# Gravity Field Model UCPH2004 from One Year of CHAMP Data using Energy Conservation

E. Howe, C. C. Tscherning

Department of Geophysics,

The University of Copenhagen, Juliane Maries Vej 30, DK-2100 Copenhagen Oe, Denmark

**Abstract.** A gravity field model has been estimated using energy conservation and fast spherical collocation.

The energy conservation method is based on knowledge of the state vector and measurements of non-conservative forces. This is or will be provided by CHAMP, GRACE and GOCE.

Here the energy conservation method is applied to calculate gravity potential values from CHAMP data measured in July 2002 – June 2003. Precise science orbits and accelerometer data are derived from GFZ Potsdam. When estimating the loss of energy due to external forces only friction is considered, calculated using the along-track acceleration. A scale-factor for the along-track accelerometer has been estimated for each half day.

Fast Spherical Collocation requires data located equidistantly on parallels. It is much faster than general least squares collocation. Fast Spherical Collocation has been used to estimate a gravity field model to degree and order 90, UCPH2004 and a model UCPHcoll to degree and order 60 has been estimated with general least squares collocation using only 1% of data.

Evaluation of the method is made by comparison with EIGEN-2 and TUM1s. Furthermore the gravity field is compared to data from the Arctic Gravity Project (ArGP). Good agreement has been found in the comparisons.

**Keywords.** Energy conservation, CHAMP, gravity potential, collocation, error correlations

## 1 Introduction

The energy conservation method has been well documented in the past few years, see for example Han et al. (2002), Gerlach et al. (2003) and Howe et al. (2003a). Every time improvements are found and new ways to calibrate the data are used. Each group has its own way to apply the energy conservation method and to estimate the spherical harmonic coefficients.

We use the energy conservation to calculate height anomalies at satellite height. When doing so we consider the tidal potential corresponding to a rigid earth of the sun and the moon (Longman (1959)), the explicit time variation of the gravity potential in inertial space (Jekeli (1999)) and friction. We subtract the earth normal potential, U, without the centrifugal term. The equation reads

$$\varsigma = \left(\frac{1}{2}v^2 - V_{moon} - V_{sun} - \omega\left(xv_y - yv_x\right) - F - E_0 - U\right)/g \quad (1)$$

where $E_0$ is an integration constant.

The data used is measured in July 2002 – June 2003. The data are kindly provided by GFZ Potsdam and are Precise Science Orbits (PSO) and accelerometer data. On November 6 and December 9 - 10. 2002, orbit manoeuvres have been made these periods are removed from the data set, see Reigber (2002). We assume that the accuracy is the same before and after the orbit manoeuvre.

Only the along-track accelerometer is used and therefore only friction calculated using this information is taken into account and no other non-conservative forces are included. The friction term reads

$$F = \int |v| a_y dt \quad (2)$$

A scale factor has been estimated for each half day in order to calibrate the along-track accelerometer. The scale factor is estimated by correlating F with the difference between the calculated gravitational potential field and an a priori gravity field model. EGM96 to degree and order 24 is used as a priori gravity field model; see Lemoine et al. (1998). The scale factors for the period are shown in Figure 1. The scale factors show an interesting pattern, which warrant further investigations.

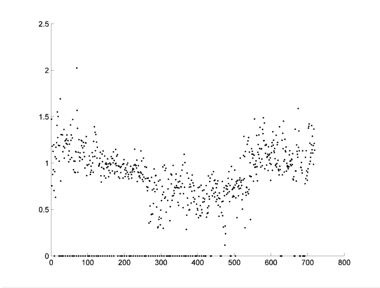


**Figure 1** The scale factors for the period. The x-axis is a number corresponding to each half day of the period. The y-axis is the scale factor. The scale factors which are 0 on the figure is days with no data.

