A COMPUTER PROGRAM FOR AN ADJUSTMENT OF COMBINED GPS/LEVELLING/GEOID NETWORKS: CASE OF STUDY: NORTH OF ALGERIA

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Abstract: The combined use of global positioning system (GPS), levelling and geoid height information has been a key procedure in various geodetic applications and provides at the same time a very attractive evaluation scheme for the accuracy of gravimetric geoid models. The main goal of this paper is to propose a tool of adjustment of combined GPS/Levelling/Geoid networks. For this kind of adjustment, generally, two main types of unknowns are estimated; the gravimetric geoid accuracy and 2D spatial field that describe all the datum/systematic distortions between the available height data sets. Accordingly, two modelling alternatives for the correction field are programmed, namely a pure deterministic parametric model, and a hybrid deterministic and stochastic model.

In this context, and for a first attempt, a Fortran program has been developed translating in application this methodology using the Hirvonen model as analytic covariance function of the reduced signals and the four parameter model to describe all possible datum inconsistencies and others systematic effects between the available height data sets. The necessary data used to test this program, the adopted methodology, the computation procedure, and the obtained results will be presented.

Key words: Covariance function, Hybrid deterministic and stochastic model, levelling.

1. Introduction

As an important result of development in high technology, satellite based positioning system has become to use in geodesy and surveying professions. These developments made the measurement works more accurate, more practical and more economic.

In practice, a high accurate gravimetrically determined geoid is often computed by using the technical Remove-Restore. Such geoid can have a very high resolution and very high relative accuracy in the sense of the difference of the geoidal height. However, its absolute accuracy in the sense of the determined geoidal height itself, is currently very poor due to the systematic errors caused by the difference in reference systems, the long wave-length errors of the geopotential model used as reference in the computation of the geoid, the biases existing in the gravity data and in the digital terrain model (DTM), etc.

In the other hand, we can now measure by means of the space techniques, on land through a combination of GPS positioning and precise Levelling and at sea through satellite altimetry, the geoid on some points on the earth's surface with very high absolute and relative accuracy, especially when the GPS coordinates have been attached directly to VLBI or SLR stations.

Consequently, the gravity solution, which has very high resolution and relative accuracy but poor absolute accuracy, and the GPS levelling solution, which has poor resolution but very high accuracy can be combined in the same adjustment (Jiang & al., 1996).

However, the fitting the gravimetric geoid to a set of GPS levelling points by using the Least Squares adjustment permits to estimate the residuals v_i which are traditionally taken as the final external indication of the network accuracy. The main problem under this method is that the v_i terms will contain a combined amount of GPS, levelling and geoid random errors that need to be separated into individual components for a more reliable geoid assessment.

The objective of this paper is to propose a tool of adjustment of combined GPS/ Levelling/ Geoid networks. For this purpose, two modelling alternatives for the correction field are programmed namely a pure deterministic parametric model, and a hybrid deterministic and stochastic model (Kotsakis & al. 1999). The program developed in this framework allows to estimate the gravimetric geoid accuracy and to compute the 2D spatial field parameters that describe all the datum/systematic distortions between the available height data sets.

2. Mathematical model for the combined adjustment

In this section, the theory of general adjustment model will be reviewed briefly in order to describe the methodology adopted in the setting of this work. For detailed aspects of combined adjustment of different heights data sets can be found in Kotsakis & al. (1999).

Let us assume that at each point P_i of a test network composed the m points, we have a triplet of height observations (h_i, H_i, N_i), or equivalently one synthetic observation:

$$\mathbf{l_i} = \mathbf{h_i} - \mathbf{H_i} - \mathbf{N_i} = \mathbf{a_i^T.x} + \mathbf{s_i} + \mathbf{v_i^h} - \mathbf{v_i^H} - \mathbf{v_i^N} \qquad \textbf{(1)}$$

where $\mathbf{h_i}$, $\mathbf{H_i}$ and $\mathbf{N_i}$ denote the available observed values for the GPS, orthometric and geoid height respectively. The $\mathbf{v_i}$ terms describe the zero mean random errors, for which a second-order stochastic model is available:

