A NEW QUASI-GEOID COMPUTATION FROM GRAVITY AND GPS DATA IN ALGERIA

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Abstract

Due to the rapid increasing use of GPS heighting, which already gives the same accuracy as levelling over some 10 to 100 km, there is an urgent need to provide the ″cm quasi-geoid″ to geodesists and surveyors. In this context, the Geodetic Laboratory of National Centre of Space Techniques has recently focused a part of the current research on the precise geoid determination using different methods. In 1997, the first determination of a preliminary geoid in a small zone in the north of Algeria was calculated by using the least squares collocation and the Gravsoft software. Nowadays, an improved quasi-geoid has been computed over the whole of Algeria. This solution was based on the validated gravity data supplied by the Geophysical Exploration Technology Ltd (GETECH), topographic information and optimal geopotential model, which were combined by using the remove-restore technique in connection with the Fast Fourier Transformation. The GLOBE 30″ digital elevation model has provisionally been chosen as the DEM to be used for computation of the effects of the topography according to the RTM reduction modelling method. However, the spherical harmonic coefficient set OSU91A, complete to a degree and an order 360, was adopted as a reference in order to eliminate the long wavelengths of the gravity field. In this paper, the main features of the Algerian quasi-geoid solution are summarized, and extended tests of this solution are undertaken using the new GPS and levelling data collected from the TYRGEONET project and the local GPS/Levelling surveys. The comparisons based on different GPS campaigns provide, after fitting by using the four-parameter transformation, an RMS differences ± 11cm especially for the north part of the country over distances of 1 to 1000 km and proves that a good fit between the new quasi-geoid and GPS/levelling data has been reached.

Key words: Fast collocation method, GPS/levelling, Geopotential model, TYRGEONET project

1. Introduction

Geoid determination is one of the most fundamental problems in geodesy. The precise model of the geoid not only enable us to transform satellite-derived heights to physically meaningful heights based on The Earth's gravity field, but also plays an important role in geophysics and oceanography. The computation of this surface can be done in a fast and efficient way by means of FFT techniques or Fast collocation which lead to geoid estimates over large areas, so avoiding sub-areas computation and patching of sub-solutions.

Furthermore, accurate global geopotential models and, over some regions, detailed DTM allows an effective reduction of the gravity data, thus implying an optimal application of the "remove-restore" procedure.

The first determination of a preliminary geoid in a small zone in the north of Algeria was done in 1997 (Benahmed Daho et al., 1998), using the Least Squares Collocation (LSC) method and the Gravsoft software. In second gravimetric solution over the whole of Algeria territory, the BGI gravity data and topographic information were included (Benahmed Daho, 2000).

Nowadays, an improved quasi-geoid has been computed over the whole of Algeria between the limits $16° \leq \varphi \leq 40°$ for latitudes and $-10° \leq \lambda \leq 14°$ for longitudes, in a grid with mesh of 5'x 5'. This solution was based on the validated gravity data supplied by the Geophysical Exploration Technology Ltd (GETECH), topographic information and tailored geopotential model, which were combined using the remove-restore technique in connection with the Fast Fourier Transformation (FFT). The computation of the topography effects according to the RTM reduction modelling method is based on a global topographic model GLOBE of 30″ x 30″. However, the spherical harmonic coefficient set OSU91A, complete to a degree and an order 360, was adopted as a reference for to remove and restore the long wavelength components of the gravity and the geoid respectively.

In order to test the quality and the accuracy of the quasi-geoid calculated, the results of this computation are compared to GPS/Levelling data collected from the international TYRGEONET project and the local GPS/Levelling surveys.

2. The available data

In order to fulfil the requests of the FFT-based algorithms used in this paper for geoid height computation, we did not restrict ourselves to the continental area of Algeria when selecting the data, but we considered also the surrounding sea region. Therefore, the data selection area is bounded by limits $16° \leq \varphi \leq 40°$; and $-10° \leq \lambda \leq 14°$. The prediction area is also bounded by same limits, but it is noted that the results at the borders of the test area are affected by errors, generally due to the lack of data around the evaluation points.

