A gravimetric quasi-geoid evaluation in the Northern region of Algeria using EGM96 and GPS/Levelling

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Abstract

The use of GPS for the estimation of orthometric heights in a given region, with the help of existing levelling data requires the determination of a local geoid and the bias between the local levelling and the one implicitly defined when the geoid is calculated which is generally based on the gravity anomalies data. The heights of new data can be collected swiftly without using the orthometric heights from levelling; it is what one calls commonly levelling by GPS.

In this framework, the Least Squares Collocation method (LSC) has been used to evaluate the quality of the available GPS-Levelling data, to determine a gravimetric geoid in the North region of Algeria and to estimate the constant datum bias.

The data used in the setting of this study are: The geopotential model EGM96, a total number of 2534 gravity anomalies, as well as 43 GPS points connected to the geodetic network levelling present on the whole North part of Algerian.

Keywords: Least Squares Collocation, GPS-Levelling, gravimetric Data, local geoid.

1. Introduction.

The use of the GPS for the determination of the orthometric heights in regions where the levelling exist requires the determination of a local geoid and the estimation of the height constant bias between the gravimetric datum and local levelling since the local geoid will have its own zero level, whereas the data of levelling can be local, or national.

The bias constant is estimated while calculating the mean of discrepancies between the differences of the GPS-Levelling and geoid. However, these values can have some irregularities in the spatial distribution; we must therefore counterbalance these discrepancies while taking in account their spatial correlations, this problem can be handled by the use of the Least Squares Collocation method (LSC).

In the present paper this work is described by the use of the available data in Northern Algeria spreads from 32° in 37° N in Latitude and -4° in 10° E in Longitude.

The main problem in this region is the one of the quality of the gravity data used and of GPS-Levelling. It has been necessary to withdraw the erroneous gravity values and to make a selection of GPS-Levelling points after a comparison between the observed values and those predicted by gravity.

2. Gravity data and GPS-Levelling.

In practice, when one determine a gravimetric geoid locally, one must represent the neighbourhood of the gravity field by the use of a geopotential model of superior degree and order, in our case of study the harmonic spherical EGM96 has been used. Its contribution must be subtracted from the local data and must be restored thereafter.

During the realization of this work, the only gravity data that were available were those provided by the International Gravimetric Bureau (BGI). We withdrew from the data file the duplicated gravity values as well as those appeared doubtful.

As Digital Elevation Model (DTM), we had used the GTOPO30, the thinnest DTM that was available, of 1kmx1km resolution.

43 GPS-Levelling points with accuracy of ±50 cm have been used; these points have the advantage to be well distributed on the zone of study, however after a comparison between the observed values and those predicted by gravity, some points gave a difference up to the doorstep fixed beforehand according to the accuracy of the GPS-Levelling and the density of points. These points have been rejected before redoing another comparison.

The EGM96contribution has been subtracted from the local gravity data. The statistics of the differences are presented in the table1.

Table 1. Statistics of the EGM96 contribution on the 2534 free air anomalies.

mGal	Mean	Std. Deviation
Observations	**28.77**	**30.18**
EGM96	**34.14**	**23.90**
Differences	**-5.37**	**23.64**

A substantial agreement between EGM96 and the local gravity data have been obtained.

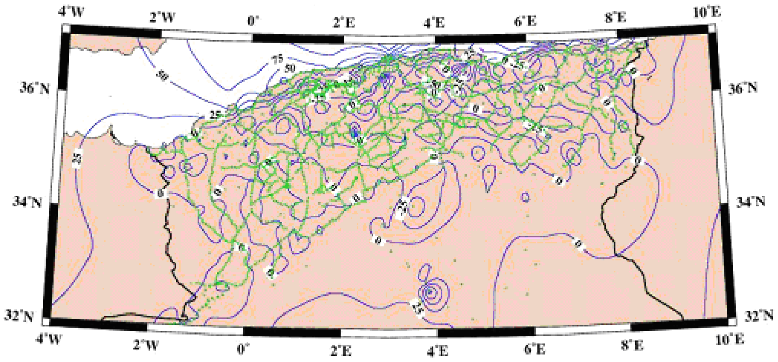

Fig.1 Free air anomalies after subtraction of the EGM96 contribution. Units mGal.

