

I.Ge.S.

BULLETIN N. 1

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FOREWORD

We finally got it!

It has been a heart-breaking four years fight with italian authorities but in the end thanks to the support of the "Istituto Nazionale di Geofisica", of the "Consiglio Nazionale delle Ricerche" and of the "Politecnico di Milano" we could collect staff and tools enough to start the activities of the International Geoid Service (I.Ge.S.).

Naturally the two researches appointed are working for the moment under a temporary contract, yet we decided to start with the hope of being able to prove that our work was so needed and maybe appreciated that it would be worthwhile to set up a definitive structure with fixed positions.

We try to explain in the next pages what the I.Ge.S. is and how we think it should work; here I just want to ask for your help.

You are the ones that will really decide whether such a structure is necessary and useful to organize worldwide a precise determination of the geoid for both scientific and technical purposes and this Bulletin could be a proper instrument to prove it.

You will find here the news about our work, our proposals and, probably in one year, precise information about our archives, how to supply or obtain data, software, expertise, etc.

But you can also send us your letters or contributions that maybe you think are not suited for major scientific journals; we will do our best to publish this material in one of the two numbers that we foresee to publish each year.

For the moment we are sending this publication with no charge, but in the next number we will add instructions on how to help us from the economical point of view since each issue is going to be rather expensive for us.

This is all for the moment. I wish to all of you a good work on the geoid and a long life to the International Geoid Service.

Fernando Sansò

INTRODUCING THE INTERNATIONAL GEOID SERVICE

F. Sansò^{*} and F. Migliaccio^{**}

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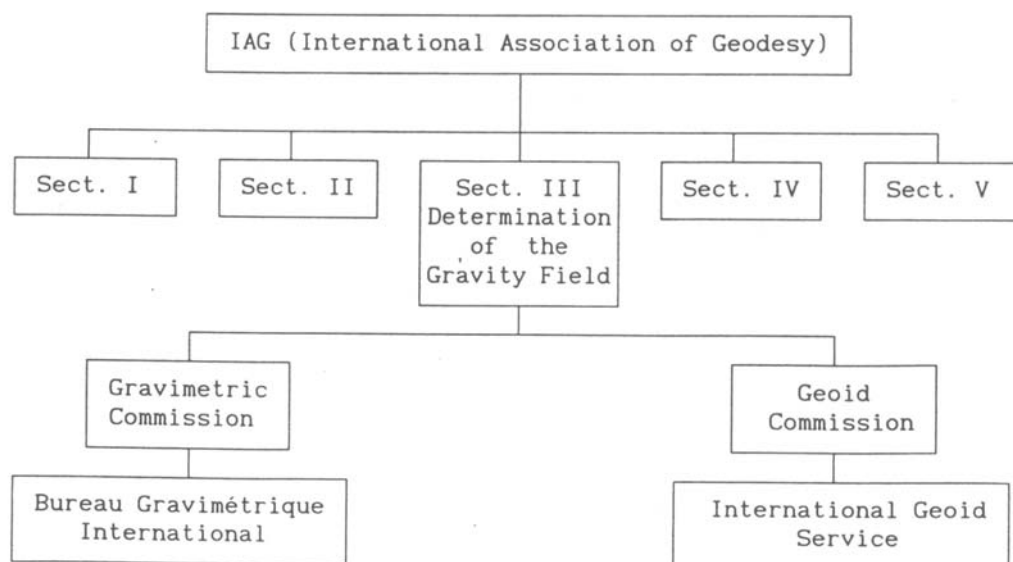
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As announced in the circular letter issued in June 1992 by Bulletin Géodésique, the International Geoid Service has finally been established in the summer of 1992, and became operational since last September.

The proposal for the new service dates back to 1988, when the International Geoid Commission was created in the frame of IAG Section III.

The President of IGC, Prof. R. Rapp, proposed to establish the IGeS as the operational group of IGC "working for the benefit of the international scientific community without any commercial interest".

The general framework in which the IGeS was therefore intended to operate can be represented in the following way:



The actual establishment of IGeS in Italy requested Prof. Sansò to work hardly during the years 1988-1992 to ensure that the new structure be scientifically and financially supported by official research organizations in Italy.

The CNR (Italian National Research Council) seemed to be the natural partner for such a sponsorship, but also ING (Italian National Institute for Geophysics) was interested in the activities of the new service.

Besides, the premises of the IGeS were envisaged to be properly located at the Politecnico in Milan, where Prof. Sansò has his research group.

So in the end the newborn service is working under the support of three distinct bodies: the CNR, the ING and the Milan Politecnico.

As a beginning, the staff of the IGeS is represented by 2 researchers and 1 technical assistant reflecting the sponsorships supporting the structure: the technical assistant is from the Politecnico, while the researchers are from the CNR and the ING.

The tasks the IGeS should perform are (as already mentioned by Prof. Sansò in his June circular letter) the following:

- 1) collecting data related to geoid determination that are not already systematically collected by other agencies or services;
- 2) making sure that all data sets given to IGeS are properly documented;
- 3) collecting the available software for the geoid determination, according to the widest sample of various methodologies, verifying that the software is properly documented and complete with test examples;
- 4) computing geoids in exceptional cases as defined by the Executive Committee of the International Geoid Commission, in support of national and scientific objectives;
- 5) working both theoretically and practically to the problem of merging regional geoids into larger solutions;
- 6) distributing the available data sets and software with the

necessary documentation upon request;

- 7) organizing, when the need arises, courses for users who would like to acquire the necessary knowledge to perform geoid computation on their own;
- 8) preparing and issuing a bulletin describing the current activities and the information available at IGeS;
- 9) pursuing any other task that the EC of the IGC would assign to it.

However, for the first term it was decided to concentrate the activities of the IGeS on the following items:

- to collect and test geopotential global models and the relative software to produce several kinds of functionals of the geopotential at prescribed locations;
- to collect and document preprocessing softwares, including the first statistical tests on data fields, the rejection of the outliers and gridding;
- to start issuing an annual bulletin providing information on the service;
- to start a first course of the International School of Technical Training for the geoid computations, concerning the use of global models;
- to participate in some outstanding international research projects, related to the determination of the geoid.

In order to collect and process data and develop software, the IGeS relies upon its own computing center.

The available hardware can be represented by the following scheme:

The programmes presently carried on by the IGeS staff on the scientific front regard two main research lines. The first is the Geomed Project in which 7 scientific groups from 6 nations (Austria, Denmark, Finland, Greece, Italy and Spain) are involved: the purpose is to set up an international cooperative effort to determine the geoid and the Sea Surface Topography on the Mediterranean sea.

The second is the study of satellite gradiometry which aims at the recovery of the anomalous gravity field directly from the data collected by a gradiometer on board a low-orbiting satellite.

Besides, a specific collaboration with the Hannover Computing Center is foreseen in order to cooperate in the determination of the European Gravimetric Geoid.

Regarding the technical aspects of the IGeS programmes, the main concern is now with collecting and validating geoid data, altimetric data and digital terrain models (DTMs) to start setting up the IGeS data base. It is clear that for gravity matters the IGeS will work in cooperation both with the Bureau Gravimétrique International (BGI) and with the West-East Europe Gravity Project (WEEGP).

The third front on which the IGeS will act in the next months is of course the educational one.

An International Technical School for the determination and use of the geoid has been announced and planned in Milan on October 25 to 29, 1993. It will focus on "The Determination and Use of Global Models of the Anomalous Gravity Potential" (see Annex).

The school will introduce the students into the problems of computing and using global models of the anomalous gravity potential, with applications to different contexts and to several functionals of the potential.

QUESTIONNAIRE

As we are starting now with our work, we feel it is essential for us to undertake a first step in getting acquainted with what is around about the geoid.

So we thought first of all to exploit the excellent work done by our Canadian colleague A. Mainville to send a questionnaire to all geoid users to know who are they and their needs.

As an example we publish also the statistics of the answer gotten in Canada, without Appendices for space reasons.

This will hopefully tell us who needs the geoid.

The next natural question is who makes the geoid and, also important from the point of view of this service, whether this geoid will be made available.

This is the reason for the second questionnarie to all geoid producers.

We hope that all the institutions and colleagues interested for any reason in the geoid will make the effort of answering and we will try then to make the point on these questions in the next number of the bulletin.

Fernando Sansò

TO ALL GEOIDS USERS

Is the Geoid Important to You ?

The geoid is the reference surface for most heights/elevations you may require. With the advent of the artificial satellite era more people need geoid information. Attached you will find a list of geoid applications and users. We can provide users with a simple, fast and computerized geoid solution that can be used in most applications, even in near real-time or large volume applications. The accuracy varies from one region to another, but should satisfy most users. We can also provide the geoidal heights and deflections of the vertical tabulated in a book from which one can interpolate. An example of a table for geoidal heights is attached. Other tables are available for deflections of the vertical, gravity anomalies, etc. As an overview, we have enclosed a map showing the separation between the geoid in Canada and the mean Earth ellipsoid.

