The performance of the space-wise approach to GOCE data analysis, when statistical homogenization is applied

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Abstract. The space-wise approach to GOCE data reduction exploits the spatial correlation of the observations by "projecting" them on a spherical grid at mean satellite altitude; to this aim a local collocation prediction based on a global covariance function of the potential T can be used. Since the implicit hypothesis of rotational invariance is not fulfilled by the gravitational potential, the global covariance function cannot describe the local characteristics of the field in the different interpolation areas. As a consequence the estimation error of the gridded values is not homogeneously distributed all over the reference sphere, but it is much higher over the Himalaya, the Alps, the Andes, etc., i.e. over areas where the random field presents an "unexpectedly" high variability. These errors may degrade the performance of the subsequent spherical harmonic analysis for the recovery of the potential coefficients, especially at very high degrees.

In the light of this reasoning, the whole procedure of the space-wise approach is expected to benefit by a priori making the analysed random field smoother and thereby more homogeneous. This can be done by first determining a global model which describes the main features of the potential distribution on the spherical grid and then by subtracting it from the observed data once and for all. The signal covariance function has to be corrected accordingly. In principle, the resulting field is more "stationary", also when it is regarded as a time-wise process.

This procedure has been applied on simulated GOCE data, showing that the errors of the estimated spherical harmonic coefficients slightly but systematically decrease. Moreover the improvement is much more significant in critical areas.

Keywords. GOCE mission, space-wise approach, gravity field regularization

1 An introduction to the problem

The paper deals with an attempt at assessing the effects of inhomogeneities of the gravity field on the space-wise approach as applied to the GOCE mission (ESA, 1999), as well as, to some extent, at repairing the subsequent drawbacks.

As it is known the same data set can be reduced following different approaches, like the direct method (Bruinsma et al., 2004; Ditmar et al., 2003) or the time-wise method (Pail and Plank, 2002; Schuh, 2000), which however are not in the focus of the present paper.

The space-wise approach (see Figure 1), as understood today, comprises three main steps, i.e.

- a time-wise Wiener filtering (Papoulis, 1984) of data along the satellite orbit;
- a local prediction of grids of suitable functionals of the anomalous potential (e.g. T, T_{rr}, $T_{NN}-T_{EE}$, where T_{NN}, T_{EE} are second derivatives in the northern and eastern directions);
- a harmonic analysis to estimate spherical harmonic coefficients by numerical integration (Migliaccio and Sansò, 1989) or by fast spherical collocation (Sansò and Tscherning, 2003).

The procedure has to be iterated both because when Wiener filtering is applied to raw data, information is lost in the low frequency band (where noise is