2.1. Gravity data

For this work, the pre-processed free air anomalies on a 5' grid in the area bounded by the limits mentioned above, derived by merging terrestrial gravity data and satellite altimetry data, have been provided by GETECH through the agreement between the National Centre of Space Techniques/Geodetic Laboratory and University of Leeds/GETECH without any information on the accuracy of different values. These data have been acquired in the framework of African Gravity Project (AGP) from Bureau Gravimétrique International (BGI), Defence Mapping Agency and from oil companies and many national academic and non-academic organisations in different countries. All of these gravity measurements are adjusted at GETECH to IGSN71 by using "Latin American Gravity standardisation Net 1977" and are referred to the Geodetic Reference System 1980. Figure 1 gives a graphical representation of the gravity data coverage in the computation area. From the figure it becomes clear that the coverage with gravity observations is not sufficient for some land areas particularly in the south of the country and new measurements are needed to accomplish a homogeneous coverage.

Furthermore and since the history of GETECH gravity data processing is not clearly understood, it is very difficult to estimate the accuracy of the resulted geoid heights. However and considering that the BGI gravity data on the Algerian territory have been included for the generation of the previous 5' grid of anomalies, we have proceed to assess the prediction accuracy by comparing the each validated BGI gravity observation with the a value predicted from the GETECH grid using the Spline interpolation. Table 1 summarises the statistics of the differences. The analysis of the results shows the large discrepancies between the original BGI gravity data and predicted ones and proves that gridded gravity set was not derived for precise geoid determination purpose, but for other geophysical applications such as regional geological interpretations.

Anomalies	Mean	Sd	Min.	Max.
Δg (Obs)	4.887	26.845	-82.590	136.200
Δg (Pred)	4.380	26.184	-89.650	107.825
Differences	4.657	0.502	-29.923	69.666

Table 1. Statistics of the differences (mGals)

2.2 Geopotential model

The tailored high-degree global geopotential model OSU91A (Rapp & al., 1991), complete to degree and order 360, was adopted as a reference in order to eliminate the long wavelengths of the gravity field. Replacing the global model OSU91A by the most recent model EGM96 do not improve significantly the present solution, because no new gravity data from Algerian territory was incorporated in EGM96. Gravity anomalies and geoid undulation can be computed in a spherical approximation from a geopotential coefficient set by:

$$\Delta g = \frac{GM}{r^2}\sum_{n=2}^{N_{max}}(n-1)\sum_{m=0}^{n}\left[\overline{C}_{nm}.\cos m\lambda + \overline{S}_{nm}.\sin m\lambda\right]\overline{P}_{nm}(\cos\theta) \quad \mathbf{(1)}$$

$$N = \frac{GM}{r\gamma}\sum_{n=2}^{N_{max}}\sum_{m=0}^{n}\left[\overline{C}_{nm}.\cos m\lambda + \overline{S}_{nm}.\sin m\lambda\right]\overline{P}_{nm}(\cos\theta) \quad \mathbf{(2)}$$

where θ, λ are the geocentric colatitude and longitude of the point where Δg and N will be determined ; $\overline{C}_{nm}, \overline{S}_{nm}$ are the fully normalised spherical geopotential coefficients of the anomalous potential ; $\overline{P}_{nm}$ are the fully normalised associated Legendre functions ; N_{max} is the maximum degree of the geopotential model.