This is a survey being carried out by:

Geodetic Survey Division
Canada Centre for Surveying
Surveys, Mapping and Remote Sensing Sector
Department of Energy, Mines and Resources
Ottawa, Canada

This survey was proposed by and prepared with the help of the Canadian Geoid Committee which advises the Minister of EMR in these matters. Feel free to contact any members of this committee; a list of names is enclosed. Please, fill in this questionnaire or have it completed by your knowledgeable staff. You should keep a copy of this questionnaire for reference. Returning the questionnaire assures you of receiving a copy of the results of this survey.

Is the Geoid Important to You ?

1. Which geoid solution(s) do you use ?

- Global satellite solution
 - GEM9 ☐
 - GEML2 ☐
 - GEMT1 ☐
 - Other: _____ ☐
- Satellite solution with global gravity
 - GEM10B ☐
 - RAPP78 ☐
 - RAPP81 ☐
 - OSU86F ☐
 - WGS 84 (American) ☐
 - GPM2 (European) ☐
 - Other: _____ ☐
- Satellite solution with local gravity
 - UNB Dec'86 ☐
 - Least-squares collocation technique ☐
 - Fast Fourier technique (FFT) ☐
 - Truncated kernel ☐
 - Other: _____ ☐
- Marine (satellite altimetry derived) solution ☐
- None but I would if :
 - it was easily available ☐
 - it was more accurate ☐
 - it was cheaper ☐
 - Other: _____ ☐

2. From what method or in what format would you like to receive geoid information : _____

Is the Geoid Important to You ?

3. For what application(s) do you plan to use the geoid?

- | | |
|--------------------------------------|--------------------------|
| • Control Surveys (for mapping) | <input type="checkbox"/> |
| • Boundary surveys / legal surveys | <input type="checkbox"/> |
| • Engineering projects | <input type="checkbox"/> |
| • Surface gravity surveys | <input type="checkbox"/> |
| • Aerial photogrammetric control | <input type="checkbox"/> |
| • Airborne geophysical surveys | <input type="checkbox"/> |
| • Satellite sensor calibration | <input type="checkbox"/> |
| • Satellite photogrammetric control | <input type="checkbox"/> |
| • Satellite remote sensing control | <input type="checkbox"/> |
| • Digital terrain model construction | <input type="checkbox"/> |
| • Inertial navigation systems | <input type="checkbox"/> |
| • Hydrographic projects | <input type="checkbox"/> |
| • Oceanographic projects | <input type="checkbox"/> |
| • Exploration geophysics | <input type="checkbox"/> |
| • theoretical geophysics | <input type="checkbox"/> |
| • Scientific interest | <input type="checkbox"/> |
| • Other: _____ | <input type="checkbox"/> |

for :

- | | |
|---------------------------------|--------------------------|
| • 3-D positioning | <input type="checkbox"/> |
| • Horizontal positioning | <input type="checkbox"/> |
| • Vertical (height) positioning | <input type="checkbox"/> |
| • Geophysical interpretation | <input type="checkbox"/> |
| • Other: _____ | <input type="checkbox"/> |

requiring :

- | | |
|------------------------------------|--------------------------|
| • Geoid values in real time | <input type="checkbox"/> |
| • Precise geoid values | <input type="checkbox"/> |
| • Large volume of geoid values | <input type="checkbox"/> |
| • Geoidal height difference values | <input type="checkbox"/> |
| • Other: _____ | <input type="checkbox"/> |

4. Please describe your geoid applications : _____

Is the Geoid Important to You ?

5. In what regions or areas do you require geoid (gravity field) values ?

- in Canada
- outside Canada
- on land
- on oceans
- at plane altitude
- at satellite altitude
- Other: _____

☐
☐
☐
☐
☐
☐
☐

6. Do you find existing geoid solutions adequate ?

Yes ☐ No ☐

If no, explain why _____

Geoidal Heights

7. What accuracy do you need in order to make use of geoidal height (differences)?

- | | | | |
|------------------------------------|--------------------------|--------------------------------------|--------------------------|
| • ± 1 cm | <input type="checkbox"/> | • ± 10 cm | <input type="checkbox"/> |
| • ± 100 cm | <input type="checkbox"/> | • \pm _____ cm | <input type="checkbox"/> |
| • ± 1 cm for length of 1 km | <input type="checkbox"/> | • ± 100 cm for length of 1 km | <input type="checkbox"/> |
| • ± 1 cm for length of 10 km | <input type="checkbox"/> | • ± 100 cm for length of 10 km | <input type="checkbox"/> |
| • ± 1 cm for length of 30 km | <input type="checkbox"/> | • ± 100 cm for length of 30 km | <input type="checkbox"/> |
| • ± 1 cm for length of 100 km | <input type="checkbox"/> | • ± 100 cm for length of 100 km | <input type="checkbox"/> |
| • ± 1 cm for length of 1000 km | <input type="checkbox"/> | • ± 100 cm for length of 1000 km | <input type="checkbox"/> |

or

- \pm _____ cm for length of _____ km ☐

or

- Other : _____

Deflections of the vertical

8. What accuracy do you need in order to make use of deflections of the vertical ?

- | | | | |
|------------------------|--------------------------|-------------------------|--------------------------|
| • ± 2 arcseconds | <input type="checkbox"/> | • ± 1 arcsecond | <input type="checkbox"/> |
| • ± 0.1 arcseconds | <input type="checkbox"/> | • \pm _____ arcsecond | <input type="checkbox"/> |
| • Other : _____ | | | |

Is the Geoid Important to You ?

Gravity / gravity vector

9. What accuracy do you need in order to make use of gravity components ?

- | | | | |
|------------------|--------------------------|------------------|--------------------------|
| • ± 2 mgals | <input type="checkbox"/> | • ± 1 mgal | <input type="checkbox"/> |
| • ± 0.1 mgal | <input type="checkbox"/> | • \pm ___ mgal | <input type="checkbox"/> |
| • Other : | _____ | | |

10. Do you feel the accuracy you quoted can be met with the solutions available today ?

Yes ☐ No ☐

If no, explain why (format, accuracy, etc.) _____

11. Do you have any use for geoidal height (difference) accuracy estimates ?

Yes ☐ No ☐

12. From whom would you like to obtain geoid data ?

- | | |
|--------------------|--------------------------|
| • Federal Gov't | <input type="checkbox"/> |
| • Provincial Gov't | <input type="checkbox"/> |
| • Private Industry | <input type="checkbox"/> |
| • Other : | <input type="checkbox"/> |

13. How important is it to you that reliable geoid values be readily available ?

- | | |
|----------------------|--------------------------|
| • Not very important | <input type="checkbox"/> |
| • Important | <input type="checkbox"/> |
| • Very important | <input type="checkbox"/> |

14. Any other comments or suggestions are welcome: _____

Is the Geoid Important to You ?

If you wish further communication concerning the geoid ☐ or wish to receive the result of this survey ☐ please provide the following information :

Name : _____

Any correction required to your address :

Establishment : _____

Address : _____

Phone : _____

Fax : _____

or attach your business card.

Thank you very much for having filled this questionnaire.

Geodetic Survey of Canada: Results on a Geoid Questionnaire

By André Mainville

Energy, Mines and Resources Canada - Survey, Mapping and Remote Sensing Sector
Canada Centre for Surveying - Geodetic Survey Division
615 Booth Street - Ottawa - Ontario K1A 0E9

Abstract

As proposed by the Canadian advisory committee on geoid matters, a questionnaire on the importance of the geoid was prepared by the Geodetic Survey of Canada with the committee's collaboration, and sent to 965 of its users in November 1989. The purpose of the questionnaire was to determine the needs for geoid information in Canada and to evaluate the direction the geoid research and development is taking. Some highlights of the results are as follows. Out of 965 questionnaires, 125 have been returned. As seen from the 109 responses to question no.12 (From whom would you like to obtain geoid data? Federal Gov't, Provincial Gov't, Private Industry or other.), it is very clear that the Canadian community expects the federal government to show leadership in geoid matters. 54% of the respondents favor receiving geoid information from the federal government only. An additional 22% favor receiving geoid information from federal and provincial sources. As seen from the 109 responses to question no.13 (How important is it to you that reliable geoid values be readily available? Not very important, Important or Very important.), the availability of reliable geoid values is considered "very important" by 31% and "important" by an additional 38% of the responders. The paper summarizes the questionnaire results. The respondents have clarified the products required to satisfy their geoid needs. According to 68% of the respondents, on-going geoid solution refinement is required until a ± 1 cm precise geoid is obtained.

Introduction

A survey questionnaire entitled "Is the Geoid Important to You ?" was initiated by the Geodetic Survey of Canada and mailed on October 4, 1989 to 965 Canadian customers. The questionnaire was proposed by a Canadian advisory committee on geoid related matters in order to develop a profile of the geoid users' needs and to compile recommendations for the development and improvement of the gravimetric geoid in Canada.