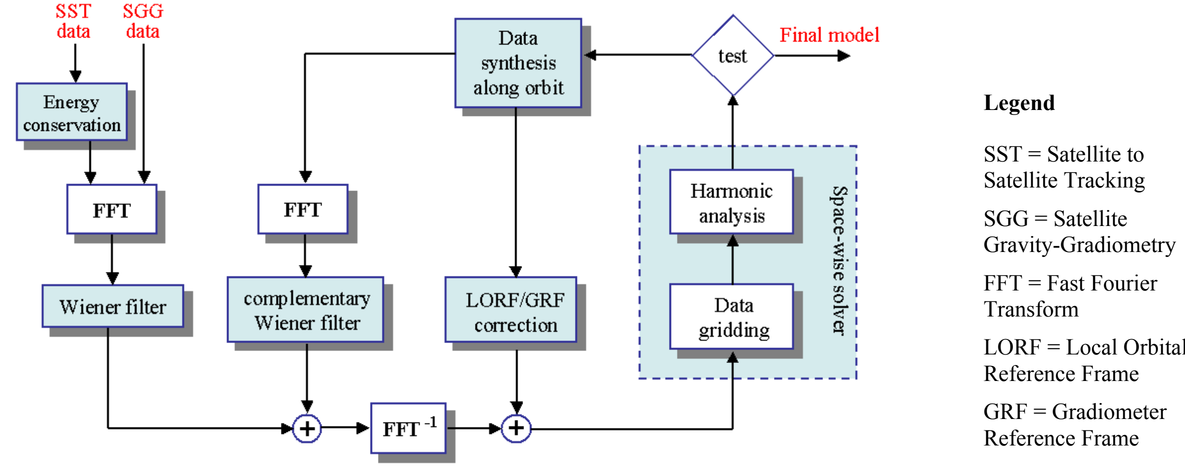

Fig. 1 Scheme of the space-wise approach to GOCE data analysis

prevailing), and in order to correct directional derivatives, due to the instrumental frame wandering; iterations provide the lost information. The procedure is known to be convergent, by numerical tests (Migliaccio et al., 2004a; Migliaccio et al., 2004b).

However, in a first phase of the mission study, when the rotations of the gradiometer were considered as controlled to the level of some arc-minutes, the grid of estimation errors after the first iteration was fairly homogeneous, showing at most some trackiness related to the strong correlation of the noise (see Figure 2).

On the contrary, when a stronger attitude dynamics has to be accepted in the GOCE design, the same procedure yields grids where a clear signature of geophysical features (e.g. the Andes, the Himalaya, etc.) was visible and persistent through the iterations (see Figure 3 and Figure 4).

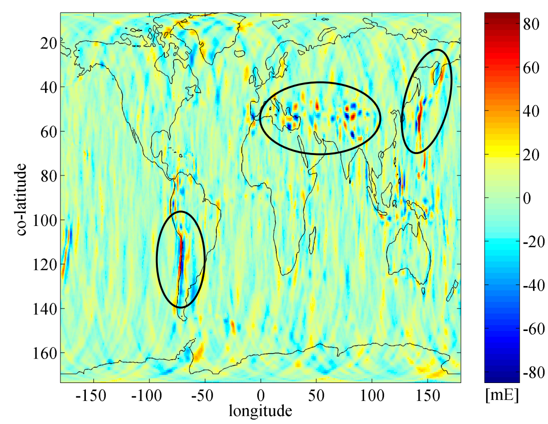

Fig. 3 Estimation error after the first iteration working with second derivatives T_{zz} along the instrumental z-axis. The critical areas are encircled

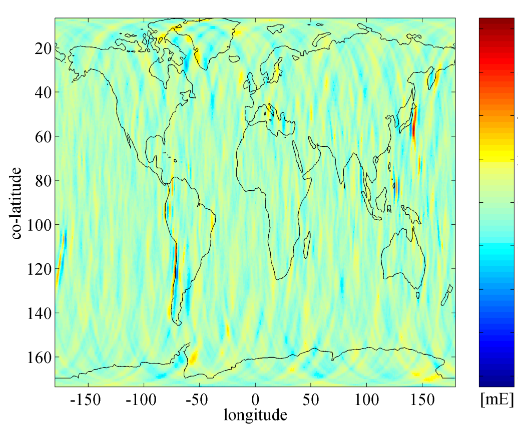

Fig. 2 Estimation error after the first iteration working with second radial derivatives T_{rr}

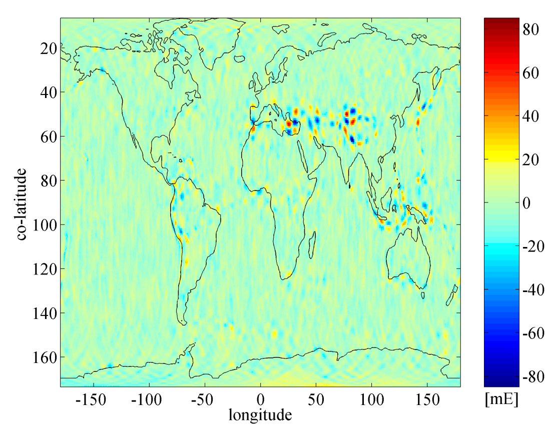

Fig. 4 Estimation error after the last iteration working with second derivatives T_{zz} along the instrumental z-axis

This of course would call for a regional gridding procedure using regionally adapted covariance functions (Arabelos and Tscherning, 2003) or for a multiscale spherical analysis (Freeden, 1999) which still remains an interesting alternative for the future; however dealing with present choices, it seemed to the authors that a kind of "homemade" multiscale analysis could be performed by:

- cutting the highest signal values (e.g. above the 2σ level, with σ computed on all the data);
- making a specific spherical harmonic expansion of a smoothed version of the "peak only" grid;
- recomputing a new data set where the effect of the peaks are removed;
- applying the space-wise approach to the regularized data set;
- adding back to the estimated spherical harmonic coefficients, those used to eliminate the peaks.

