Latest Geoid Determinations for the Republic of Croatia

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Abstract. *Due to several improvements that have been done in comparison to the previous solution, like the usage of better terrestrial gravity data at the Adriatic Sea, much more GPS/leveling points available, wider topography integration area and better residual terrain modeling procedure, one-block collocation for entire area, and finally, four times denser computation grid, have resulted in the official geoid solution for Croatia HRG2000. This solution is based on the long wavelength gravity field structures from EGM96 global geopotential model.*

Recent CHAMP and GRACE satellite missions are defining new standards in modeling of the Earth's gravity field, improving global gravity field models especially in long- and medium-wave range. This was confirmed for the Croatian territory thanks to the undertaken comparison of several such models with GPS/leveling and HRG2000 geoid data.

Based on these facts and the availability of a new and much denser point gravity data set over the land area, newest geoid computation for the Republic of Croatia become possible. This paper offers a detailed description of the applied computation procedure, the geoid quality estimation using GPS/leveling points, and the presentation of specially developed computer program made for the purpose of geoid interpolation in any area of the state.

Keywords. *Geoid, gravity, GPS/leveling, terrain modeling, satellite missions, HRG2000 geoid solution*

1 Introduction

First serious attempts in determination of geoid surface in this region are connected with the establishment of independence of the Republic of Croatia. Nevertheless, a significant improvement was realized in 1998, when the first HRG98 solution was presented at the EGS 23rd General Assembly in Nice (Bašić et al. (1999)), and immediately after that HRG98A modified solution at the 2nd Joint Gravity and Geoid Meeting in Trieste (Bašić and Brkić (1999)).

During the year 2000 next step forward was made in the Department for Geomatics at the Faculty of Geodesy University of Zagreb, resulting with the most recent geoid solution HRG2000, see Bašić (2001). Since HRG2000 was proclaimed by the State Geodetic Administration as the official geoid surface of the Republic of Croatia, in the continuation the short presentation of this solution and the special computer program for interpolation purpose is given, followed by an overview of the latest efforts in the preparation for the calculation of the new solution HRG2005.

2 Computation of HRG2000 geoid

2.1 Previous investigations

In the frame of preparatory work (Hećimović (2001)), the numerical investigation with 14 global Earth Geopotential Models (EGM) was undertaken with the goal to find the model that best fits the Earth gravity field on the territory of Croatia. As reference values, GPS/leveling geoid undulations on 121 points were used. It was found out that global geopotential models EGM96 and GFZ97 fit best the Earth gravity field on our territory (see Table 1). In order to determine the existence of constant vertical shift between the Croatian vertical datum and EGM96 and GFZ97, two transformation models were defined. The transformation model which includes zero-undulation N_o fits better the real data and improves the existence of constant vertical displacement between GPS/leveling undulations and EGM96 and GFZ97 geoid of –1.37 m and -1.28 m respectively, for more details see Hećimović and Bašić (2003).

Another interesting investigation was the examination of the influence of the resolution change in reference Digital Terrain Model (DTM) on the calculation of short-wavelength effects (Residual Terrain Modeling - RTM) in gravity anomalies (Hećimović (2001), which showed that the use of 20′x30′ reference DTM yielded residual gravity anomalies with the best statistical properties to apply in the collocation (small and smooth residual gravity field).

Table 1. The statistics of differences between GPS/leveling and different EGM geoid undulations (in m).

Model	Min	Max	Mean	St.
EGM96	-2.43	0.21	-1.33	0.44
GFZ97	-2.44	0.22	-1.21	0.47
OSU91A	-4.43	2.07	-0.62	1.16
IFE88E2	-3.42	1.41	-0.55	0.93
GFZ93A	-2.97	1.74	-0.62	0.85
GFZ93B	-3.08	1.67	-0.69	0.85
GPM2	-4.45	2.08	-0.80	1.47
GRIM4	-3.37	2.28	-0.53	1.14
GEM-T3	-4.21	0.76	-1.65	1.11
JGM-1S	-3.85	1.07	-1.99	1.00