$$\mathbf{E}\{\mathbf{v_h v_h^T}\} = \mathbf{C_h} = \sigma_h^2\mathbf{Q_h}\ ,\ \mathbf{E}\{\mathbf{v_H v_H^T}\} = \mathbf{C_H} = \sigma_H^2\mathbf{Q_H} \ \text{ and } \ \mathbf{E}\{\mathbf{v_N v_N^T}\} = \mathbf{C_N} = \sigma_N^2\mathbf{Q_N} \qquad \textbf{(2)}$$

and Q_h, Q_H and Q_N denote the cofactor matrices.

For the orthometric height, the covariance matrix $\mathbf{C_H}$ is determined from the adjustment of the levelling network. In the same way, $\mathbf{C_h}$ can be computed from the adjustment of the GPS surveys performed at the levelled benchmarks. However, the covariance matrix C_N is determined by means of the error propagation from the original noisy data used in the geoid solution (Kotsakis & al., 1999).

By means the matrix notation, the equation **(1)** can be written under the form:

$$\mathbf{L} = \mathbf{AX} + \mathbf{s} + \mathbf{Bv} \qquad \textbf{(3)}$$

where $l = [l_1 .... l_i .... l_m]^T$; vector of measured quantities set.

$\mathbf{s} = [\mathbf{s_1 .... s_i .... s_m}]^T$; vector of the signals.

$\mathbf{v} = [\mathbf{v_h^T \ \ v_H^T \ \ v_N^T}]^T$; residual random noise. $\mathbf{B} = [\mathbf{I_m \ \ \text{-}I_m \ \text{-}I_m}]^T$; I_m : m x m unit matrix.

A is a given (m x n) matrix expressing the effect of the parameters **X** on the observation $\mathbf{l_i}$; it is sometimes called ″ sensitivity matrix″. The expression **AX** is usually obtained by linearizing an originally non-linear function of the (n) parameters; it corresponds in our case to all necessary reductions that need to be applied to the original data in order to eliminate the datum inconsistencies and other systematic errors in heights data sets. Another function s, the ″signal″, irregularly oscillating about zero; it is assumed that this quantity has an expected value equal to zero. Finally, X is a (n x 1) vector of unknown non-random parameters.

The problem is to determine this curve AX + s by means of discrete observation l, which are furthermore affected by observational errors v. It clearly appears that the adopted general adjustment model is analogous to collocation model with parameters; it is a synthesis between adjustment and prediction.

The solution of the general adjustment model (3) satisfy the minimum condition:

$$\mathbf{s}^T\mathbf{Q}_s^{-1}\mathbf{s}+\mathbf{v}_h^T\mathbf{Q}_h^{-1}\mathbf{v}_h+\mathbf{v}_H^T\mathbf{Q}_H^{-1}\mathbf{v}_H+\mathbf{v}_N^T\mathbf{Q}_N^{-1}\mathbf{v}_N=\mathbf{min} \qquad (4)$$

with $\mathbf{Q}_s^{-1}$ being an appropriate weight matrix for the unknown correction signals.

One of the main difficulties in this approach is that the mean value $\mathbf{m_s}=\mathbf{E}\{\mathbf{s}\}$ of the stochastic signals will not necessary be zero, due to the systematic behaviour that is supposed to exist in their values. In order to avoid such problem, one can initially solve the system (3) using (4) with a unit signal weight matrix. The initial solution for the signal part (Kotsakis & al., 1999),

$$\begin{aligned}
\mathbf{Q} &= \left(\mathbf{Q_h}+\mathbf{Q_H}+\mathbf{Q_N}+\mathbf{I_m}\right)^{-1}\\
\mathbf{W} &= \mathbf{I_m}-\mathbf{A}\left(\mathbf{A}^T\mathbf{Q}\mathbf{A}\right)^{-1}\mathbf{A}^T\mathbf{Q}\\
\hat{\mathbf{S}}_{\mathbf{init}} &= \mathbf{QWl}
\end{aligned} \qquad (5)$$