We hoped that such a remove-restore procedure, by providing a more homogeneous and smooth data set to the system, would in the end produce a more accurate estimate of *T*.

2 The experiment and its results

In order to test the performance of the iterative space-wise approach when the previously described regularization procedure is applied, a simulation has been performed in a typical GOCE framework.

The EGM96 model $\{T_{nm}\}$, from degree 25 to 300, is used to simulate the potential values *T* and the second derivatives T_{zz}, along the instrumental *z*-axis oscillating according to the following long-period sinusoidal laws:

$$\theta_y = A_y \sin \omega_y t \qquad A_y = 3° \qquad \frac{2\pi}{\omega_y} \cong 2\,\text{hours}$$

$$\theta_r = A_r \sin \omega_r t \qquad A_r = 1° \qquad \frac{2\pi}{\omega_r} \cong 1\,\text{hours}$$

where θ_y is the rotation around the *y*-axis (pitch) and θ_r around the radial direction (roll).

The observations are simulated every second along a circular orbit, at an altitude of 250 km and with an inclination of 96.5°, for a total number of data $N = 2^{23} \cong 8 \cdot 10^6$, corresponding to a mission length of about 100 days.

A coloured noise, specified by the spectrum in Figure 5, is added to the T_{zz} observations, while a white noise with a standard deviation of $0.3\text{m}^2/\text{s}^2$, corresponding to an orbital error of about 3 cm, is added to the potential *T* values (see Figure 6).

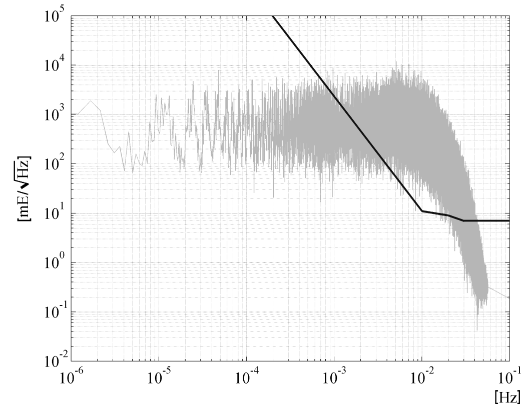

Fig. 5 Signal (in grey) and noise (in black) power spectral density of the second derivatives T_{zz}

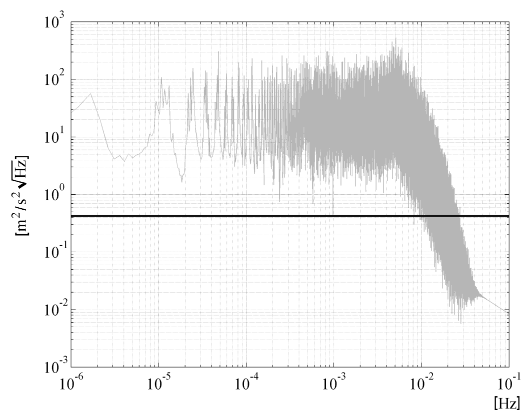

Fig. 6 Signal (in grey) and noise (in black) power spectral density of the potential *T*

A multiple-input multiple-output (MIMO) Wiener filter is designed by using the empirical spectra of T_{rr} and *T* as well as the empirical cross-spectrum between T_{rr} and *T*, all computed from the EGM96 model.

After filtering, the data are locally interpolated on a spherical grid at satellite altitude, with a cell size of 0.72°×0.72°; in fact this allows for an almost exact integer partitioning of the latitude interval covered by the observations, excluding the polar gaps. The local gridding is implemented by collocation, cell by cell, on a moving window of double size with respect to the grid cell.