2.2 Calculation method

The strategy for high-resolution local gravity field determination is using three parts of the gravity field information: the long-wavelength part is taken from the global geopotential model, the medium-wavelength part originates in terrestrial point gravity field observations like free-air gravity anomalies and GPS/leveling data, and the short-wavelength part is taken from the high-resolution digital terrain model. In the simple remove-restore technique, the reduced observations are thus written as linear functional of the anomalous gravity potential T (Bašić (1989)):

$$x_i = \mathrm{L_i}(T) - \mathrm{L_i}(T_{EGM}) - \mathrm{L_i}(T_{RTM}) + n_i \, . \qquad (1)$$

The least squares collocation determines the approximation:

$$\tilde{T}'(P) = \mathbf{C}_P^T \, (\mathbf{C} + \mathbf{D})^{-1} \, \mathbf{x} \, . \qquad (2)$$

Here P is a point in space, matrix **C** contains the signal co-variances between the observations, $\mathbf{C}_P$ contains the signal co-variances between the observations and predicted $\tilde{T}'$ value in the point P, and **D** is the variance-covariance noise matrix.

The least square collocation technique results in predictions $L_j(\tilde{T})$. To obtain the desired results, the effect of the anomalous masses and the effect of the geopotential model need to be added back through the restore procedure:

$$L_j(\tilde{T}) = \mathrm{L_j}(\tilde{T}') + \mathrm{L_j}(T_{EGM}) + \mathrm{L_j}(T_{RTM}) \, . \qquad (3)$$

We decided to use collocation technique because the territory of Croatia is a relatively small area, so the huge and extensive numerical operations were possible to be done in one-step due also to their flexibility in handling heterogeneous irregular spaced data. In addition, we preferred to have the error estimates of predicted quantities (Bašić (1989)).

2.3 Data sets used

Since the global geopotential models EGM96 fit best the Earth gravity field on our territory (Table 1), it is used for definition of long-wavelength structures. This model consists of spherical harmonic coefficients complete to degree and order 360 (Lemoine et al. (1996)).

In performing the residual terrain modeling, three grids were put to use: the detailed model of topography 4"x5" (approx. 120x110 m), covering an area from 41° to 48° in latitude, and 12° to 21° in longitude, the coarse 5′x5′ grid of relief heights covering an bigger area from 32° to 57° and from 0° to 33° in latitude and longitude respectively, and 20′x30′ RTM reference grid of the same area as the coarse one. For the topographic masses the constant density of 2670 kgm^{-3} is assumed. These digital terrain models were applied in well-known Forsberg's TC software (Forsberg (1984)) for computation of terrain effects on gravimetric quantities.

For the calculation of free-air gravity anomalies point gravity data has been applied over the continental part of Croatia, Slovenia and Bosnia and Herzegovina, as well as Hungary and Italy (but rare). The gravity anomalies over the Adriatic Sea have been derived in 5′x5′ grid from map 1:750 000 (Morelli et al. (1969)), while the data covering Serbia and Montenegro have been recalculated in 5′x5′ grid from Bouguer anomaly maps 1:200 000. In this way a gravity data bank with more than 7500 free-air anomalies was created (Bašić (2001)).

In Table 2 the main statistics of gravity anomalies is presented, where the effects of the applied remove procedure decreasing the standard deviation from 34.53 mgal for observed anomalies to 12.40 mgal for residuals (Δg_{OBS}-Δg_{EGM96}-Δg_{RTM}) is evident. A significant reduction of the mean value from 8.70 mgal to 0.47 mgal (good centered data) can be recognized too (1 mgal = 10^{-5} ms^{-2}).

Table 2. Statistics of gravity anomalies and their residuals (in $10^{-5}ms^{-2}$)

	Δg_{OBS}	Δg_{EGM}	Δg_{RTM}	Δg_{RES}
Mean	8.70	14.27	-6.04	0.47
St.Dev.	34.53	29.15	20.21	12.40
Min	-103.07	-95.59	-119.35	-47.04
Max	180.14	109.33	106.80	64.31

A priori information about the variation of the local gravity field is introduced through the empirical covariance function calculated using residual gravity anomalies. In our case the variance of the empirical covariance function has the value of 154.09 $mgal^2$ and the first zero-value occurs already at the distance of 50 km (Bašić (2001)).