The RTM terrain effect reduction has been computed using a detailed 30”x30” grid and the outer zone height grid 5’x5’ from the reference height grid 30’x30’.

After the subtraction of the EGM96 contribution and RTM terrain effect we obtain the residual anomalies by the following relation:

$$\Delta g_{Rés} = \Delta g_{fa} - \Delta g_{EGM96} - \Delta g_{RTM} \tag{1}$$

Table2. Statistics of the RTM subtraction from the reduced anomalies. Units mGal.

#	Min	Max	Mean	Std. Dev.
$\Delta g_{\mathrm{Re}\,sidual}$	**-112.66**	**102.32**	**0.79**	**22.48**

3. Least Squares Collocation.

The LSC solution is gotten under the following shape:

$$T(P) = \sum_{i=1}^{N} b_i \cdot \mathrm{cov}(T(P), L_i),$$

$$(b_i) = (\mathrm{cov}(L_i, L_j) + \sigma_{ij})^{-1} \cdot (x_j) = \overline{C}^{-1} \cdot x_j \tag{2}$$

Where T is the local approximation of the anomalous potential, x_j are the observations and σ_{ij} are the errors of covariance. The covariance is represented by the expression that follows in which the constant R (Radius of the Bjerhammar sphere), ***a*** and ***A*** are determined from the local gravity data.

$$\begin{aligned}\operatorname{cov}(P,Q) &= \operatorname{cov}(r,r',\psi) \\ &= a\cdot\sum_{k=2}^{K}\sigma_{EGM}^{2}\left(\frac{R^{2}}{rr'}\right)^{k+1}P_{k}(\cos\psi)+\sum_{k=K+1}^{+\infty}\frac{A}{(k-1)(k-2)(k+4)}\left(\frac{R^{2}}{rr'}\right)^{k+1}P_{k}(\cos\psi)\end{aligned} \quad \textbf{(3)}$$

P and Q are two points between a spherical distance, and r, r' are the distances of the two points from the origin.

Initially an empiric covariance function has been determined of the reduced gravity anomalies.

The estimated values have been adapted in that case to a covariance model by an iterative adjustment with the 3 parameters R, $\boldsymbol{a}$ and $\boldsymbol{A}$.

The limit summation has been fixed to 250, the coefficients bigger than 250 didn't give reliable information in the region. The depth of the Bjerhammar sphere ($R - R_B$) has been estimated to -3.736 km and the total variance of gravity anomalies to 603.36 mgal².

The Figure2 shows the empirical and analytical covariance functions.

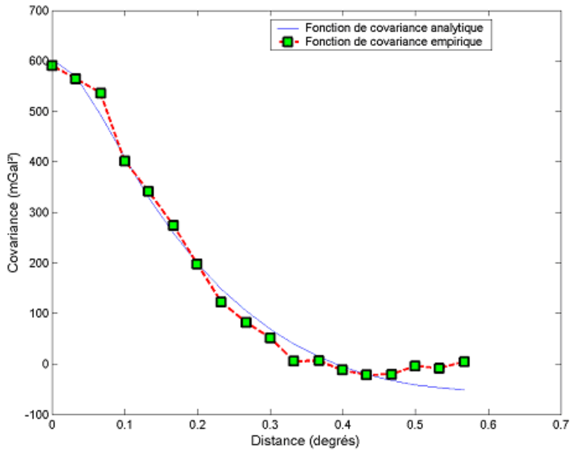

Figure 2. Covariance functions of reduced gravity anomalies.

The empirical covariance function of the residual gravity anomalies has as well been determined and the empirical values have been adapted to a covariance model.

The limit summation for this time has been fixed to 260, The depth of the Bjerhammar sphere ($R - R_B$) has been estimated to -9.961 km and the total variance of gravity anomalies to 432.83 mgal².