At this point, we set up the homogenization procedure, selecting the T_{rr} gridded data which are outside the global 2σ value (in our simulation σ is equal to 143.7 mE). Note that the thresholding is only based on the T_{rr} values and not on the potential *T*, which is much smoother and thus more homogeneous.

Figure 7 shows the T_{rr} grid as obtained by the local gridding procedure, while Figure 8 represents the “peak only” grid, referred back to the zero level; in other words, we have considered a field only in the peak areas, with values ($T_{rr} - 2\sigma$) where $T_{rr} > 2\sigma$ and ($T_{rr} + 2\sigma$) where $T_{rr} < -2\sigma$.

After a moving-average smoothing to avoid Gibbs effects, we get a spherical harmonic global model $\{\delta T_{nm}\}$ (by numerical integration), which reasonably describes the “peak only” grid.

This model has to be subtracted from the original one, in order to obtain a more homogeneous data set, i.e.

$$T_{nm}^{new} = T_{nm} - \delta T_{nm}$$

$$(T_{rr}, T)^{new} = (T_{rr}, T) - (\delta T_{rr}, \delta T)$$

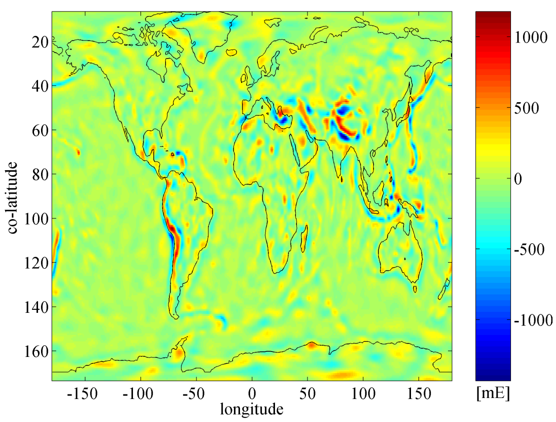

Fig. 7 T_{rr} estimated grid after the first step of the iterative space-wise approach

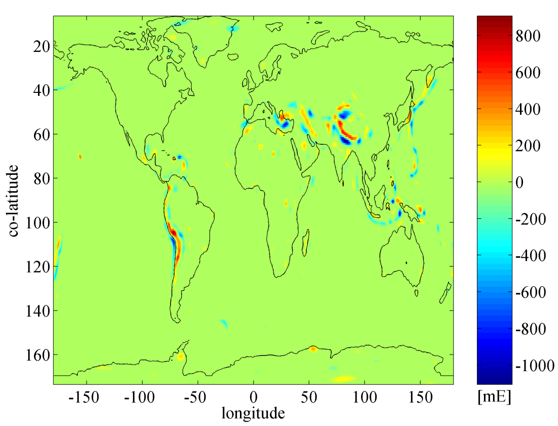

Fig. 8 Grid of the highest T_{rr} values, above the 2σ threshold; the values are referred back to the zero level

Note that the signal degree variances of the new model, and hence the corresponding global covariance functions, remain practically the same, as shown in Figure 9.

Starting from the new regularized data, we first apply the space-wise approach and then, at the end of the iterative procedure, add back the “peak only” model.

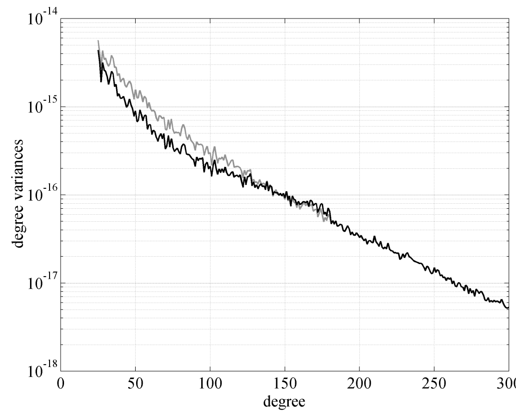

Fig. 9 Degree variances of the EGM96 model (in grey) and of the new one (in black) after regularization

Let us come to the results of this technique, comparing the cases with and without statistical homogenization.