For the purpose of correct absolute orientation of the calculated geoid surface, a limited number (138) of GPS/leveling points distributed across the Croatia has been used. The statistics are presented in Table 3, where an apparent remove effect is present again, but it should be noted that the value of the mean N_{RES} = -2.16 m, most likely originates from the discrepancy between the used EGM96 model and the definition of national vertical datum, which is related to the tide gauge in Trieste.

Table 3. The statistical characteristics of geoid reduction effect in 138 GPS/leveling points (in m)

	$N_{GPS/l}$	N_{EGM9}	N_{RTM}	N_{RES}
Mean	44.41	45.78	0.81	-2.16
St.Dev.	1.22	1.07	0.23	0.33
Min	39.65	40.35	0.61	-3.18
Max	46.88	47.63	1.67	-1.50

2.4 Geoid prediction

The actual computation area has been chosen to cover the entire territory of Croatia: from 42.0° to 46.6° in latitude, and 13.0° to 19.5° in longitude, with calculation grid of 1′x1.5′ (approx. 1.8x2.0 km) or total 72 297 prediction points. As a final product selected HRG2000 with 36 184 points was defined covering strictly the Croatian territory (Fig. 1). Table 4 gives the main statistics for the selected HRG2000 geoid, their error estimates and belonging height information. For the predominant part of selected area the geoid undulations have the standard deviations of 1-2 cm and only in the Central Adriatic Sea they are up to 10 cm (Bašić (2001)).

The comparison of HRG2000 with former HRG98 solution (Bašić et al. (1999) shows significant differences resulting from the application of more reliable gravity anomalies over the Adriatic Sea, considerably more GPS/leveling points (138), better residual terrain modeling (20′x30′ reference DTM and farther integration up to 2000 km), as well as successful realization of one step collocation.

Furthermore, calculation grid in HRG2000 solution is four times denser. Although we met dramatically huger demands in numerical processing of topography data as well as in collocation, everything was successfully done on an average personal computer.