Now, we can create the following reduced observations and signals:

$$\mathbf{l_r}=\mathbf{l}-\mathbf{m_s} \quad \text{and} \quad \mathbf{s_r}=\hat{\mathbf{s}}_{\mathbf{init}}-\mathbf{m_s} \qquad (6)$$

It is now clear that the reduced signals s_r have zero mean. Furthermore, and in order to compute the cofactor matrix Q_{s_r} and to predict the signal s at points other than the measuring points, it is necessary to have an analytical covariance function of signals. In this context, the numerical values s_r will be used for an empirical determination of a covariance function model describing the average spatial behaviour of the reduced signals s_r and to select consequently the appropriate analytical covariance model.

In this way, we can repeat the adjustment of the model (3) using a new version for the stochastic model of the correction signals:

$$\begin{aligned}
&\mathbf{l_r}=\mathbf{AX}+\mathbf{s_r}+\mathbf{Bv}\\
&\mathbf{E}\{\mathbf{s_r}\}=\mathbf{0} \quad \textbf{and} \quad \mathbf{E}\{\mathbf{s_r}\mathbf{s_r}^T\}=\mathbf{C}_{\mathbf{s_r}}=\sigma_{\mathbf{s_r}}^2.\mathbf{Q}_{\mathbf{s_r}}
\end{aligned} \qquad (7)$$

Finally, the solution of the adjustment model (1) by using (7) will be given by the following unbiased estimators:

$$
\begin{aligned}
Q &= \left(Q_h + Q_H + Q_N + Q_{s_r}\right)^{-1} \\
W &= I_m - A\left(A^T Q A\right)^{-1} A^T Q \\
\hat{X} &= \left(A^T Q A\right)^{-1} A^T Q l_r \\
\hat{v}_h &= Q_h Q W l_r \,,\ \hat{v}_H = -Q_H Q W l_r \,,\ \hat{v}_N = -Q_N Q W l_r ,\ \hat{S}_r = Q_{s_r} Q W l_r
\end{aligned}
\tag{8}
$$

We note that the deterministic approach is obtained from the collocation approach by omitting the presence of the residual correction signals s. In this case, the general adjustment model (3) will be reduced to the form:

$$L = AX + Bv \tag{9}$$

The final solution of equation (9) is deducted of the solution given by (8) while putting $\mathbf{Q}_{s_r} = 0$. This solution satisfies two different but equivalent minimum conditions, both of which have been given already by Gauss: least squares and minimum variance. The well known least squares condition for the adjustment model (9) is:

$$\mathbf{v}_h^T \mathbf{Q}_h^{-1} \mathbf{v}_h + \mathbf{v}_H^T \mathbf{Q}_H^{-1} \mathbf{v}_H + \mathbf{v}_N^T \mathbf{Q}_N^{-1} \mathbf{v}_N = \mathbf{min} \tag{10}$$

3.Correction surface model

In practice, the various wavelength errors in the gravity solution may be approximated by two kinds of functions in order to fit the quasigeoid to a set of GPS levelling points. The first model is based on a general 7-parameter similar datum shift transformation with its simplified 4-parameter model for only the geoid determination purpose. The second is a polynomial regression with its simplified case, a planar regression.

In this work and in the aim to minimise the long wavelength errors, the systematic datum differences between the gravimetric geoid and the GPS/levelling data were removed by using the following four-parameter transformation equation:

$$\mathbf{a}_i^T \mathbf{x} = \mathbf{x}_0 + \mathbf{cos}(\varphi_i).\mathbf{cos}(\lambda_i)\,\Delta\mathbf{X} + \mathbf{cos}(\varphi_i).\mathbf{sin}(\lambda_i)\,\Delta\mathbf{Y} + \mathbf{sin}(\varphi_i)\,\Delta\mathbf{Z} \tag{11}$$

where $\mathbf{x}_0$ is the shift parameter between the vertical datum implied by the GPS/levelling data and the gravimetric datum, and $\Delta\mathbf{X}$, $\Delta\mathbf{Y}$ and $\Delta\mathbf{Z}$ are the three translation parameters in X, Y, Z axes.