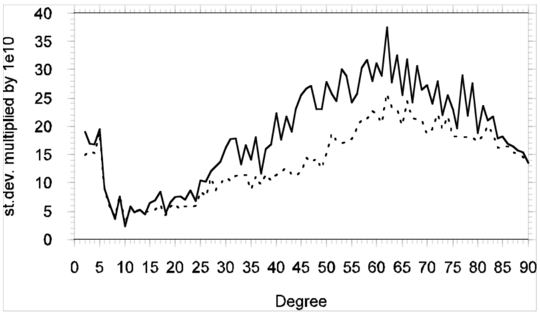


**Figure 2** The standard deviation of the old model (solid) and of the new model (dotted)

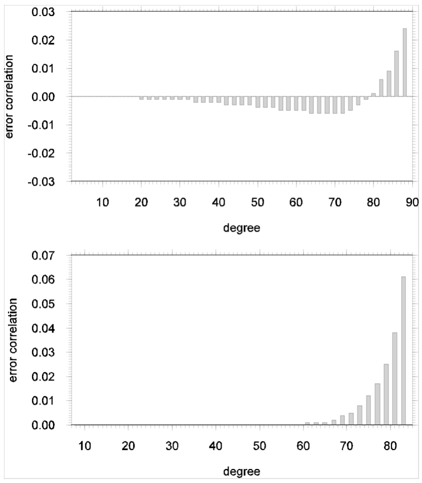


**Figure 3** Typical error correlations of the data. The top figure is for degree 90 and fixed order 2. The bottom figure is for degree 85 and fixed order 7.

Fast spherical collocation has then been used to estimate spherical harmonic coefficients to degree and order 90. A detailed description of Fast Spherical Collocation can be found in Sansó and Tscherning (2003). Comparison have been made with coefficients estimated by general least squares collocation using 1 % of the data, EIGEN-2 (Reigber et al. (2003)), TUM1s (Gerlach et al. (2003b)) and data from the Arctic Gravity Project (ArGP) (Forsberg (2002)).

Bias parameters have been estimated as a method similar to a cross-over analysis of our data and error correlations have been calculated.

## 2 The gravity field model UCPH2004

EGM96 to degree 24 is subtracted from our data in order to make them statistically more homogeneous. The residual potential values are then up-/downwards continued to a common height of 413 km above the ellipsoid. The up-/downwards continuation is performed using gravity disturbances calculated from EGM96. Fast spherical collocation requires that data are located equidistantly on parallels and at the same altitude. The residual potential values are therefore gridded with 0.5° spacing using collocation. From this grid we estimate the spherical harmonic coefficients and their associated errors using fast spherical collocation. After the estimation EGM96 to degree 24 is added to get a complete set of spherical harmonic coefficients.

### 2.1 Analysis of the model

Previously 1 month of data was used when estimating the gravity field model UCPH2003_03 which had good agreement with other post-CHAMP models; see Howe et al. (2003b). The standard deviation of the different degrees of the old model and the new model are compared to see if an improvement is obtained by having more data, see Figure 2.

It can be seen that more data gives a significant improvement of the gravity field model above degree 25. In both models EGM96 to degree 24 is subtracted when estimating the model and then added again. No significant difference is expected between the two models below degree 25.

Fast spherical collocation can be used to compute the error co-variances of the spherical harmonic coefficients, see Tscherning (2004). Figure 3 illustrate some typical error correlations for fixed order. The top figure is for degree 90 and order 2 and the bottom figure is for degree 85 and order 7. The values of the error correlations show interesting patterns. But further investigations are needed in order to explain them.

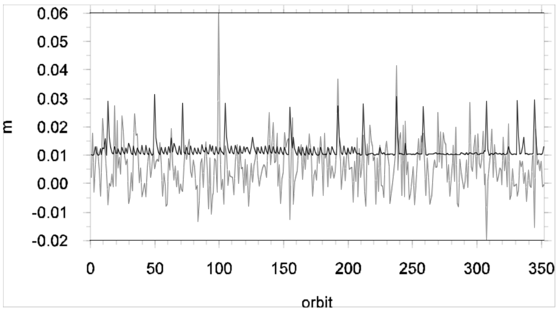


**Figure 4** Bias parameters for each day (grey) and the error estimates (black)

### 2.2 General least squares collocation

Using general least squares collocation we need to solve a system of as many equations as unknowns each having a matrix with combinations of all of the observations. We have approximately 1 year of 10 sec data, 2567422 data points and for a useful test at least 30000 data points are needed. This gives a factor 10 between data and coefficients determined up to degree 60. There are many ways of selecting data. In this study every 100th point is selected. Another way would be to take a mean value of 100 points or to select more data at places with much gravitational variation and less data at places with little gravitational variation.

The result UCPHcoll is compared to UCPH2004. A mean difference of 0.004 m and a standard deviation of 0.50 m are seen. It was not expected to give better result than UCPH2004 since less data have been used. It can be seen though that general least squares collocation gives very good results with only a small amount of data. Further investigations with larger data sets could lead to an improvement of the model UCPH2004.