The figure 3 shows the empirical and analytical covariance functions.

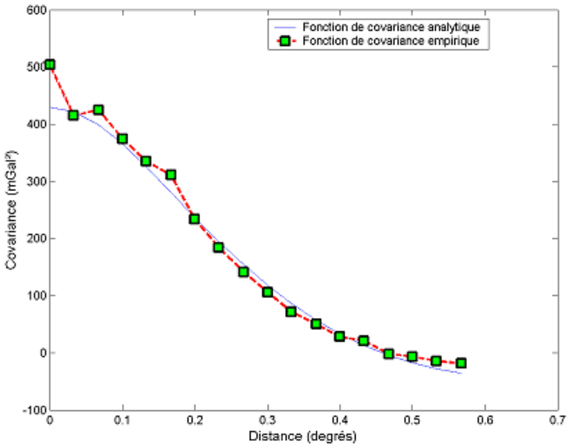

Figure 3. Covariance functions of residual gravity anomalies.

The 2534 reduced and residual gravity anomalies have been used therefore to determine an estimated of T, from which the estimates of the geoid heights on the 43 benchmarks of GPS-Levelling have been computed. The results of the differences between observed values and predicted are presented in the table3.

Table 3. Statistics of the differences between predicted and observed values. Units m.

43 points	Min	Max	Mean	Std.Dev.
GPS-Levelling	32.48	49.82	44.53	3.93
Prediction by reduced gravity	32.24	50.60	44.09	4.12
Differences	-1.40	3.05	0.44	1.10

Table 4. Statistics of the differences between predicted and observed values. Units m.

43 points	Min	Max	Mean	Std.Dev.
GPS-Levelling	32.48	49.82	44.53	3.93
Prediction by residual gravity	33.09	50.68	44.66	4.07
Differences	-2.54	2.93	-0.13	1.35

The observation must be rejected if:

$$\left| N_{GPS} - N_{\Delta g} \right| \geq 3 \cdot (C_{pp} - C_{pi}^{\ T} \left\{ \overline{C}_{ij} + Err_{ij}^{\ 2} \right\}^{-1} C_{pj})^{½}$$ and ≥ to the fixed doorstep.

This has given several suspected errors, six observations have been rejected, the results of differences between the retained observed values and predicted are presented in the tables 5 and 6.

Table 5. Statistics of the differences between predicted and observed values. Units m.

37 points	Min	Max	Mean	Std.Dev.
GPS-Levelling	**32.48**	**49.82**	**44.41**	**4.13**
Prediction by reduced gravity	**32.24**	**50.60**	**44.27**	**4.35**
Differences	**-1.40**	**1.47**	**0.14**	**0.85**

Table 6. Statistics of the differences between predicted and observed values. Units m.

37 points	Min	Max	Mean	Std.Dev.
GPS-Levelling	**32.48**	**49.82**	**44.41**	**4.13**
Prediction by residual gravity	**32.27**	**50.74**	**44.28**	**4.31**
Differences	**-1.52**	**1.68**	**0.12**	**0.92**

The residual geoid undulations were determined by LSC, where the required auto and cross-covariance functions are computed by covariance propagation from the modelled local covariance function.

The residual quasi-geoid is represented in the figure 4.

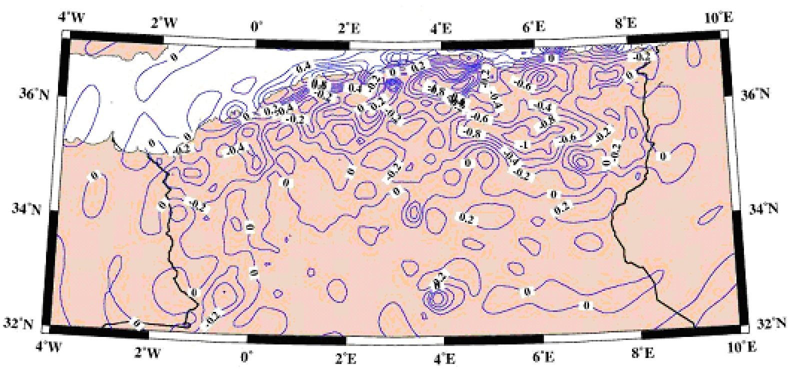

Figure 4. Residual quasi-geoid. Units m.