Furthermore, and in order to perform the general adjustment model using the collocation approach in which the signal part is considered as additional stochastic parameters, an analytical expression of the covariance function of reduced signals is more convenient. For this purpose and in our case, the Hirvonen model is adopted as optimal analytic covariance function of the reduced signals, which is given by:

$$C(\psi) = \frac{C_0}{1 + \left(\frac{\psi}{\zeta}\right)^2} \tag{12}$$

where C_0 : variance of the reduced signals,

ζ : denote the correlation distance.

The use of this model as a local covariance function requires the estimation of two parameters: the variance of the reduced signals and the correlation distance. These parameters are obtained by fitting the Hirvonen function to a number of empirical covariance values employing the least squares adjustment.

The empirical covariance function of the reduced signals was computed with a new program developed in the framework of this work using the following formula:

$$C_{ss}(\psi) = \frac{1}{M}\sum s_i.s_j \qquad (13)$$

where M is the number of combinations, ψ is the spherical distance between Q_i and Q_j and s is the reduced signal.

The summation was made for all the combinations of the data points Q_i and Q_j whose distance was comprised between $(\psi - \Delta\psi / 2)$ and $(\psi + \Delta\psi / 2)$, and here $\Delta\psi = 0.5$ minutes. The value of the sampling interval size $\Delta\psi$ represents the minimum distance between the benchmarks stations.

The obtained results of the empirical covariance function are identical to these calculate by the **EMPCOV** program of the **GRAVSOFT** software.

4. Numerical tests

4.1. Description of program

A Fortran program **ADJ_GLG** has been developed at National Centre of Space Techniques by using the general adjustment model described above. This program has two main objectives. The first aim is to adapt the gravimetric quasigeoid to a set of levelled GPS reference points, and the second aim is to proceed to meticulous and reliable analysis gravimetric geoid accuracy.

The present version will perform three different adjustments by using:

- The Deterministic approach,
- The collocation approach, and
- The Least squares adjustment without information a priori on the accuracy of height data sets.

In the last case, only the 2D spatial field that describes all the datum/systematic distortions between the available height data sets were estimated.

Special attention has been paid to the organisation of data of gravity geoid grid and the GPS levelling point to establish the observation equation matrix, and to save memory and to speed up computation.

The input data necessary to perform the **ADJ_GLG** program were: the grid of the geoid in standard and binary format, the levelled GPS benchmarks coordinates, and the variances-covariance matrices of the GPS/levelling and Geoid networks if they are available, otherwise the uniform accuracy of the ellipsoidal, orthometric and geoid heights will be used.

4.2. Data used

4.2.1. GPS surveys

Many GPS campaigns have been carried out in the past years in Algeria. Furthermore, in the framework of the TYRGEONET (*TYRhenian GEOdetic NETwork*) project, two sites located in the North of Algeria have been determined in the WGS84 system, which have been used later for the densification and improvement of accuracy of the local geodetic networks. The number of stations GPS used in this investigation was 41, which 18 are benchmarks of the first order levelling network, and the others belong to the second levelling network. All of these points are located in the north of Algeria whose 34 points are close to the station of Arzew (see fig. 2). The GPS observations were performed with four ASHTECH Z-12 dual frequency receivers with baseline length ranging from about 1 to 1000 km, and the BERNESE software with precise ephemerides was used to process the GPS data. The computed ellipsoidal heights were referred to WGS84 system and their standard deviations do not exceed 3 cm. So, in order to make possible the estimation of N (geoid undulation) in these points, all GPS stations have been connected to the national levelling network, which consists of orthometric heights. The accuracy of the levelling heights may be estimated to about 6 cm depending on the type of connection measurements.