The results of the questionnaire are tabulated and graphically represented by histograms and pie charts in the following. The questionnaire was distributed according to Table 1.

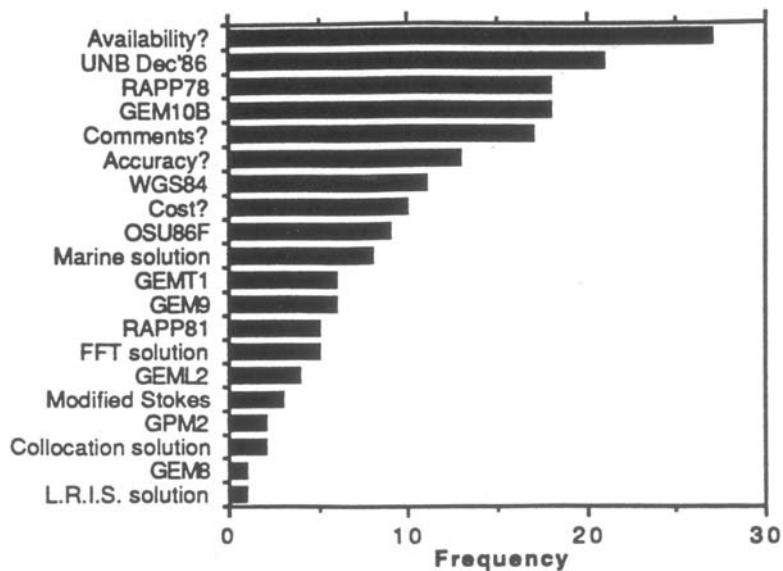
Table 1: Distribution of the questionnaire by regions.

Province	Questionnaires sent		Questionnaires returned		Questionnaires returned over sent
	#	%	#	%	%
NFLD	9	1	0	0	0
N.S.	27	3	4	3	15
P.E.I.	4	0.4	1	0.8	25
N.B.	39	4	9	7	23
Quebec	101	10	12	10	23
Ontario	241	25	19	15	8
NCR	226	23	22	18	10
Man.	49	5	10	8	20
Sask.	38	4	6	5	16
Albarta	132	14	21	17	16
B.C.	84	9	11	9	13
N.W.T.	8	0.8	1	0.8	13
Yukon	7	0.7	1	0.8	14
Unknown			8	6	
Total	965	100	125	100	13

In Table 1, NCR means the National Capital Region which represents public servants in the region of Ottawa. It includes 70 colleagues of the Geodetic Survey of Canada, people from EMR and other departments. The Ontario count does not include the NCR count. The questionnaire was sent to people/agencies who have requested vertical or horizontal coordinates from our Data Services section. Not all of these customers use geoid information. This list was thus updated to include people/agencies who have requested geoid information directly from the Systems Development section (SDS) where the geoid information is developed in-house. According to Table 1, out of 965 questionnaires 125 were returned, which results in a 13% return. On the other hand 48 out of the 78 people i.e. 62% of those who have dealt directly with SDS have returned the questionnaire. Out of 125 respondents only 5 did not identify themselves. A list of the respondents is given in appendix A. The list is sorted from east to west.

Question 1: Which geoid solution(s) do you use ?

Figure 1 - Answers to question 1



Out of 125 respondents, 91, or 73%, have answered this question. The histogram (Figure 1) for question no.1 summarizes the answers. Here is the analysis of each answer given in descending frequency of each answer.

1 - People question the availability of geoid solutions. More publicity should be done, by federal and perhaps by the provincial institutions too. When a standard geoid will have been determined, then it will be a simple matter to publicize it.

2 - The UNB Dec'86 solution (Vanicek et al, 1986) is more used than other solutions in Canada. The RAPP78 solution (Rapp, 1978) comes next because it was used in the continental horizontal network readjustment called NAD83. The GEM10B solution (Lerch et al., 1981) is next because it was used previously to RAPP78 in the beginning of the Doppler satellite era. These 3 solutions have been the most publicized solutions in Canada. The UNB solution was widely distributed because it is more accurate than RAPP78 and GEM10B and because it covers most parts of Canada. Different people have evaluated it and have had success using it. Its popularity shows that a more accurate solution and one that covers all Canada is favoured by Canadians.

3 - The following comments were received indicating interest:

- We intend to use it in the future.
- None presently.
- In the future.
- When needed.
- Not applicable at this time.
- When compatible with Macintosh computers (this is now available).
- Will use when we begin doing our own GPS surveys.

- My customers wanted it.
- Whatever the contractors require.
- Not currently in use, but under investigation.

4 - The accuracy is a top priority.

5 - WGS84 was chosen often, probably because people knows that GPS results refer to WGS84. In fact the first coefficients of the WGS84 geopotential model are used in deriving broadcast and precise ephemerides. It is believed however that no canadian has the complete set of geopotential coefficients or the geoid values of WGS84.

6 - The cost conscious people didn't know that geoid data has always been provided free of charge. It should be free as with information on geodetic control stations. Perhaps cost recovery for the diskette and reproduction could be considered.

7 - Other answers come from a few people who are very specialized geoid users.

In conclusion, people are mostly questioning the availability of geoid solutions. Geoid solutions are available but an official geoid is yet to be accepted. When it is, then wider publicity could be considered.

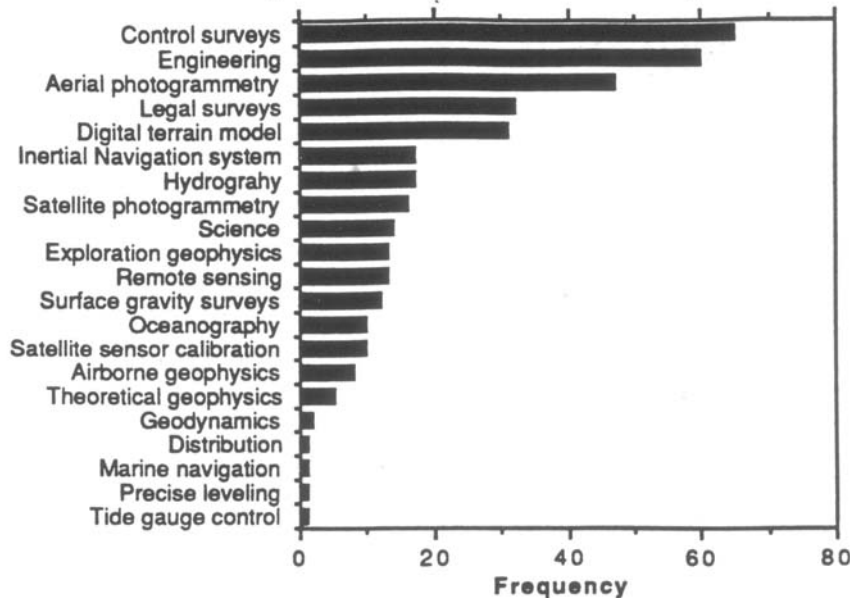
Question 2: From what method or in what format would you like to receive geoid information.

Out of 125 respondents, 77 i.e. 61% answered this question. Appendix B lists the answers. In general a solution already exists for the needs identified. In the appendix, the text **marked in bold** identifies a need for which a solution would require new development. Some parts were underlined to emphasize already available products from Geodetic Survey of Canada. In summary, the users require:

- hard copy,
- computerized grid,
- computer independent (Is Fortran on IBM and Macintosh sufficient ?)
- graphics,
- ellipsoid transformation,
- initial observations,
- larger scale geoid map (Coming!!!)
- function of easting and northing, UTM/MTM (???)
- with estimated standard deviation,
- with vertical deflections,
- a consistent model within Canada.

Question 3: For what application(s) do you plan to use the geoid?

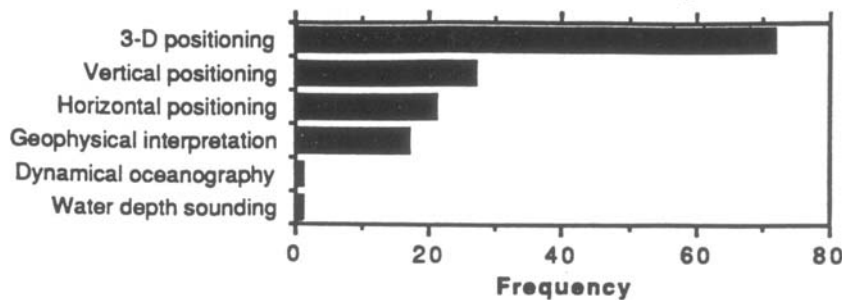
Figure 2 - Answers to question 3(a)



Out of 125 respondents, 94 i.e. 75% answered the first part of question no.3. The histogram to question 3(a) summarizes the answers. The respondents plan to use the geoid mostly for the following applications; control surveys, engineering projects, aerial photogrammetry, boundary/legal surveys and digital elevation model construction. As seen from the histogram the geoid is required for many other applications: inertial navigation systems, hydrography, satellite photogrammetry, scientific interest, exploration geophysics, remote sensing, surface gravity surveys, oceanography, satellite sensor calibration, airborne geophysics, theoretical geophysics, geodynamics, marine navigation, precise leveling, tide gauge control and for distribution. Answers to question 4 describe in more details some of these applications.