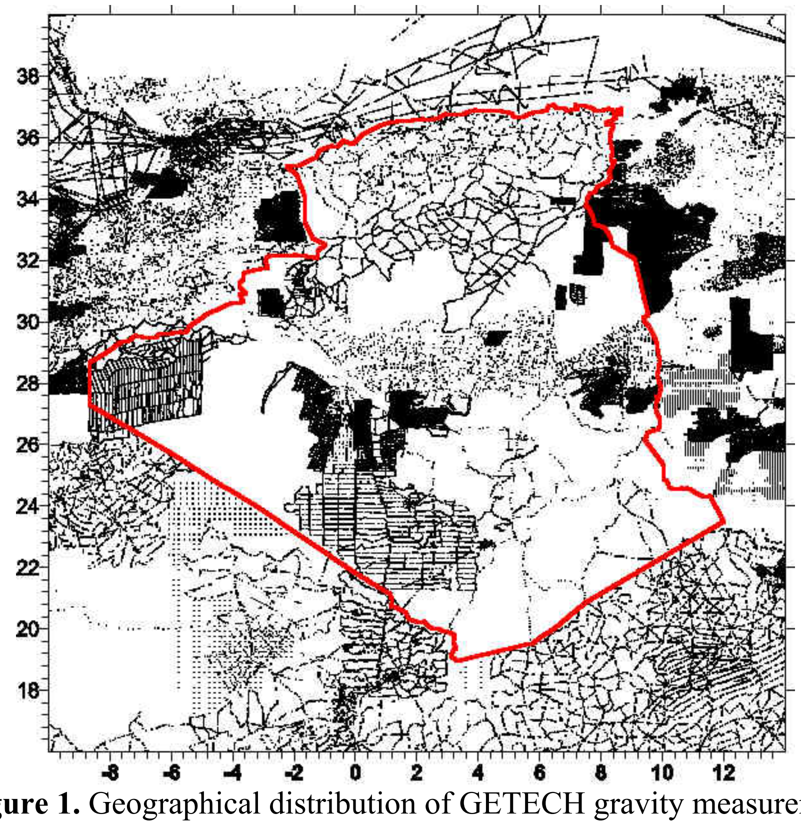

Figure 1. Geographical distribution of GETECH gravity measurements.

2.3 Topographic data

The computation of the terrain effects on the quasi-geoid required a detailed DTM model. For this purpose, The GLOBE 30″ digital terrain model has provisionally been chosen as the DTM to be used for computation of the effects of the topography according to the RTM reduction modelling method.

3. Computation procedure

The computations were done using the "GRAVSOFT" package, developed during a number of years at the National Survey and Cadastre (KMS). For actual solution, the Fast Fourier Transformation (FFT) and the Remove-Restore procedure were used to compute the quasi-geoid estimate. In order to smooth the gravity field, the previous gridded gravity data must be corrected for the effect of the atmospheric and reduced for the effect of the spherical harmonic model and the topography. The spherical harmonic coefficient set OSU91A has been used to remove the long wavelength component of gravity data. The computation of the effects of topography according to the RTM reduction modelling method is based on global topographic model GLOBE of 30″ x 30″ which were used up to a distance of 200 km. The reference surface of 10' x 10' needed for the RTM reduction has been obtained by means of a moving average applied to the detailed one. Table 2 shows the statistics of the reduced gravity data.

Anomalies	Mean	Sd	Min.	Max.
Δg_{Obs}	2.43	28.81	- 172.96	218.84
$\Delta g - \Delta g_{OSU91A}$	-2.29	14.94	-122.90	192.28
$\Delta g_r = \Delta g_{Obs} - \Delta g_{OUS91A} - \Delta g_{RTM}$	-1.55	14.62	-120.12	823.63

Table 2. Statistics of reduced gravity data (mGals).

From the results of Table 1, it is obvious that the OSU91A reference field fits well the gravity in the area under consideration, and the smoothing of the gravity data is considerable after the removal of the topographic effect if we take into account only the mean and standard deviation values. However, we will note that all the gravity residuals values are less than to 277.08 mGals excepting the maximum value which is too large with respect to the statistics of the observed gravity. Probably, this is due to the errors produced by the FFT technique and the global topographic model GLOBE used in the computation of the topographic correction at borders of the test area.

The residual quasi-geoid (ζ_r) has been evaluated using FFT technique, implemented in the GEOFOUR program written by Rene Forsberg. The statistics of residual undulations in a 5' x 5' grid (289 x 289 values) are presented in Table 3 while the plot of the values is shown in Figure 2.