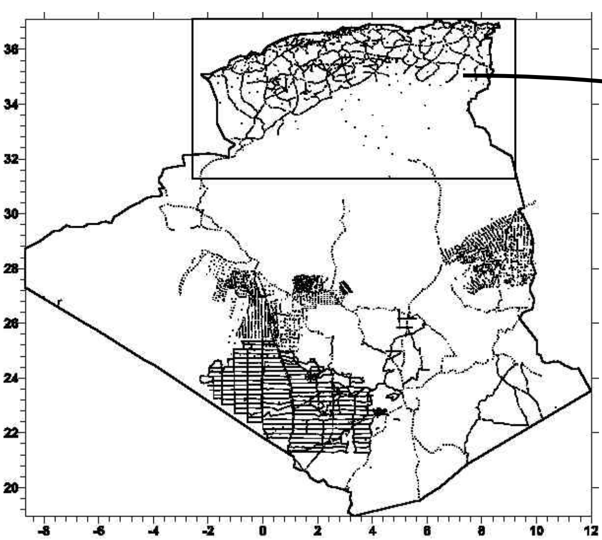

Fig 1. Geographical distribution of BGI gravity Measurements.

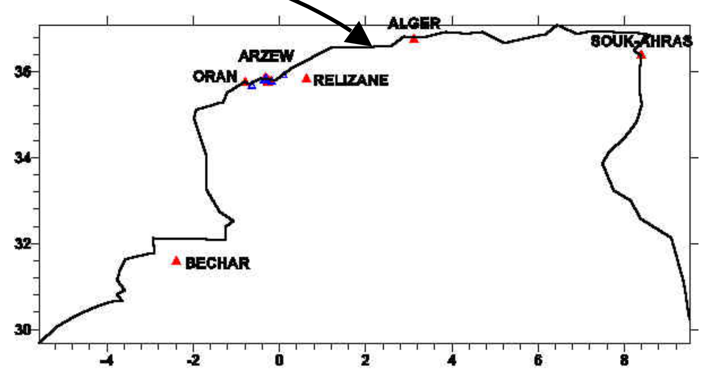

Fig 2. Geographical distribution of GPS stations (Δ : Benchmark, + : Control point)

4.2.2. Local geoid

In view the use of the GPS for the orthometric height computation, the National Centre of Space Techniques through the national projects of research, has recently focused a part of the current research on the precise geoid determination using different methods. In 1999 a new gravimetric geoid published in the IGes Bulletin, was computed over the whole of Algeria by the present author (Benahmed, 2000). This solution is based on the validated gravity data supplied by the BGI, topographic information and the optimal geopotential model OSU91A, which were combined using the remove-restore technique in connection with the Fast collocation method. The final result is a gravimetric geoid on a 5’ x 5’ grid in the area with the limits $20° \le \varphi \le 37°$ and $-7° \le \lambda \le 10°$. The Fig.3 shows a map of the quasi-geoid solution in Algeria contoured with 1m interval.

The selected zone to test this program is located in north of Algeria. The choice was emphazised by the relatively high density of the gravity points, and by the availability of the precise GPS stations covering the whole of the area (see fig. 1.).

