), Sea level rise during past 40 years determined form satellite and in-situ observations, *Science*, 294, pp. 840 - 842.

Cazenave, A., K. Dominh, F. Ponchaut, L. Soudarin, J.F. Cretaux, and C. Le Provost (1999), Sea level change from Topex-Poseidon altimetry and tide gauges, and vertical crustal motion from DORIS, *Geophys. Res. Lett.*, 26(14), pp. 2077 - 2080.

Ekman, M. and A. Stigebrandt (1990), Secular Change of the Seasonal Variation in Sea Level and of the Pole Tide in the Blatic Sea, *J. Geophys. Res.*, 95(C4), pp. 5379 - 5383.

Ekman, M. (1996), A consistent map of the postglacial uplift of Fennoscandia, *Terra Nova*, 8, pp. 158 - 165.

Ekman, M. (1999), Climate changes detected through the world's longest sea level series, *Global and Planetary Change*, 21, pp. 215 - 224.

Jekeli, C. and P. Dumrongchai (2003), On monitoring a vertical datum with satellite altimetry and water-level gauge data on large lakes, *J. Geodesy*, 77, pp. 446 - 453.

Keller, W. (2000), Applications of wavelets, Lecture notes, The University of Calgary, Calgary, Canada, pp. 78 - 82

Kuo, C.Y., C.K. Shum, A. Braun, and J.X. Mitrovica (2004), Vertical crustal motion determined by satellite altimetry and tide gauge data in Fennoscandia, *Geophys. Res. Lett.*, Vol.31, L01608, doi:10.1029/2003GL019106.

Lambeck, K., C. Smither, and M. Ekman (1998), Tests of glacial rebound models for Fennoscandinavia based on instrumented sea- and lake-level records. *Geophys. J. Int.*, 135, pp. 375 - 387.

Mallat, S. (1998), A wavelet tour of signal processing, Academic Press, New York, USA.

Nerem, R.S. and G.T. Mitchum (2001), Observations of sea level change from satellite altimetry. *Sea Level Rise: History and Consequences*, B.C. Douglas, M.S. Kearney, and S.P. Leatherman, Eds., Academic Press, pp. 121-347.

Nerem, R.S. and G.T. Mitchum (2002), Estimates of vertical crustal motion derived from differences of TOPEX/POSEIDON and tide gauge sea level measurements, *Geophys. Res. Lett.*, 29(19), 1934, doi:10.1029/2002GL015037, 2002.

Shum, C.K, C. Kuo, and J.X. Mitrovica (2002), Glacial isostatic adjustment in the Great Lakes region inferred by tide gauges and satellite altimetry, *EOS Trans.*(Abstract), Spring AGU Meeting, Washington D.C., May 28-31.

Wróblewski, A. (2001), Lunar nodal tide in the Baltic Sea, *Oceanologia*, 43(1), pp. 99 - 112, 2001.