Mean	Sd	Min.	Max.
0.00	1.42	-7.05	5.69

Table 3. Statistics of the quasi-geoid residuals ζ_r (m)

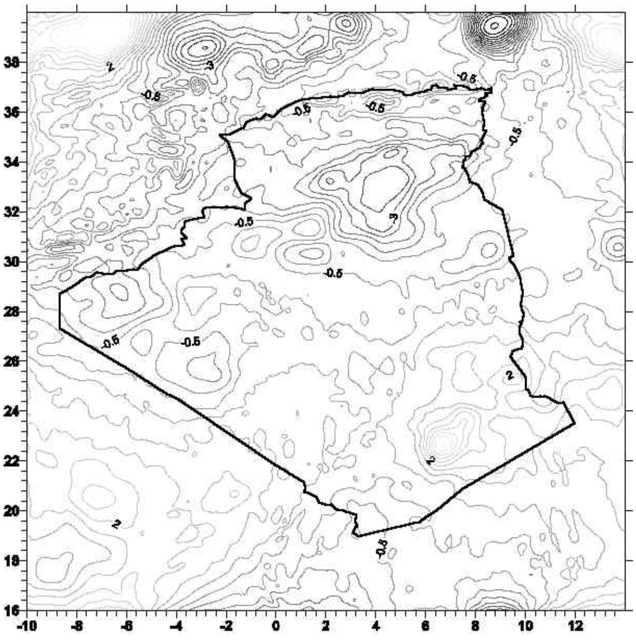

Figure 2. Residual quasi-geoid (m)

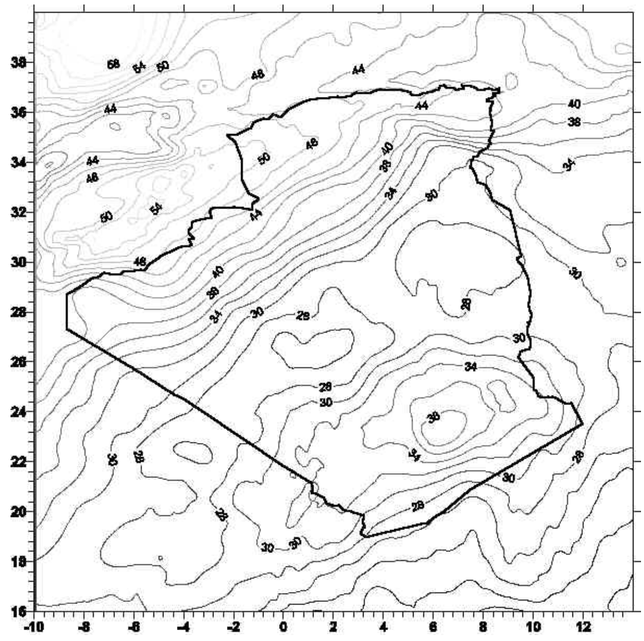

Figure 3. Quasi-geoid solution in Algeria (m)

The final quasi-geoid was obtained by adding the model and the residual terrain effect on the 5' x 5' residual quasi-geoid grid. The values of the geopotential model range from 15.85 m to 56.0 m and yield the major part of the quasi-geoid. The standard deviation and maximum values of the contribution from the gravity data are 1.42 m and 5.69 respectively, while the corresponding values of the RTM effects are 0.07 m and 0.86 m. Figure 3 shows a map of the quasi-geoid solution in Algeria. The statistics of the total quasi-geoid values are summarised in Table 4.