Do you plan to use the geoid for 3-D, horizontal, or vertical positioning, for geophysical interpretation or other reasons.

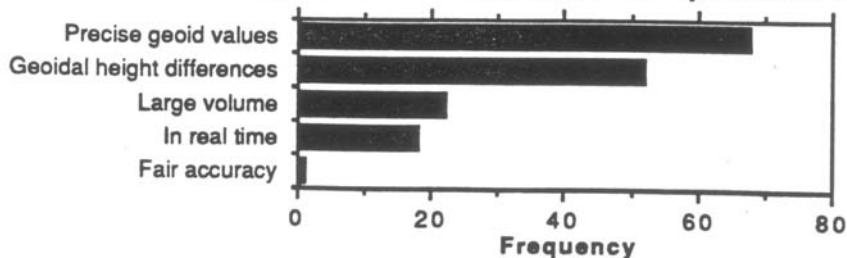
Figure 3 - Answers to question 3(b)



Out of 125 respondents, 118 i.e. 94% answered the second part of question no.3. The histogram to question 3(b) summarizes the answers. Most of the 118 respondents, 72 i.e 61% plan to use the geoid for 3-D positioning. Doing 3-D positioning basically means here using GPS satellite positioning technique. 15% will use the geoid for geophysical interpretation. Then there are those in the business of vertical or/and horizontal positioning. Some users mentioned more specialized applications such as dynamical oceanography and water depth sounding.

Will your application require geoidal height differences, geoid values in real time, precise, in large volume or other.

Figure 4 - Answers to question 3(c)



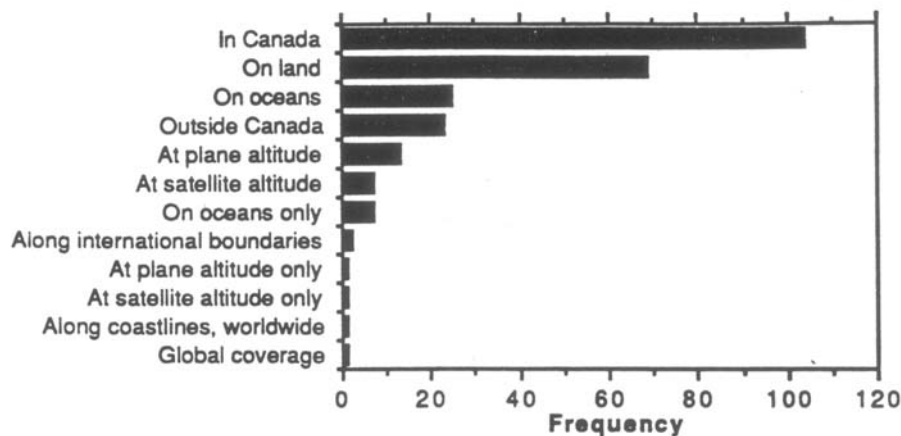
As seen from the histogram to question no.3(c) people are again requesting precise geoid values. The respondents need geoid height differences again because relative geoid information is more precise than absolute geoid information (with respect to the Earth's centre). A relatively significant number of respondents (15%) foresee using geoid data in large volume as well as in real time applications. Of these, 3 need ± 1 m precision, 4 want ± 10 cm and 7 require ± 1 cm precise geoid values.

Question 4: Please describe your geoid applications.

Out of 125 respondents, 86 i.e. 69% answered the question. They described their geoid applications. The other 31% don't use the geoid yet. It is very instructive to look at the answers received. These are given in appendix C. In general, geodetic applications such as satellite GPS positioning for mapping control are described. **The text marked in bold** identifies remote sensing applications. **The text outlined** describe oceanic applications. **The text shadowed** pertains to geodynamics and geophysics exploration. The text underlined identifies other engineering applications.

Question 5: In what regions or areas do you require geoid (gravity field) values ?

Figure 5 - Answers to question 5



Of course Canadians will use the geoid mostly in Canada and for positioning on land. But 20% of the respondents, which is considered an important number of users, require geoid values over oceans. Geodetic Survey must thus start studying in more detail geoid solutions over oceans. In addition, to be of use to oceanographic projects, it will help understanding and improve the geoid along the coasts. It could also be a contribution to global change studies. And another 20% require geoid values outside Canada for projects in other countries. Geodetic Survey must thus continue to study and provide the existing "global" geoid solutions to the Canadian industry. The histogram to question no.5 shows also some very specific needs such as along world-wide coastlines, and at plane and satellite altitude.

Question 6: Do you find existing geoid solutions adequate ? If no, explain why.

Figure 6 - Answers to question 6

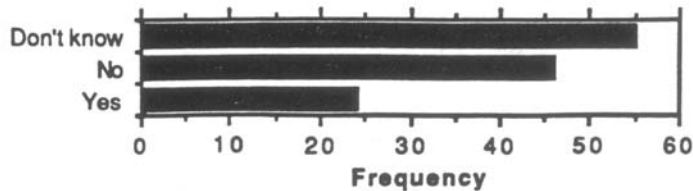


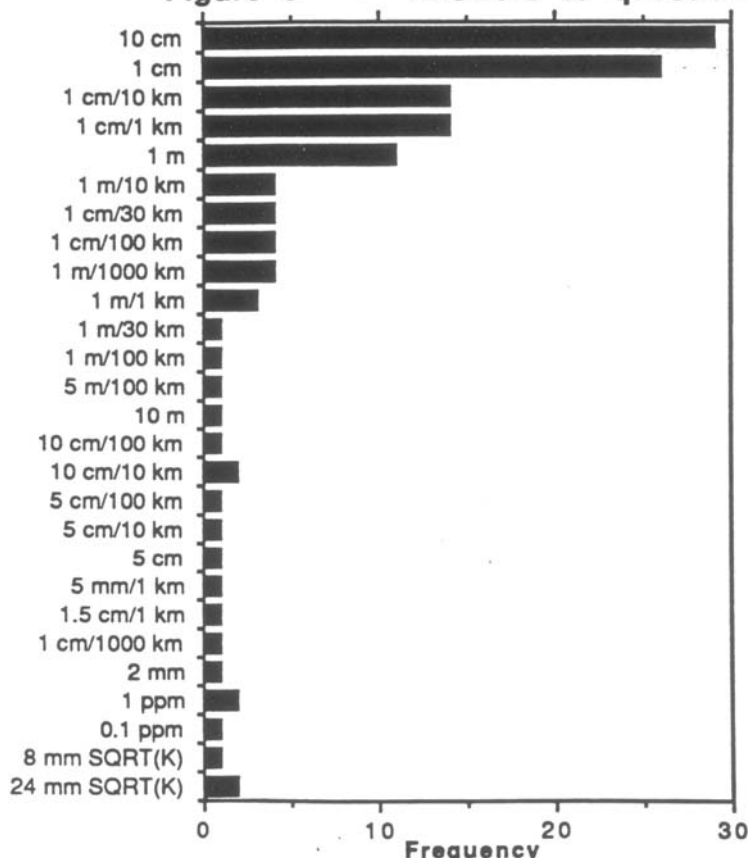
Figure 7 - Answer to question 6



The histogram and pie chart to question no.6 summarize the answer. Out of 125 respondents, 44% don't know if the existing solutions are adequate. This is not surprising since 31% have never used any geoid model. The other 13% have not had enough experience with the geoid models to evaluate them in a satisfactory manner. Those who find the existing geoid solutions inadequate represent 37% against 18% for those who find them adequate. The 58 comments received are given in appendix D. In general, the answers indicate that limited accuracy was experienced by the respondents. **The text marked in bold in the appendix identifies improvements requested by the respondents.**

Question 7: What accuracy do you need in order to make use of geoidal height or geoidal height differences ?

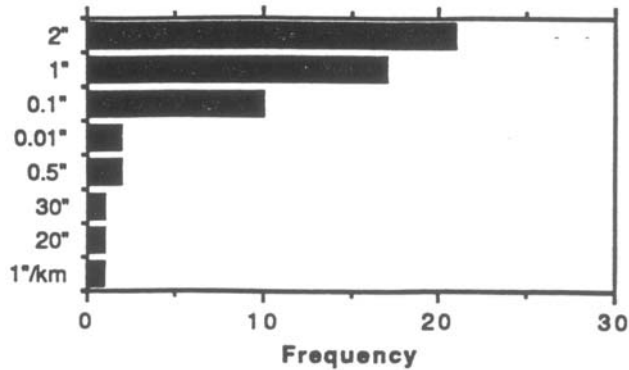
Figure 8 - Answers to question 7



The histogram to question no.7 provides a breakdown of the answers. Out of 125 respondents, 98 i.e 78% answered this question. In order to make use of geoidal heights or geoidal height differences the respondents indicated 22 times that ± 1 metre accuracy is required, 42 times that ± 10 cm accuracy is required and 67 times that ± 1 cm accuracy is required. The users of ± 1 metre accuracy were mostly those dealing with satellite remote sensing applications. The users of ± 10 cm accuracy were mostly those dealing with control survey and mapping industry. Those involve with engineering projects and oceanographic projects are requesting ± 1 cm accuracy. But not only them since 68% of the respondents have indicated a need for ± 1 cm accuracy. It is so probably because 1 cm is the magnitude of precision that common people can deal with easily (1 millimetre being too fine, 1 metre too coarse). It would indicate that ± 1 cm accuracy should be a goal in improving our knowledge of the geoid and of the "Figure of the Earth". \pm One cm is also the goal of geodesists interested in plate motions/deformations. VLBI goal is ± 1 mm (a dream!)