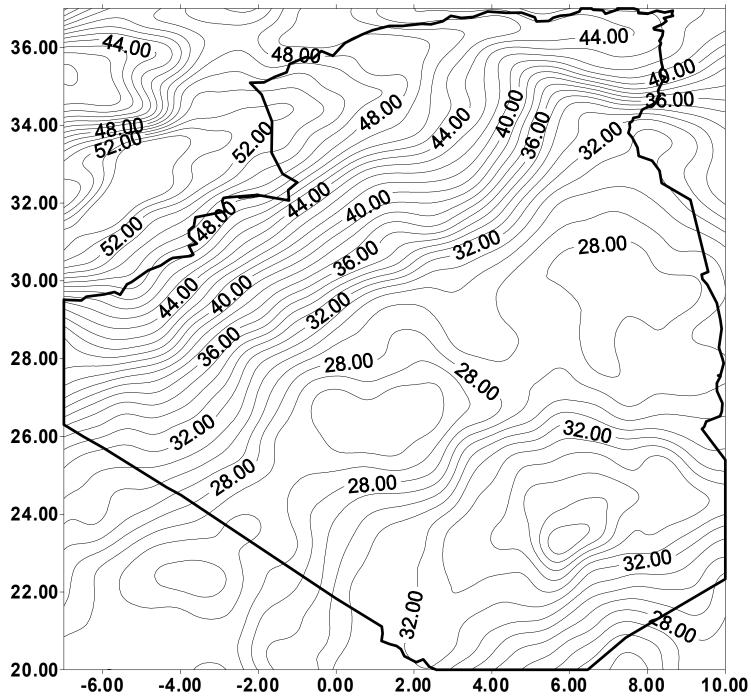

Fig. 3. Quasi-geoid solution in Algeria (m)

4.2.3. Practical results

Before applying the general adjustment model, a first computation by removing the long length effects using the four parameters transformation permits to confirm the existence of one suspect blunder in the GPS levelling measurements. The result of the pre-adjustment shows that the combined method is very effective for detecting the blunder errors in the GPS levelling measurements a condition that the short wavelength of the geoid are very modelling.

After one rejection, 40 GPS levelling points have been selected but only 18 well distributed GPS levelling points are used as benchmarks points, and all the other points were excluded from the combined adjustment in order to estimate the real accuracy given by the comparison between the adjusted values and the known ones. Moreover, and since the variance-covariance matrix of the GPS and levelling networks adjustments necessary for this kind of combined adjustment are not available, we have used the a priori uniform accuracy of the ellipsoidal, orthometric fixed to 3 cm, 6 cm respectively according to networks accuracy. Also and in absence of the prior geoid error model, we have used a unit weight matrix and get an estimate for the a posteriori unit geoid variance.

The following tables give the obtained results of the general adjustment model by using the collocation approach. For the very long-wavelength errors, the four-parameter similitude datum shift transformation model was determined. The values of parameters are presented in the table 1. The fig. 4 shows the empirical and analytic covariance functions of the reduced signals in benchmarks points. The bad behaviour of the empirical function is principally due to statistical character of the reduced signals in benchmarks and to the total number of the GPS stations used in the computation of the empirical values, which is too small relatively to experimental area size. Furthermore, the most GPS points are close to Arzew station.