After restoring the long and short wavelength signals we obtain the final quasi-geoid presented below in the figure 5.

$$N = N_c + N_{EGM96} + N_{RTM} \quad \textbf{(4)}$$

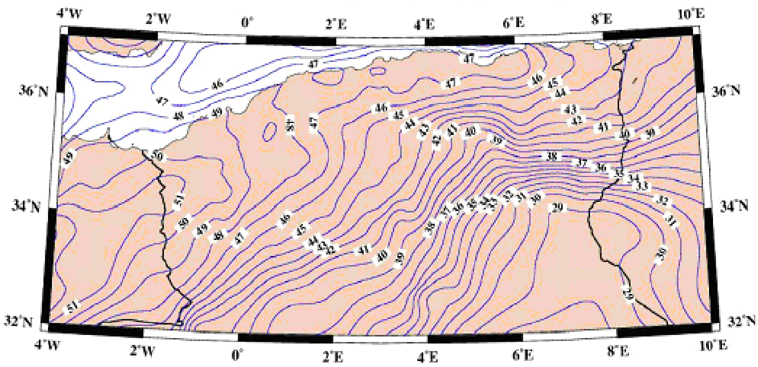

Figure 5. Final quasi-geoid. Units m.

4. Constant bias.

The parameters of which depend the used data, as the difference between the datum of the geoid and the local levelling, can be determined by LSC. The observations are tied to T and the vector of the X parameters by the following equation:

$$x_k = L_k(T) + A_K X + \varepsilon_k \qquad \textbf{(5)}$$

Where L_k is associated to the observation, A_k is a vector with the elements 0 or 1, X is the vector parameter and ε_k is the error of observation.

So :

$$X = (A^T \overline{C}^{-1} A)^{-1} \cdot A^T \overline{C}^{-1} x_i \qquad \textbf{(6)}$$

The constant bias between the gravimetric datum and the local levelling has been estimated, using residual data.

The results are presented in the tables below.

Table 7. Statistics of differences between the residual gravimetric quasi-geoid undulations and GPS-levelling at 37 control points (in meters) before fitting.

Differences ζ rtm observed – ζ rtm predicted (m)	Before fitting
Mean	-.382
Std.Dev	1.079

Table 8. The parameter transformation model.

Data	Parameter		
Units : m		**Values**	**Error estimates**
No	Constant bias	.128	.108

Table 9. statistics of differences between the residual gravimetric quasi-geoid undulations and GPS-levelling at 37 control points (in meters) after fitting.

Differences ζrtm observed – ζ rtm predicted (m)	After fiting
Mean	.000
Std.Dev	.244

4. Conclusion.

The gravimetric geoid has been computed by the Least Squares Collocation method.

The subtraction of EGM96 gave the expected results, the variance and the mean value decreased significantly. The gravity RTM subtraction didn't reduce the variance a lot on the other hand the mean value has been reduced. This is due probably to the quality of the DTM used.

The expected errors of the GPS-Levelling data are (±0.5 m), due mainly to the errors in the levelling.

However, after the EGM96 subtraction and prediction by gravity, large differences (around 3 m) have been obtained between observed and predicted values for six stations. It might be antenna height problems or erroneous identification of the levelling points. It might also be due to tectonic movements in the period between the levelling and the GPS.

Through this study we could see that the Least Squares Collocation method offers an important alternative to achieve an optimal evaluation in a stochastic process and to detect subsequently the gross errors of gravity data and GPS-Levelling.

In addition and at last, the constant bias No between the gravimetric datum and the local levelling has been estimated to (0.128 m) with error estimate of (0.108 m).

The local gravimetric geoid determined in the setting of this study deserves to be improved, by a densification of the gravimetric cover, the use of a more precise DTM and GPS-Levelling data with better quality.

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