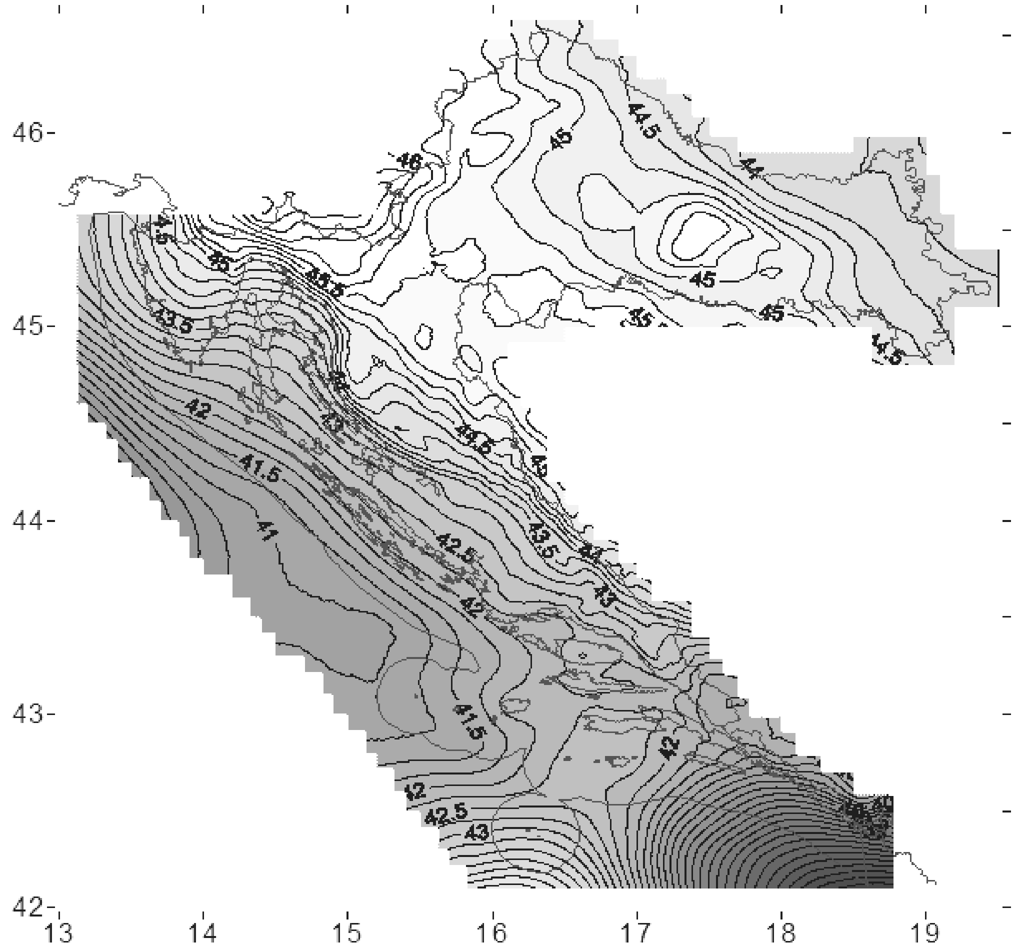

Fig. 1 Selected HRG2000 geoid surface (m).

Table 4. Statistics for selected HRG2000 geoid undulations, their standard deviations and corresponding heights (36184 data, in m)

	N	St. Dev.	$H_{HRG2000}$
Mean	43.2	0.02	118.9
St.Dev.	1.9	0.02	347.9
Min	36.3	0.01	-1169.0
Max	47.0	0.11	1781.0

3 Computer program for interpolation

Resulting from an increasing number of GPS-technology users in Croatia, a special scientific and professional project has been made by the State Geodetic Administration and the Department for Geomatics at the Faculty of Geodesy that resulted with the computer program: IHRG2000. The program is written in Microsoft Visual Basic 6.0 and supports all the latest Windows platforms. The basic purpose of the program is the interpolation of HRG2000 geoid (Fig. 1) and the presentation of the results on screen, their storage on a disc and a print out. In the IHRG2000 program there are two possibilities of interpolation: bilinear and spline (Bašić and Šljivarić (2003)).

After initiating the program IHRG2000, the *initial form* in Croatian language is shown on the screen (Fig. 2). The program is ready for data input and for the changes in computation parameters. The most important input values are latitude and longitude in DEG (degrees and decimal degree parts) or DMS (degree, minutes and seconds) units. They are entered in three ways: by keyboard, disc, and from a map.

Fig. 2 Initial form of IHRG2000 program

The utility programs IHRG2000, created in the Department for Geomatics, to be used by the State Geodetic Administration of the Republic of Croatia should find their application in the practice, especially because the application of modern geodetic technologies requires that. Without knowing the accurate geoid surface it is impossible to make the connection between ellipsoid heights that are today very accurately provided by GPS technology, and orthometric heights that are used in practice. The licensing system of the program by the State Geodetic Administration provides the uniqueness and official character for this computer program, as well as for the data processed and realized using this program.

4 Preparations for the calculation of the new geoid

In 2004 the work was continued on finding even better solution for the geoid surface in Croatia. Within the frame of these efforts, the analysis of CHAMP and GRACE geoid solutions on our territory was made (Hećimović and Bašić (2004a)), the new digital terrain model Shuttle Radar Topography Mission (SRTM) was used for computing topographic effects of the Earth gravity field and compared with the so far used DTM (details in Hećimović and Bašić (2004b)), and the most important of all, the situation in gathering and quality control of the new gravity data has been significantly improved in this area.

4.1 CHAMP and GRACE geoid models

Recent CHAMP and GRACE satellite missions define new standards in modeling gravity field of the Earth. Gravity signals that were out of sensitivity band of previous measurement techniques are becoming clearly recognized. CHAMP and GRACE are improving global gravity field models especially in long-wave and middle-wave range.

To estimate how well CHAMP and GRACE geopotential models fit gravity field in Croatia, the comparison of seven CHAMP and three GRACE models with GPS/leveling undulations (121 points) and HRG2000 geoid in 1'x1' raster has been made. GRACE models show better fitting of gravity field than CHAMP models (see Tables 5 and 6). The differences from gravity field with CHAMP models are strongly correlated with topography. The differences of GRACE gravity models are containing higher amount of short-wave topography gravity signal than CHAMP models, but the discrepancies from a more precise topography structure can be recognized. GRACE model GGM01C is the best fitting gravity field (Fig. 3 and Table 6), details in Hećimović and Bašić (2004a).