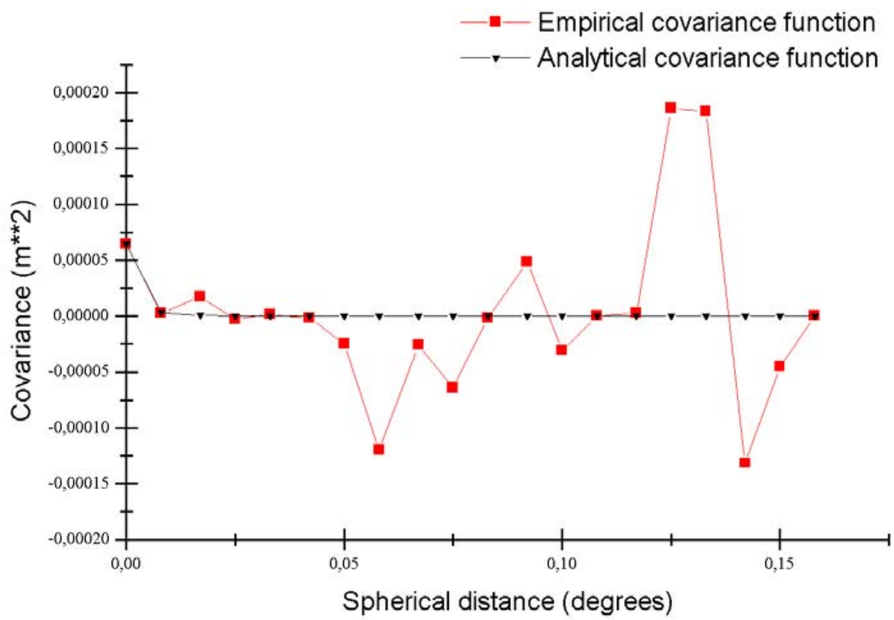

Fig 4. Covariance functions of reduced signals

The tables 2 and 3 summarise respectively the statistics of the differences between the geoid height determined by GPS/Levelling and gravimetric quasigeoidal heights after fitting out the systematic biases using a four-parameter transformation, and the individual components of adjusted GPS, levelling and geoid residuals. The first statistics show that a good fit between the gravimetric quasigeoid and GPS/Levelling has been reached, but we do not believe that the rms value is the real accuracy of the determined geoid, this provided proof that the combined adjustment can optimally fit the gravity geoid to the GPS levelling point in the least square sense. However, the second statistics prove that the residuals in benchmarks are due particularly to gravimetric quasigeoid errors.

The standard deviation of the discrepancies between the gravimetric quasi geoid solution and GPS levelling geoid undulations at control points amounts to ± 5cm after fitting. This fact confirms the good fit in the test area between the Algerian quasigeoid and GPS/levelling data has been reached.

Finally, we note that the three approaches described above give the similar values of parameters and the residuals. It is due to the fact that the variance-covariance matrices of different heights data sets used in this investigation are the diagonal matrices and the estimated signals are the very small quantities.

Parameters	Value	RMS
ΔX (m)	-750.482	14.925
ΔY (m)	-40.376	1.149
ΔZ (m)	-497.213	9.155
Scale factor	0.00014090	0.000003

Table 1. Four-parameter transformation model.

Mean	Min.	Max.	RMS
0.000	-0.030	0.055	0.020

Table 2. Residuals after fitting the Gravimetric geoid to 18 GPS Levelling points (in meters)

	Mean	Min	Max	RMS
v_h	0.000	-0.001	0.001	0.000
v_H	0.000	-0.003	0.002	0.001
v_N	0.000	-0.051	0.028	0.017

Table 3. Individual components of GPS, levelling and geoid residuals (in meters).

Mean	Min	Max	RMS
0.017	-0.074	0.104	0.047

Table 4. Statistics of differences between the gravimetric geoid undulations and GPS levelling at 22 control points (in meters).

Conclusion

This paper presents the new tool for an adjustment of combined GPS/levelling/geoid networks. The proposed methodology is interesting specially when we envisage to use the GPS techniques for levelling purposes with respect to a local vertical datum, and when we want to proceed to a meticulous and reliable analysis of relative observations to different heights. Two modelling alternatives for the correction field are programmed namely a pure deterministic parametric model, and a hybrid deterministic and stochastic model. So, the developed program can be used for testing the reliability of preliminary geoid error models, which have been derived via internal error propagation from the source data and their noise used in the gravimetric solution.

However, the difficult step in the application of general adjustment model using the collocation approach is the estimation of the covariance function of reduced signals and subsequently the selection of its corresponding analytic representation. In this context, the special attention will be made for the optimal choice of a local covariance model of the reduced signals capable to describe their spatial behaviour.

In the field experiment introduced above, the results of the numerical test show that:

- The estimated signals don't have any influence on the obtained results since their magnitude varies between 10^{-6} and 10^{-7} meters.
- A good fit between the Algerian quasigeoid and GPS/levelling has been reached, it proves clearly that the combination of GPS/levelling and geoid models is capable to produce orthometric heights with an accuracy acceptable for the low order levelling network densification
- The residuals in benchmarks are due mainly to gravimetric quasigeoid errors in the test area.

Finally, the results obtained were satisfactory, so in the near future the new adjustment will be performed for a more reliable geoid assessment. This will include an accurate gravimetrically geoid computed in the whole of Algerian territory by integrating the maximum of new gravimetric, topographic and geodetic informations and the new data of GPS/levelling. Furthermore, it is necessary to add in the general adjustment model the variance-covariance matrices of different height data sets in order to take account of the correlation between data.

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