Question 8: What accuracy do you need in order to make use of deflections of the vertical ?

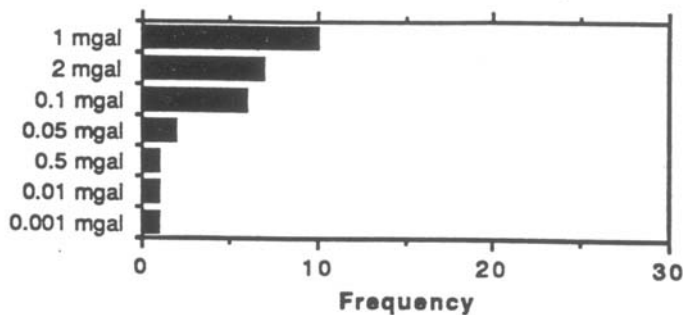
Figure 9 - Answers to question 8



The histogram to question no.8 summarizes the answers. Only geodesists and geophysicists have answered this question about deflections of the vertical. Out of 125 respondents, 54 i.e. 43% answered the question. 17%, 14% and 8% respectively need deflections accurate to 2 arcsecond, 1 arcsecond and 0.1 arcsecond. The geophysicists are requesting the most accurate values. One geophysicist would make use of 0.01 arcsecond.

Question 9: What accuracy do you need in order to make use of gravity components ?

Figure 10 - Answers to question 9



The histogram to question no.9 summarizes the answers. Few people have answered this question. Out of 125 respondents, 34 i.e. 27% answered the question. 8%, 6% and 5% respectively need gravimetric observations accurate to 1 milligal, 2 milligal and 0.1 milligal. Exploration and theoretical geophysicists and those studying the Earth geodynamics are requesting the most accurate values and are even interested in 0.05, 0.01 and 0.001 milligal accuracy. Some people mention their concern for gravimetric coverage rather than precision.

Question 10: Do you feel the accuracy you quoted can be met with the solutions available today ? If no, explain why (format, accuracy, etc.).

Figure 11 - Answers to question 10

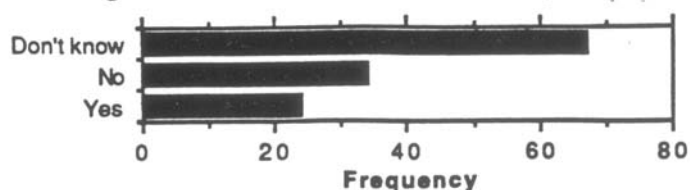
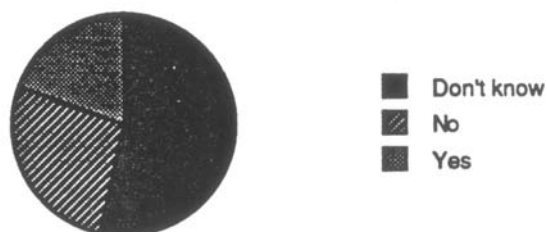


Figure 12 - Answer to question 10



Out of 125 respondents, 65 i.e. 52% answered the question. 54% don't know if the accuracy they quoted can be met with the solutions available today, 27% said no, and 19% said yes. Of those who said yes, 5 quote that they need ± 1 metre accuracy but 16 quote a ± 10 cm accuracy requirement. A histogram and a pie chart summarize the answer. The 32 comments received are given in appendix E. In general, the respondents indicate that limited accuracy was experienced. The text marked in bold identifies domains where work or technology development is required. Some parts were underlined to indicate work known to be in progress at the Geodetic Survey or Geophysics divisions of the Department of Energy, Mines and Resources.

Question 11: Do you have any use for geoidal height (or geoidal height difference) accuracy estimates ?

Figure 13 - Answers to question 11

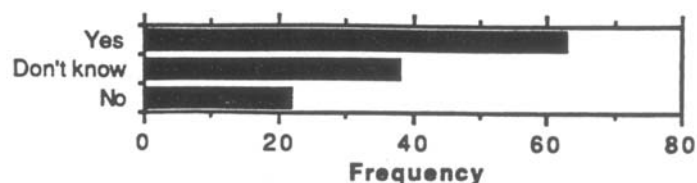
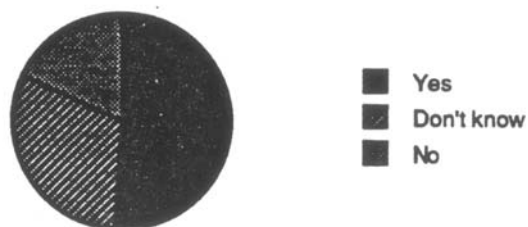


Figure 14 - Answer to question 11



A histogram and pie chart summarize the answer. Out of 125 respondents, 87 i.e. 70% answered the question. 31% don't know if they have a use for geoidal height accuracy estimates, 19% said no, and 50% said yes. It was felt that some of those who said yes only need to know the order of magnitude of the accuracy and not the individual accuracy for each geoid value. Individual accuracy values have never been requested from Geodetic Survey.

Question 12: From whom would you like to obtain geoid data ? The federal gov't, the provincial gov't, the private industry or other.

Figure 15 - Answers to question 12

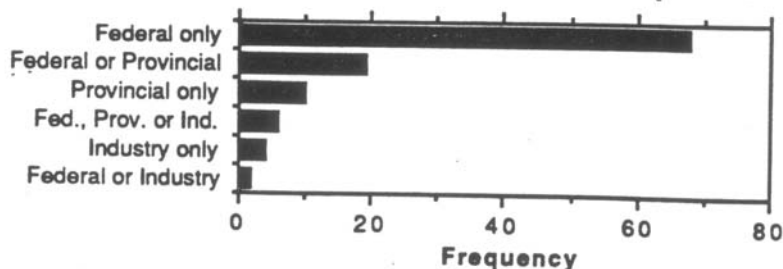
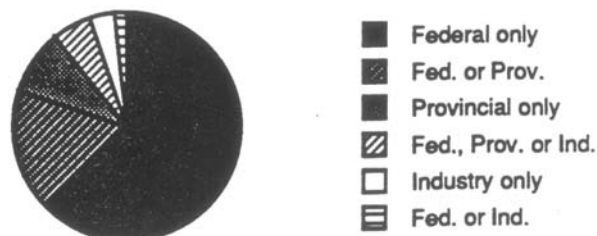


Figure 16 - Answer to question 12



Out of 125 respondents, 109 i.e. 87% answered the question. 54% of the respondents prefer receiving the geoid data from the federal government. An additional 22% would not mind receiving it from the Federal, the Province or the Industry. The breakdown is given in the histogram and on the pie chart to question no.12.

Question 13: How important is it to you that reliable geoid values be readily available ? Not very important, important, very important.

Figure 17 - Answers to question 13

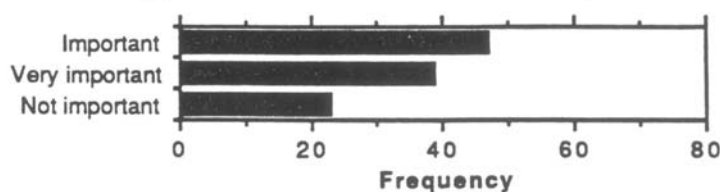


Figure 18 - Answer to question 13



Out of 125 respondents, 109 i.e. 87% answered the question. 38% of the respondents felt it was important that reliable geoid values be readily available. And additional 31% felt it was very important. The geoid is not very important for 18% of the respondents. The histogram and pie chart show these results.

Question 14: Any other comments or suggestions are welcome.