Least squares collocation gives the opportunity to make a kind of a cross over analysis, where the data do not have to be in the exact same point. It is a good test of the accuracy. A bias parameter is estimated for each day, see Figure 4. It can be seen that the estimated bias parameters are close to their error estimates. This shows consistency in the technique.

## 3 Evaluation of UCPH2004

The gravity field model is evaluated by comparison with other state-of-the-art gravity field models and with data from the Arctic Gravity Project.

From error analysis it can be seen that there is not much information left above degree 60. The comparisons are on this basis only made using coefficients up to degree 60. UCPH2004 is compared to Eigen-2 and TUM1s. The results are listed in table 1.

**Table 1** Differences between UCPH2004 and EIGEN-2 and TUM1s in metre

| | Mean | St. dev | Min | Max |
|---|---|---|---|---|
| EIGEN-2 | -0.008 | 0.52 | -2.76 | 2.57 |
| TUM1s | 0.08 | 0.54 | -2.84 | 2.51 |

UCPH2004 is within half a meter of the two gravity field models used here. Further analysis show that the main differences are in the polar regions. This could be due to only taking friction in the direction of the velocity vector into account and not sun pressure and cross winds, which are strongest at the poles.

A comparison between the gravity field models and data from the Arctic Gravity Project is conducted to further test the accuracy of the model. If the comparison is made over the entire arctic area (64.0°N – 89.9°N, 179.9°W – 179.9°E) EIGEN-2 and TUM1s fits slightly better than UCPH2004, see table 2. The differences in how well the three models fit the arctic data are within the standard deviation for the three models. The gravity field model estimated with general least squares collocation UCPHcoll fits as well as UCPH2004 even though there is a difference of 0.5 m between the two. A new comparison is made in an area where it previously is seen that the three models disagree (70°N -80°N, 50°E - 70°E), see table 3. Here UCPH2004 has a smaller mean difference to the arctic data than EIGEN-2 and TUM1s and a slightly better standard deviation. UCPHcoll has a very small mean difference of less than 1 mGal to the arctic gravity data but the standard deviation is bigger.

**Table 2** Comparison with ArGP data in the entire arctic region. Units are mGal..

| | Mean | St. dev | Min | Max |
|---|---|---|---|---|
| UCPH2004 | -0.91 | 24.29 | -216.23 | 217.40 |
| EIGEN-2 | -0.25 | 23.53 | -209.31 | 221.72 |
| TUM1s | -0.16 | 23.47 | -209.80 | 221.26 |
| UCPHcoll | -0.86 | 24.11 | -218.74 | 213.34 |

**Table 3** Comparison with ArGP data in the region (70-80°N, 50-70°E). Units are mGal.

| | Mean | St. dev | Min | Max |
|---|---|---|---|---|
| UCPH2004 | -1.42 | 16.11 | -45.93 | 79.29 |
| EIGEN-2 | -3.05 | 16.54 | -53.21 | 81.73 |
| TUM1s | -2.98 | 16.63 | -52.90 | 80.56 |
| UCPHcoll | -0.63 | 17.11 | -45.77 | 81.69 |

## 4 Concluding remarks

It is possible with CHAMP data to estimate a state-of-the-art gravity field model using energy conservation and collocation.

We find a difference of 0.5 m between UCPH2004 and EIGEN-2 and TUM1s. The three models all have a standard deviation of about 24 mGal compared to the arctic gravity data. This shows that our gravity field model UCPH2004 is comparable to other state-of-the-art gravity field models.

In order to enhance the gravity field model the entire acceleration vector should be taken into account. Further investigations are needed. The estimated scale factors show that it would induce an error if the scale factor was assumed to be constant. It may even be a good idea to estimate a scale factor more often than each half day, one per revolution for example.

The accuracy has been improved compared to our older models due to more data. Comparison with general least squares collocation has been made. It shows that the techniques are consistent. Furthermore it is seen that general least squares collocation can improve our spherical harmonic coefficients. The only drawback is the large computer power needed and it is rather time consuming.

**Acknowledgement.** Thanks to the CHAMP data centre for providing the data. Thanks to Louise Sørensen and Henriette Hjorth for data processing. This paper is a contribution to the SAGRADA project sponsored by the Natural Science Council of Denmark

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