The error degree variances of the new solution, shown in Figure 10, are globally systematically better. The GOCE requirement of solving the gravity field up to degree 200 can be safely reached.

The amount of the improvement in the harmonic coefficients estimation can be evaluated by the following relative differences

$$\varepsilon_n = \frac{\delta\sigma_n - \delta\sigma_n^{new}}{\delta\sigma_n}$$

where $\delta\sigma_n$, $\delta\sigma_n^{new}$ are the error degree standard deviations after the last step of the space-wise iterative procedure, without and with regularization respectively. This index is displayed in Figure 11, degree by degree, clearly showing a systematic improvement of the order of about 5%. Let us also note that, apart from one value, $\delta\sigma_n > \delta\sigma_n^{new}$.

Nevertheless the real advantage of this technique can be better perceived locally. To this aim we concentrate on a critical area, e.g. in the Himalaya ($78° < \lambda < 92°$, $60° < \theta < 68°$), where the grid estimation errors are very high.

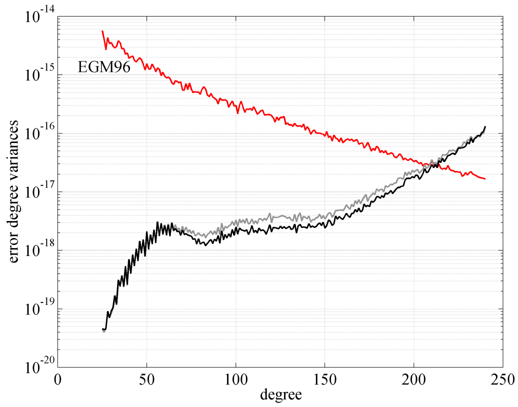

Fig. 10 Error degree variances of the estimated model with regularization, after the first (in grey) and last (in black) iteration. The reference model is EGM96

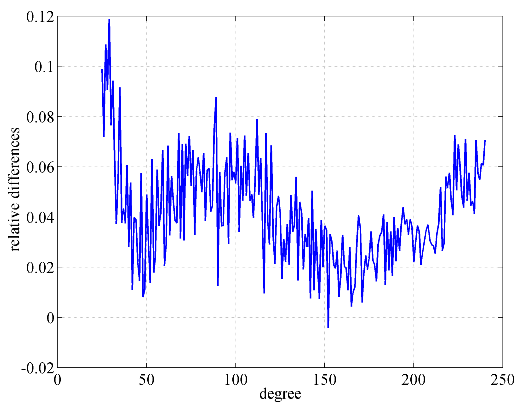

Fig. 11 Relative differences of the error degree standard deviations without and with regularization, after the last step of the iterative procedure

The final errors, in terms of geoid undulation *N*, are shown in Figure 12 and Figure 13, without and with the statistical regularization, respectively. The same errors for the gravity anomalies Δg are displayed in Figure 14 and Figure 15. These errors have been computed as differences with respect to the "true" EGM96 model, up to degree 200. It is clear that the new estimated model produces a much more homogeneous error distribution, reducing the "local peak" down to 30%. Some statistical indicators are reported in Table 1 and Table 2. The values are rather large considering the global goal of the GOCE mission of 1 cm in the geoid and 1-2 mgal in the gravity anomalies at about 1 degree wavelengths. However, note that we have simulated only 3 months of data and we have not used all the components provided by the GOCE gradiometer. Their additional use may improve the solution considerably (Tscherning, 2003).

Table 1. Maximum and r.m.s. error of the geoid undulation, in the Himalaya region, without and with the regularization

max error of N		r.m.s. error of N	
without reg.	with reg.	without reg.	with reg.
1.071 m	0.722 m	0.336 m	0.229 m
relative difference		relative difference	
32.57%		31.80%	

Table 2. Maximum and r.m.s. error of the gravity anomalies, in the Himalaya region, without and with the regularization

max error of Δg		r.m.s. error of Δg	
without reg.	with reg.	without reg.	with reg.
23.28 mgal	16.53 mgal	7.74 mgal	5.52 mgal
relative difference		relative difference	
29.01%		28.75%	