Out of 125 respondents, 41 provided additional comments to the questionnaire, comments which are very appropriate. These are listed in appendix F. In general, the comments indicate the importance of a gravimetric geoid to support satellite technology. **The text marked in bold in the appendix F identifies politics or needs identified by respondents.**

Conclusion

According to the answer to question no.1 there is no surprise about the geoid solutions which have been used up-to-now in Canada. The most precise and at the same time the ones that cover all Canada have always been the most popular. According to question no.2 people wish to have the geoid in the form and format in which the Geodetic Survey has provided it in the past. Thus we believe that tabulating the geoid values and picturing the extent of coverage of the solution as for the UNB Dec'86 geoid solution, and providing a rapid and simple grid interpolation software on a desktop computer, has fulfilled the user geoid requirements. Question no.3 and 4 have permitted users to describe in detail various applications of the geoid. While the needs of a geoid for control surveys and mapping projects are well known, engineering applications, satellite remote sensing, exploration geophysical and oceanographic projects should be kept in mind when refining geoid products. From question no.5 and 10 we have identified that Geodetic Survey must start studying in more details geoid solutions over oceans. In addition to being of use to oceanographic projects, it will help understanding and improve the geoid along the coasts. It would also be a contribution to global change studies. Geodetic Survey must also continue to provide "global" geoid solutions to the Canadian industry for use with projects in other countries. In question no.6 it would seem that people are divided in deciding if the geoid solutions are adequate or not. This is quite natural if we consider that it is only since spring 1989 that we have obtained better than ± 10 cm precise geoid results in parts of Canada (Mainville and Veronneau, 1989). This is quite recent and it will take few years to obtain the same results in different regions of Canada. The most unexpected results of the questionnaire come undoubtedly from question no.7 where 68% of the respondents have indicated a need for geoid values accurate to ± 1 cm. This is probably because 1 cm is the limit of accuracy that common people can deal with easily (1 millimetre being too fine, 1 metre too coarse). It would indicate that ± 1 cm accuracy should be a goal in striving to improve our knowledge of the geoid and of the "Figure of the Earth". Answers to questions no.8 and 9 indicate that deflections and gravity information are of interest to fewer experts than is the geoidal height information. For this reason their requests are still dealt with on an individual basis. The accuracy of the geoid is the biggest concern of the respondents. Since 50% indicated at question no.11 their need for accuracy estimates. This is an area where the Geodetic Survey needs some investigation to make the information available in a simple fashion. According to the answers to no.12, the Geodetic Survey should continue to show leadership in developing, computing and providing geoid information. As demonstrated by question no.13 and 14 there is no doubt the geoid information is becoming more and more important with the advent of satellite techniques and global approaches to a better understanding of the planet Earth.

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THE GEOMED PROJECT

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As the I.Ge.S. is a newborn structure, we decide for the first time, in which the service is in the trial stage, to concentrate our efforts on international projects. One of them is the Geomed (GEoid in the MEDiterranean sea), which is a project supported by the EEC, aiming at studying the problem of the determination of the geoid and the sea surface topography in closed seas with particular regard to the Mediterranean.

The Geoid, or better some equipotential surface of the gravity field suitably fitting the physical surface of the Mediterranean, is determined as the basic surface in Geodesy to which orthometric heights are referred; a good knowledge of the geoid could for instance allow for a much better reattachment of the different national height systems, usually conventionally referred to some tide gauges as zero points.

The SST, i.e. the stationary height of the sea above the ellipsoid, is a fundamental parameter of physical oceanography strongly related to the steady circulation pattern involving surface as well as deep water stream (geostrophic flow) (cfr. e.g. C. Wunsch [1992]).

These two surfaces can nowadays be separated because the geoid can be determined by measurements related to the gravity field only; while the physical surface of the ocean can be achieved by the radaraltimetric measurements performed by dedicated satellite missions (like the now flying ERS1 and Topex Poseidon satellites) after several corrections (firstly the radial orbital correction) and time averaging are applied (cfr. G. Balmino, [1992] and V. Zlotnicki, [1992]).

The state of the art of the computation of the geoid and SST in the Mediterranean sea, before the start of the Geomed Project, is represented by a now old analysis of Seasat data only by M. Barlier with the cross-over adjustment and the subsequent estimate of the

geoid under the hypothesis that there is no sea surface topography. An altimetric geoid has been computed from the same data by Cruz and Rapp as well in 1983.

More recent analyses have been performed by Arabelos and Tziavos in the whole Mediterranean. Also further analyses have been performed in limited areas, like for instance the Sicily Channel.

From what is specified before, it is clear that new scientific research is going to be performed in the framework of Geomed, specially in the field of:

- validation of regionally distributed or time distributed data;
- optimal estimation of the gravity field from different data types;
- gravity inversion.

The expected achievements are described in terms of the main products, i.e. improved maps of the gravity field, of the geoid and of the SST in the Mediterranean area.

The possible by products of our research are in the direction of:

- giving a precise reference surface for marine cartography (bathymetry);
- giving a precise reference surface for describing the sea dynamics and whence stationary currents, sea level rise, interaction with the coasts (erosion), etc.;
- giving an improved gravity field for the purposes of exploration geophysics.

The groups involved in the project are:

- I.Ge.S., Dept. of Environmental Engineering of the Politecnico of Milan (Italy)
- Inst. of Mathematical Geodesy of the Technical University of Graz (Austria)
- Inst. of Astronomy and Geodesy of the University of Madrid (Spain)
- Dept. of Geodesy and Surveying of the University of Thessaloniki (Greece)
- Geophysical Institute of the University of Copenhagen (Denmark)
- National Survey and Cadastre of Denmark
- Finnish Geodetic Institute of the University of Helsinki (Finland).

Several targets are involved in this project at the level of data collection and methodology.

Referring to the first point we start collecting and validating the following data:

- 1) Marine Gravity Data: these are F.A. anomalies given on the sea surface, mainly derived by digitizing the famous maps by Morelli (cfr. C. Morelli, 1970; T.D. Allan and C. Morelli, 1971; C. Morelli et al., 1975a; C. Morelli et al., 1975b; C. Morelli et al., 1975c; D. Arabelos, 1980; D. Arabelos, 1987; D. Arabelos and C.C. Tscherning, 1988; D. Arabelos and I.N. Tziavos, 1989), although other gravity files are now available and in future they will be compared with the above for the scope of validation. By the way the actual data have all been scrutinized and essentially submitted to internal validation, so that dubious or possibly spurious data are now properly flagged in our files.
- 2) Land Gravity Data: we have (available) the national archives of Spain, Portugal and Italy; the Greek gravity data are not open, however they can be used by the Thessaloniki University in computations of the geoid in the Eastern Mediterranean. To these a few more data must be added mainly provided by the Bureau Gravimétrique International which is considering to deliver a set of low resolution gravity for France. Moreover there is a possibility to get similar data for Northern Africa from the University of Leeds (WEEPG Project).
- 3) Digital Terrain Models: these include both topographic heights on land and the bathymetry of Mediterranean. As for land data only a small part of what would be needed with the proper resolution is available. On the other hand on the whole region, including bathymetry, we have two global models, namely TUG87 and ETOPOSU both with a resolution of 5'x 5'. Moreover the bathymetric maps of Morelli (resolution 5'x 7'.5", equidistance 200 m) are also available in a digitized form thanks to the work of the Thessaloniki group. Some work has been already done by using the TUG87 Model; however there are several doubts about its effectiveness due to

a recent experience in the computation of the geoid in Italy where it was shown that there are large discrepancies with the national DTM, particularly in Southern Italy. Furthermore, looking at any countour map, it seems quite obvious that it is unrealistically too flat in the whole central Mediterranean; on the other hand the good point is that there seems to be a fair agreement with the shore line and with the islands locations, proving that in applying a remove-restore technique the highest frequency contribution to the geoid should be possibly guessed. Some work has been done in order to obtain an improved bathymetry by merging the existing data (see in the next pages the paper "DTM comparison").

- 4) Altimetric Data: we have collected the available altimetric data for Mediterranean concerning the Seasat mission as well as the Geosat mission, the last restricted to the (ERM) Exact Repeat Mission (of period 17 days) for the first 22 repetitions. These data have already been cleaned and processed in global adjustments by the OSU and, in particular for Geosat, the radial orbital error has been corrected for (Y.M. Wang and R.H. Rapp, 1990).

A new altimetric data set is now in the process of being collected and validated, namely that produced by the ERS1 mission; for the moment our files include the ERM(1-4) with a 35 days period and the ERM(1-4) with a 3 days period.

- 5) Global Geopotential Models: many global models are available in the Geomed Files, including IfE88, OSU78, GPM2, OSU81, OSU86E, OSU86F, OSU89A, OSU89B, OSU91A, DGFI92A, GEM10C. All these models have been tested statistically in the area of interest against gravity or altimetric data to decide which one could conveniently represent the data locally. At the end the choice has been for OSU91A as, although its performance was comparable to that of IfE88, it is credited to have superior global representativity.

Beyond these data which are essential either in computing the gravimetric geoid or the stationary sea surface, other two data sets are currently collected in the Geomed Project as, so to say, subsidiary data, namely:

- 6) Tide Gauge Data: there are currently corrected by the Madrid and the Thessaloniki groups and attempts are now made to set up an empirical tidal model for the Mediterranean, split into 3 basins (Western, Central, Eastern).
- 7) Geophysical Data: in particular we have collected information on the Moho depth in order to be able to smooth as much as possible the gravity field and to be able to predict it as accurately as possible. The Graz group has already performed some experiments in this direction.

