Proposal for the precise definition of mean values of gravity field quantities

by

C.C. Tscherning

Department of Geophysics,
University of Copenhagen.
Juliane Maries Vej 30,
DK-2100 Copenhagen Ø., Denmark
e-mail: cct@gfy.ku.dk

1. Introduction.

Some of the products of the GOCE mission are mean values of gravity anomalies and geoid heights. In order to avoid ambiguities a precise definition of these quantities is needed. First mean values are defined in a general manner, so it can be used for any gravity field related quantity, and finally the definition of specific GOCE mean values are proposed.

2. Mathematical definition of a mean value.

A mathematical definition of a mean value is the mean of values of a function or of a functional applied on a function,

$$f : \Omega \subseteq R^n \to R$$

The mean must be taken over a continous or discrete, bounded subset of Ω. The subset must have dimension less than or equal to n, which for the gravity field are 4 for time dependent quantities and otherwise 3. See for example Heiskanen and Moritz (1967, eq. (7-76)).

<u>Examples</u> are for

1D: discrete or contineous mean along a flight, ship or satellite track bounded in time or space.
2D: - discrete or contineous mean over a 2D surface bounded by parallels and meridians having a fixed ellipsoidal height (equi-angular or equal-area mean values).
- mean over an area bounded by a closed curve, such as over a lake.
3D: discrete or contineous mean over a volume defined by the coordinates of the corners of a convex area such as a sphere or a box.

DMA/NIMA 1^{o} mean gravity anomalies are for example defined as the mean of 6x6 values in an equi-angular area (see DMAAC, 1973). The associated altitude is the mean topographic height.

3. Definition of a gravimetric quantity.

Let

- W be the gravity potential of the Earth,
- Φ the centrifugal potential
- U is the Somigliana-Pizetti normal potential of a specific reference system
- V_N a linear combination of N harmonic functions such as spherical or ellipsoidal harmonics to degree (and order) $\sqrt{(\mathbf{N}+1)^2}$ without the centrifugal potential, and
- $T_N = V_N + \Phi$ - U
- T=W-U

We do not require the zero and first order harmonics of T or T_N to be zero. V_N may also be a linear combination of potentials of point-masses or of covariance functions used in Least-Squares Collocation.

A gravimetric quantity is here

1) a functional applied on W, V_N , T or T_N , including or excluding effects of the atmosphere and the tides. Normal gravity γ in a point Q will be the magnitude of the gradient of U in the point.
2) an observed quantity like (a) the potential W(P) in a specific point P in a given datum, (b) g(P) the magnitude of the gravity vector in a point P, in a given reference system (one for P and one for g), (c) the second order radial derivative of W, W_{rr}(P) in a given point P. The altitude associated with P is always the ellipsoidal height.
3) an anomalous quantity, i.e. a functional applied on T or T_N .

For GOCE products anomalous quantities will be derived from $T_N = V_N + \Phi$ - U.

A GOCE gravity anomaly Δg_N is then the difference between the norm of the gradient $\gamma_{\mathbf{N}}$ computed from $U_N = V_N + \Phi$ and normal gravity γ in a point Q on the same ellipsoidal normal, and where U_N (Q)= W(P). The linearized expression for the gravity anomaly will be for example,

$$\Delta g_N = -\frac{\partial T_N}{\partial n} - \frac{\partial \gamma_N}{\partial h} T_N / \gamma_N$$

The derivative must be computed in the direction of the vertical as defined by U_N . The height anomaly ς_N is the distance along the ellipsoidal normal between the points P and Q. Note that P is on or outside the surface of the Earth, while Q may be inside.

The geoid height is obtained from the height anomaly at the surface of the Earth using a fixed conversion formula, such as the one which include the Bouger anomaly, see Heiskanen and Moritz, 1967, eq. (8.103).

A height anomaly may be computed from observed quantities at the surface of the Earth, e.g. from normal heights and ellipsoidal heights obtained using GPS. In contrast to geoid heights they do not include any hypothesis about the internal mass-distribution of the Earth. Such hypotheses are different from country to country, mainly due to different mass density values used to compute the Bouguer anomaly.

4. Computational considerations

Using the proper definitions of GOCE height anomalies and gravity anomalies it will not be difficult to evaluate these quantities at specific points, and subsequently compute a mean value. However in order to evaluate the quantities they must be compared with observed quantities. This is more problematic. Existing mean gravity anomalies have not been computed using procedures corresponding to the definitions above, see e.g. Wilcox(1974), Jones(1980).

First of all different procedures have been used to reduce the observed anomalies to a surface of constant height. Furthermore this height is generally taken as the mean topographic height, i.e. the anomalies are partly defined inside the masses. This should be avoided. But this means that mean gravity anomalies to be used for GOCE calibration must be recomputed. However, it is not the purpose of this note to discuss this aspect. But what is important is that the procedures must be rather easy to implement on a computer, and that the procedure adopted corresponds to the procedures to be used for the computation of GOCE height or gravity anomalies.

4. Conclusion. Proposed definitions of mean values

For computational use it is important that T_N (as determined by GOCE) is harmonic outside the surface of the Earth, i.e. it should not include any tidal or atmospheric effects.

Mean values of GOCE gravity anomalies and height anomalies should be defined as weighted sums of quantities associated with points which all are on or outside the surface of the Earth. However mean values in general may be computed from any of the 4 types of quantities discussed in section 2.

For 2D means, the mean values should be over surfaces with constant ellipsoidal heights. Area means are "equi-angular" means. A standard such as a 5x5 or 6x6 point set should be selected in accordance with NIMA definitions.

If geoid mean values have to be computed, the basis is height anomalies at the surface of the Earth. The corresponding geoid heights must then be computed using the conversion formula used by the country in which the block resides. For blocks overlapping two countries which use different hypothesis, they should not be computed. Note, that the conversion generally require that also the Bouguer anomaly is known for the points used to compute the mean value.

References:

DMAAC: Computational methods for determining 1 deg. x 1 deg. mean gravity anomalies and their accuracies. DMAAC Ref. Publ. 73-001, 1973.

Heiskanen, W.A. and H. Moritz: Physical Geodesy. W.H. Freeman & Co, San Francisco, 1967.

Jones, S.: A comparative study of the economy and accuracy of various methods of predicting 1 deg. x 1. deg. mean gravity anomalies. Thesis Washington University, Saint Louis, 1980.

Wilcox, L.: An analysis of gravity prediction methods for continental areas. DMAAC Ref. Publ., No. 74-001, 1974