Mean	Sd	Min.	Max.
35.18	8.83	17.69	60.65

Table 4. Statistics of the quasi-geoid (m)

4. Comparison with GPS/Levelling data

Many GPS campaigns have been carried out in the past years in Algeria. Furthermore, in the framework of the TYRGEONET project, two sites located in the North of Algeria have been determined in the WGS84 system, which have been used later for the ALGEONET (*ALGerian GEOdynamical NETwork*) project and for densification and improvement of accuracy of the local geodetic networks. The number of stations GPS used in this investigation was 258, which 16 are benchmarks of the first order levelling network, and the others belong to the second levelling network. All of these points are located in the north of Algeria and distributed as in Figure 5 whose the most are close to the station of Arzew. So, in order to make possible the estimation of N (geoid undulation) in these points, all these GPS stations are connected to the national height system through spirit levelling. The GPS observations were performed with four ASHTECH Z-12 dual frequency receivers with baseline length ranging from about 1 to 1000 km, and the BERNESE software with precise ephemerides was used to process the GPS data.

Among 258 GPS levelling points only 16 well distributed GPS levelling points are used as benchmarks points, and the others were excluded in order to estimate the real accuracy given by the comparison between the adjusted values and the known ones. These values have been compared also to the BGI solution which is based on a set of 12183 validated point free air gravity anomalies supplied by the BGI, two elevation grids; 1 km x 1 km digital terrain model for the north of Algeria and the ETOPO5 for the rest of the area, and the OSU91A geopotential model, which were combined using the remove-restore technique in connection with the Fast collocation. The final result was a gravimetric geoid on a 5’ x 5’ grid in the area bounded by limits $20° \leq \varphi \leq 37°$ and $-7° \leq \lambda \leq 10°$ (Benahmed Daho, 2000). These two computations show differences even up to 4 m and the standard deviation value is about 1.35 m. The high discrepancies are attributed principally to the two difference computation methodologies relative to the technique used for the geoid heights estimation and to the DTM employed for the topographic correction computation.

The statistics of the differences for both solutions before and after fitting out the systematic biases and tilts between the gravimetric geoid and the GPS/levelling data by using the appropriate four-parameter transformation are summarised in table 5 and show that no significant improvements have been reached in the GETECH solution comparative to the BGI solution. Probably, it is due to the fact that the data provided by BGI are included in the GETECH database which have been used to generate of previous free air anomalies grid.

Furthermore, and in order to estimate the real accuracy, the GPS/Levelling undulations at 242 control points are compared at adjusted ones. The figure 4 shows the histogram of these differences. We can see that the combination the gravimetric BGI_solution with GPS/levelling gives the best results. Unfortunately, and considering the number of GPS/Levelling stations used in this investigation and their distribution, the obtained accuracy can not be generalised for the all north part of the country.

	Geoid models	Mean	Min.	Max.	Stand. dev.
Before Fitting	**BGI Solution**	-1.431	-1.732	0.306	0.617
	GETECH Solution	0.608	0.455	1.531	**0.321**
After Fitting	**BGI Solution**	0.000	-0.034	0.048	**0.020**
	GETECH Solution	0.000	-0.194	0.135	0.104

Table 5. Comparison of GPS/Levelling undulations to gravimetric quasi-geoidal heights (m)