Table 5. The main statistical characteristics of differences between GPS/leveling and recent global geoid models (121 points; in m)

Model	Min	Max	Mean	St. Dev.
EIGEN-2	-4.70	2.30	-1.34	1.35
EIGEN-3p	-3.40	1.89	-1.07	1.04
TUM-1S	-3.74	1.57	-1.26	1.15
TUM-2Sp	-4.16	1.98	-0.62	1.29
ITG-CHAMP01E	-3.50	1.88	-0.91	1.07
ITG-CHAMP01S	-4.11	1.72	-0.89	1.13
ITG-CHAMP01K	-4.13	1.69	-0.88	1.16
GRACE01S	-2.51	1.13	-1.08	0.80
GGM01S	-2.59	1.33	-1.06	0.82
GGM01C	-2.02	0.17	-1.00	0.45

Obtained wavelengths of differences are in short wave ranges depending on the terrain behavior. In the higher mountain area the differences are bigger than in the flat area. As the new CHAMP and GRACE global geopotential models represent much better the long and medium wave gravity field structures at the Croatian territory than the older models, they could certainly be used for the computation of the new geoid solutions in the future.

Table 6. The main statistical characteristics of differences between global geoid models and HRG2000 (53135 grid points; in m)

	Min	Max	Mean	St. Dev.
EIGEN2 - HRG2000	-0.51	5.74	2.54	1.61
GRACE01S - HRG2000	-0.99	3.38	1.23	0.78
GGM01S - HRG2000	-1.20	3.72	1.18	0.83
GGM01C - HRG2000	-0.29	2.50	1.20	0.46

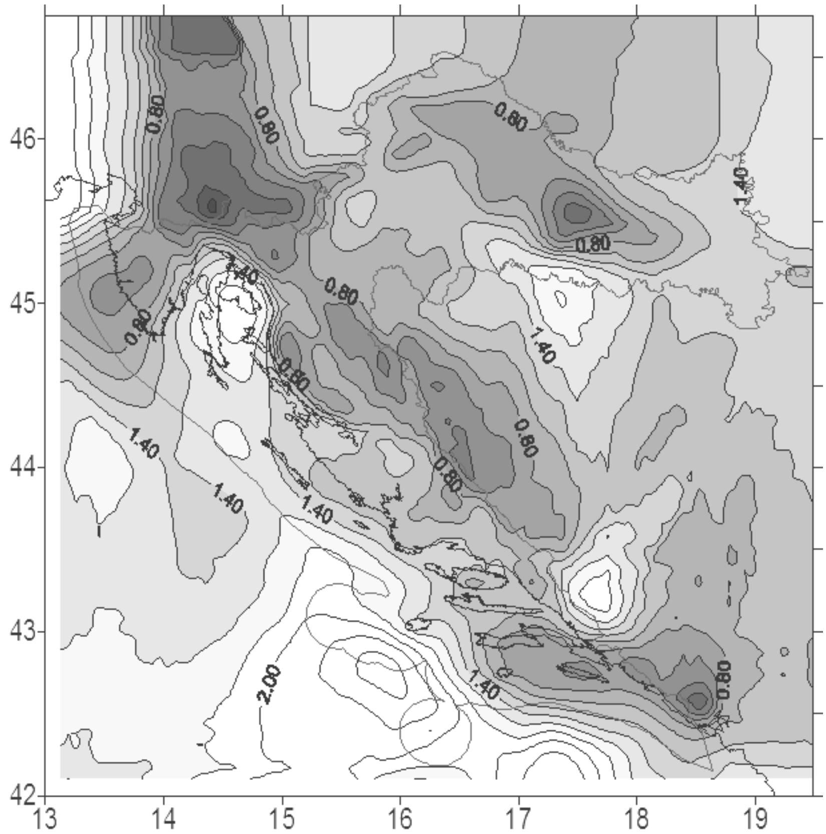

Fig. 3 Differences between GGM01C and HRG2000 geoid solutions (m)

4.2 New Gravity Data Base

In the meantime, a new gravity data base with over 46700 items has been successfully established, out of which over 41000 point gravity values cover the land part of former Yugoslavia (see Fig. 4).