The collection and validation of the above data sets may be not only useful for the Geomed project, but also will provide an excellent data base for any scientist interested in the gravity field, the geometry of the geoid, the sea surface topography and the geophysical structure of the Mediterranean area.

As for the second point, that is methodology, we have different problems related to the estimate of the radar altimetric surface and the geoid.

The main problem related to radaraltimetric observations is the removal of the radial orbital error. This treatment is essentially an adjustment of cross-over values based on the observation equations:

$$h = (N + t) + (\xi_r + \tau) + \nu$$

where: h is the measured height of the sea;

N is the above the ellipsoid geoidal height;

t is the stationary sea surface topography;

ξ_r is the radial error for the flying altimetric satellite;

τ is the time varying sea surface topography;

ν is a noise.

If the radial orbital error ξ_r is approximated by a linear function of time (cfr. E.J.O. Schrama) and we consider only the crossovers equation, a rank deficiency problem exists (R. Barzaghi et al., 1990).

In order to remove it various methods have been tested (R. Barzaghi et al., 1992) proving that the most convenient constraint is to

minimize, with convenient weights, the sum of the differences

$$h_i - N_{MOD} - (a_i \lambda + b_i) = \Delta_i$$

where: N_{MOD} is a geoidal height computed by a global geopotential model;

λ is the latitude.

Referring to the computation of the geoid as a first consideration we must point out that the optimal "on ground" combination of available data is still matter of discussion among scientists: integral methods (in the mode of a boundary value problem) and collocation are among the most commonly utilized.

In either cases it is convenient first of all to modify the gravity data set (particularly when we consider not only the sea, but also the intruding/surrounding land) by the remove-restore process (cfr. R. Forsberg, 1985), which has the effect of smoothing and regionalizing the gravity field. In this frame the use of the most proper global geopotential model is also a subject not well assessed; in particular the calculation of a realistic error of global models is still questionable and the recently introduced taylorized models still deserve a more through discussion.

Another crucial point is the unavailability of a sufficiently detailed digital terrain model for the whole Mediterranean area.

For the sake of solving this problem, as we have mentioned in the previous pages, we have started merging the ETOPO5U model with the more detailed national digital terrain models.

The tests on the computation of the geoid performed till now are the following:

- geoid in the Western ($0^\circ \leq \lambda \leq 10^\circ$; $37^\circ \leq \phi \leq 50^\circ$) and in the Eastern Mediterranean ($14^\circ \leq \lambda \leq 25^\circ$; $34^\circ \leq \phi \leq 40^\circ$) computed using Stokes formula by FFT.

The result of the experiment in the Eastern Mediterranean is displayed in Fig. 1.

- geoid in the Western Mediterranean (cfr. Fig. 2) computed by fast collocation, a new technique implemented by G.P. Bottoni and R. Barzaghi (1992).
- geoid computed on the whole Mediterranean sea splitting it into

330 ($1^\circ \times 1^\circ$) zones on each of which the prediction was performed from a ($2^\circ \times 2^\circ$) block covering the estimation area (cfr. Fig. 3). Referring to the SST it has been computed for the moment only in the Western Mediterranean subtracting the gravimetric geoid from the altimetric one; the result is shown in Fig. 4.

As one can see we have a surface waving from -0.80 m along the African coast to -0.20 m along France and 0.20 m in Cataluña. These variations seem to be certainly higher than the noise we expect in the geoid, which is of the order of 5 cm.

In order to test our geoids we have started the comparison with geoids computed by other groups in the same area.

At the moment the only external geoid we have available is supplied by the Bureau Gravimétrique International and was computed by J.P. Barriot (1987) over a large window ($-15^\circ \leq \lambda \leq 28^\circ$; $25^\circ \leq \phi \leq 55^\circ$) by applying a truncated Stokes formula, with 6° cap, on gravity data not reduced by the terrain effects.

The differences on the Western Mediterranean (Fig. 5) between the BGI and the Geomed geoids are very high and systematic (only positive signs) on land, probably due to the different treatment of the contribution of the topography, but become quite flat in the marine area.

In fact the average and the RMS of the differences decrease respectively from 0.49 m to 0.20 m and from 0.81 m to 0.54 m if we restrict our analysis only on points on sea.

We also tried to analyze if it possible to see a datum shift between the two geoids (only on sea), using the transformation:

$$N(P_i)^{\text{GEOMED}} - N(P_i)^{\text{BGI}} = a \cos\phi_{P_i} \cos\lambda_{P_i} + b \cos\phi_{P_i} \sin\lambda_{P_i} + c \sin\phi_{P_i} + v_i$$

The results we obtained on the 6009 points on sea are:

$$\hat{a} = 0.1305 \text{ m} \quad \hat{b} = 1.3384 \text{ m} \quad \hat{c} = -0.2336 \text{ m}$$

$$\text{while } \hat{\sigma}_0 = 0.4099 \text{ m}.$$

The conclusion for the moment is that the last transformation decreases significantly the sqm but not strongly.

The comparisons with other geoids as well as the collection and validation of new data sets and the computation of the geoid in the Central Mediterranean will be goals of the Geomed groups for the next period.

After these intermediary goals will be reached we will have a geoid and a SST over the whole Mediterranean. The problems at that point will be to homogenize the solutions and to proceed to their interpretation at least in terms of geostrophic circulation.

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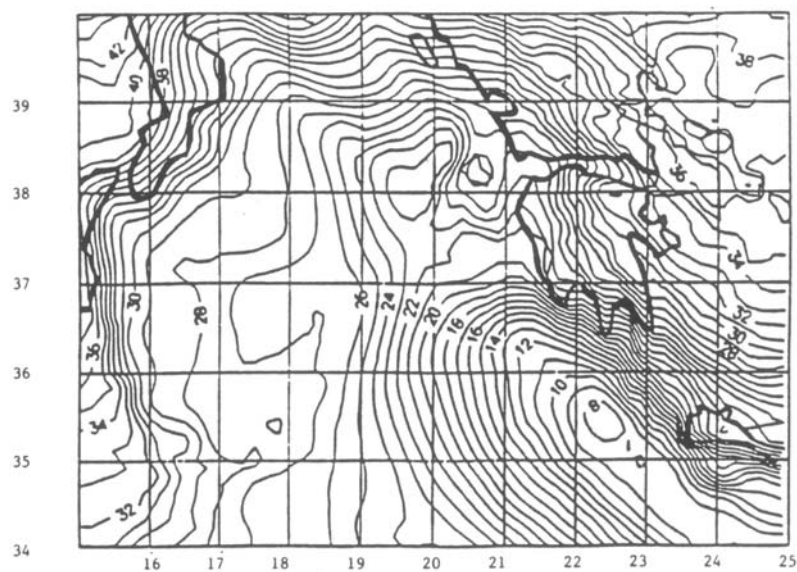


Fig.1 - Geoid computed by FFT techniques in the Eastern Mediterranean (countor interval 1 m).

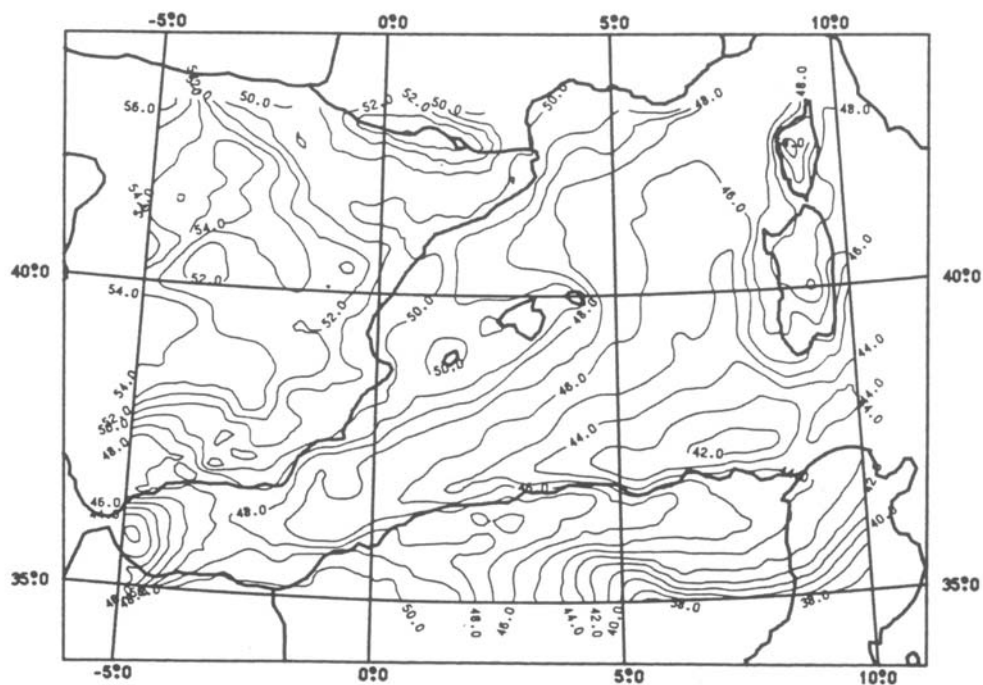


Fig. 2 - Geoid computed by fast collocation in the Western Mediterranean (countor interval 1 m).