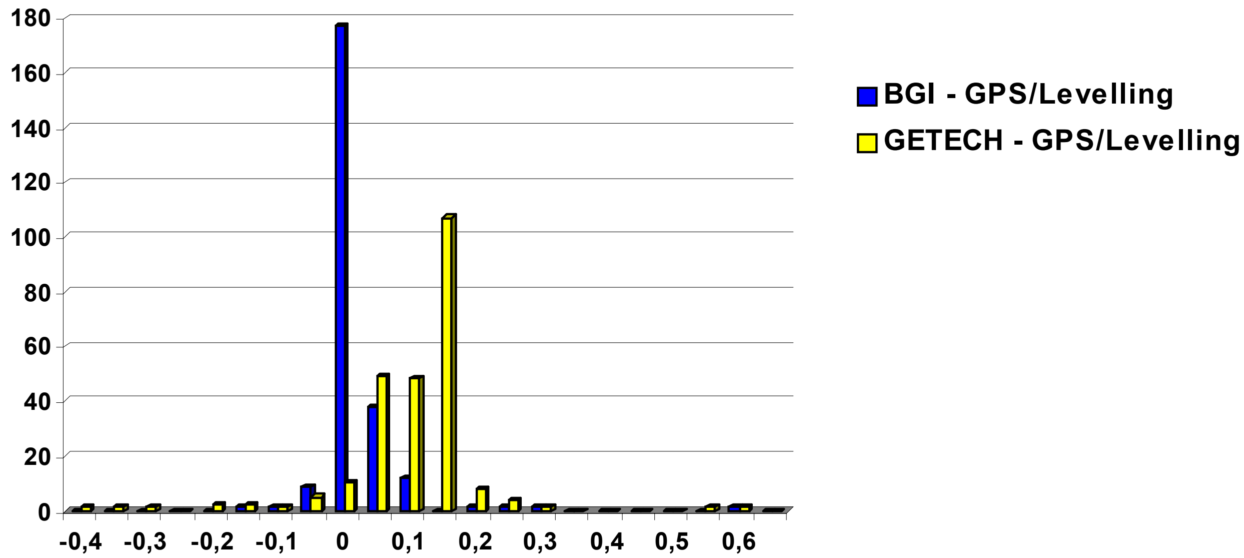

Figure 4. Histogram of the differences with GPS/Levelling geoid undulations at control points (m)

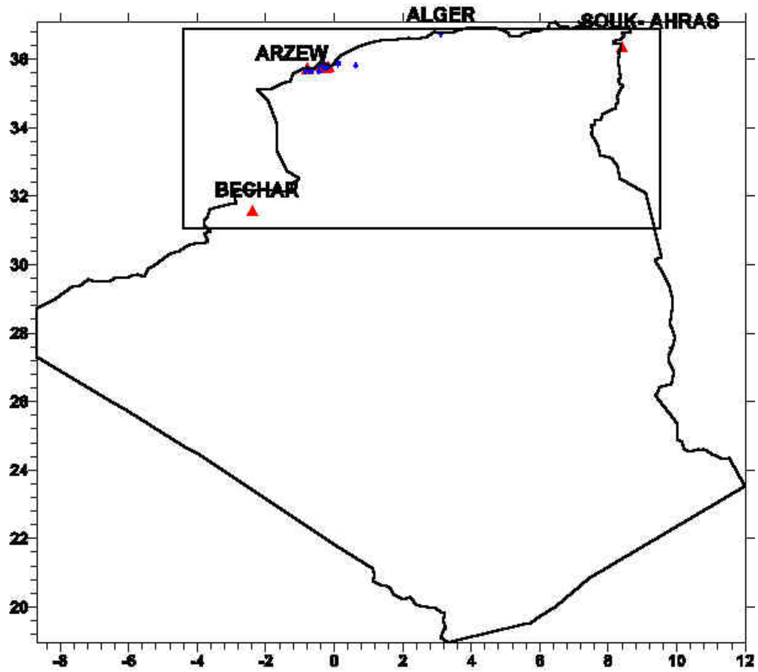

Figure 5. Geographical distribution of GPS stations
(+: Control point, Δ: Benchmark point)

Conclusion

The new Algerian quasi-geoid was computed via Fast Fourier Transformation using the Remove-Restore procedure by integrating the new gravity data supplied by GETECH through the agreement between the National Centre of Space Techniques/Geodetic Laboratory and University of Leeds/GETECH. The comparisons of the new quasi-geoid with GPS/Levelling data provide, after fitting, an RMS differences about ± 11 cm for the north part of Algeria over distances of 1 to 1000 km and prove that good fit in the test area between the new quasi-geoid and GPS/levelling data has been reached. Unfortunately, the non-availability of GPS levelling data on the set of the country with a homogeneous distribution and sufficient density didn't allow to make a more reliable assessment on the quality of the computed geoid.

Finally, the results obtained were satisfactory, so in the near future, the new solutions will be proposed and the additional comparisons should be made for a complete error assessment of the new Algerian quasi-geoid. These will include new gravity data, topographic informations, and the new data of GPS/levelling in order to reach an acceptable accuracy on all the country.

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