We deal here with essentially larger number of available gravity data than it has been the case so far (more than 10 times). Using the method of prediction by least squares, the quality of these data was first checked comparing all measured gravity values with those predicted on the basis of adjacent points. In Fig. 5 one can see the points where the differences were obtained between the measured and predicted anomalies being larger than the three times of standard deviations (206 differences up), i.e. larger than the two times of standard deviations (1372 differences down), both obtained from the prediction. It is obvious that a very small number of measurements are problematic in Croatia, and the majority of such measurements are outside of our area where an additional analysis of these data will be necessary in order to find the causes.

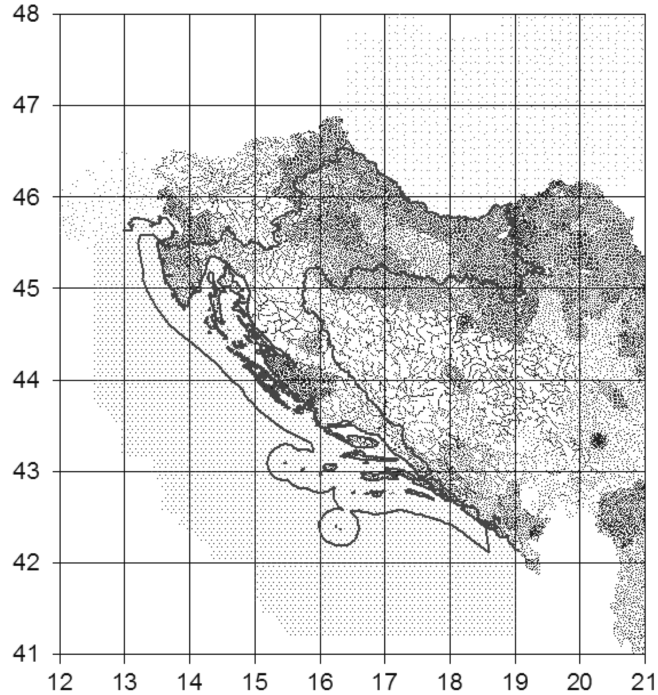

Fig. 4 New available gravity data base

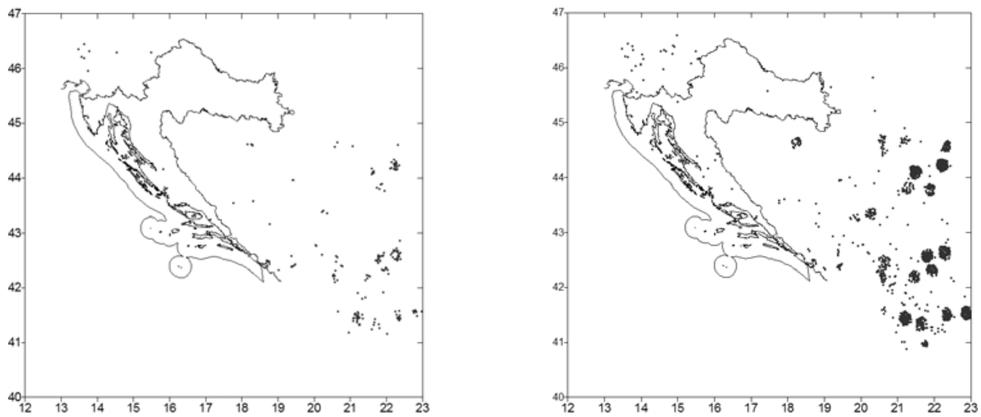

Fig. 5 Differences between measured and predicted gravity anomalies (left 206 diff. > 3·std, and right 1373 diff. > 2·std, obtained from the preduction)

5 Conclusion

Since we are now in the situation that most preparatory work has been done, determination of the new geoid solution for the territory of Croatia is expected in the nearest future, using also other methods of calculation, like FFT and integral formulas. We are also happy to be involved in the international **E**uropean **G**ravity and **G**eoid **P**roject (within IAG Commission 2) serving as a regional data and computing center. Therefore, we would like to develop a good cooperation with all surrounding countries.

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