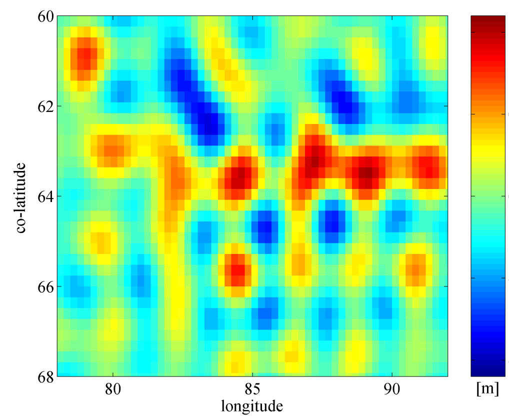

Fig. 12 Estimation errors of the geoid undulation *N* in the Himalaya region without applying the regularization

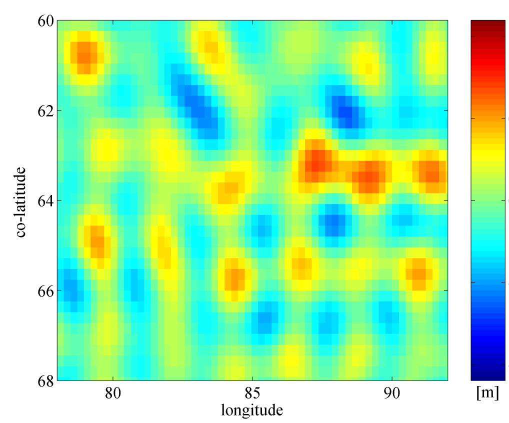

Fig. 13 Estimation errors of the geoid undulation *N* in the Himalaya region after applying the regularization

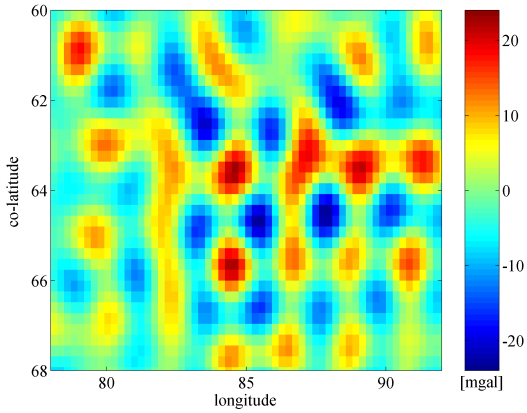

Fig. 14 Estimation errors of the gravity anomalies Δg in the Himalaya region without applying the regularization

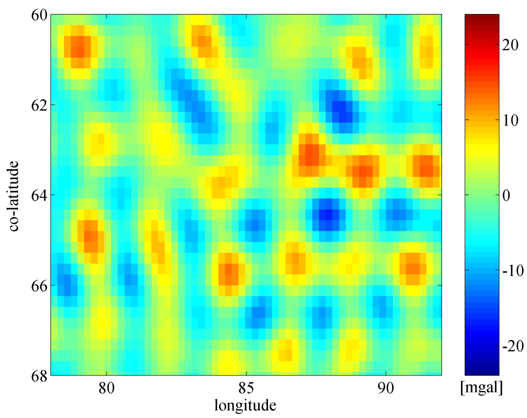

Fig. 15 Estimation errors of the gravity anomalies Δg in the Himalaya region after applying the regularization

3 Conclusions

Although the use of this remove-restore approach to smooth the high level inhomogeneities before applying collocation has not a very large impact on the accuracy of harmonic coefficients estimation, we think that:

- the improvement is systematic and therefore worthwhile;
- the local effect of this correction in critical areas is very significant, thus justifying the effort of its implementation.

The reason why such a strong signature of the errors was not visible in previous simulations of the space-wise approach, stems in our opinion from the fact that, by introducing strong rotational dynamics of the satellite, we make the hypothesis of stationarity, on which the Wiener filtering is based, much less realistic (Albertella et al., 2004). As a result the areas of strongest signal introduce an error with a signature directly proportional to the signal strength. Would this hypothesis be confirmed, one should also think of applying a similar procedure to the time-wise approach.

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