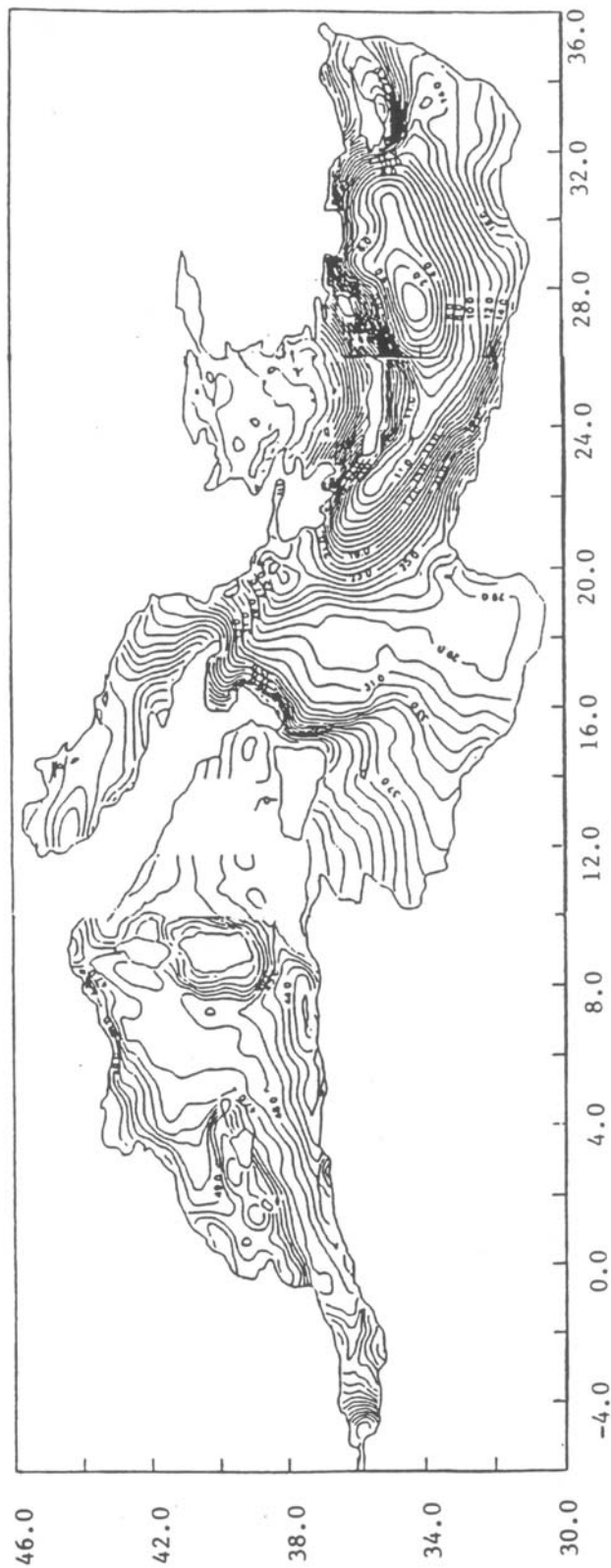


Fig. 3 - Geoid computed by pure collocation in the Mediterranean sea.

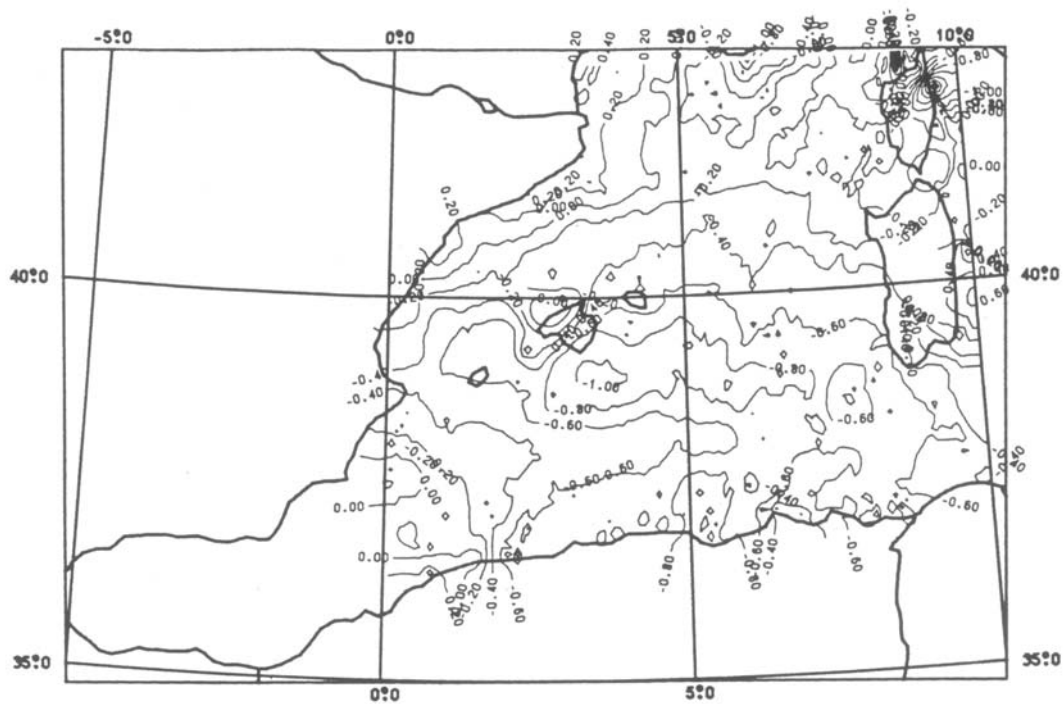


Fig. 4 - Sea surface topography in the Western Mediterranean sea

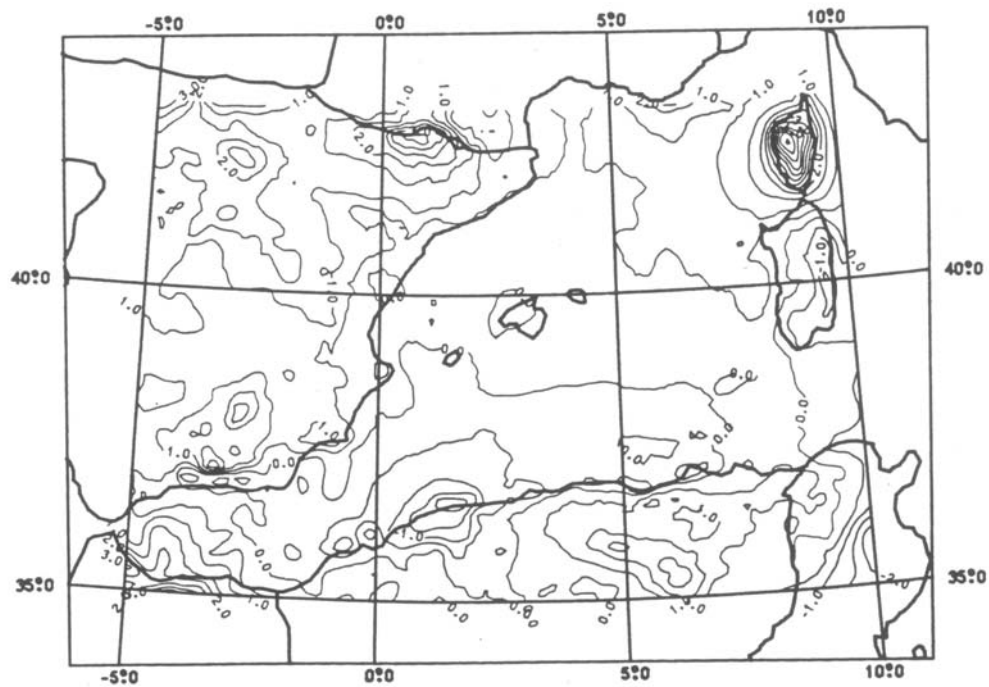


Fig. 5 - Differences between Geomed geoid and the geoid supplied by the BGI in the Western Mediterranean (countor int. 0.5 m)

THE BUREAU GRAVIMETRIQUE INTERNATIONAL

G. Balmino
Director of B.G.I.
Secretary General of I.U.G.G.

1. INTRODUCTION

In geodesy, gravity values play a great part in the modelling of the Earth gravity field, which is of permanent use for the computation of precise satellite orbits. It is also an essential information for the determination of the geoid, and for the definition of the ocean mean surface used for the study of the global circulation (Balmino et al., 1986). In geophysics, the interpretation of the gravity field anomalies allows to study density variations in the lithosphere or the mantle, with applications in oil and mineral prospecting (Balmino, 1986a).

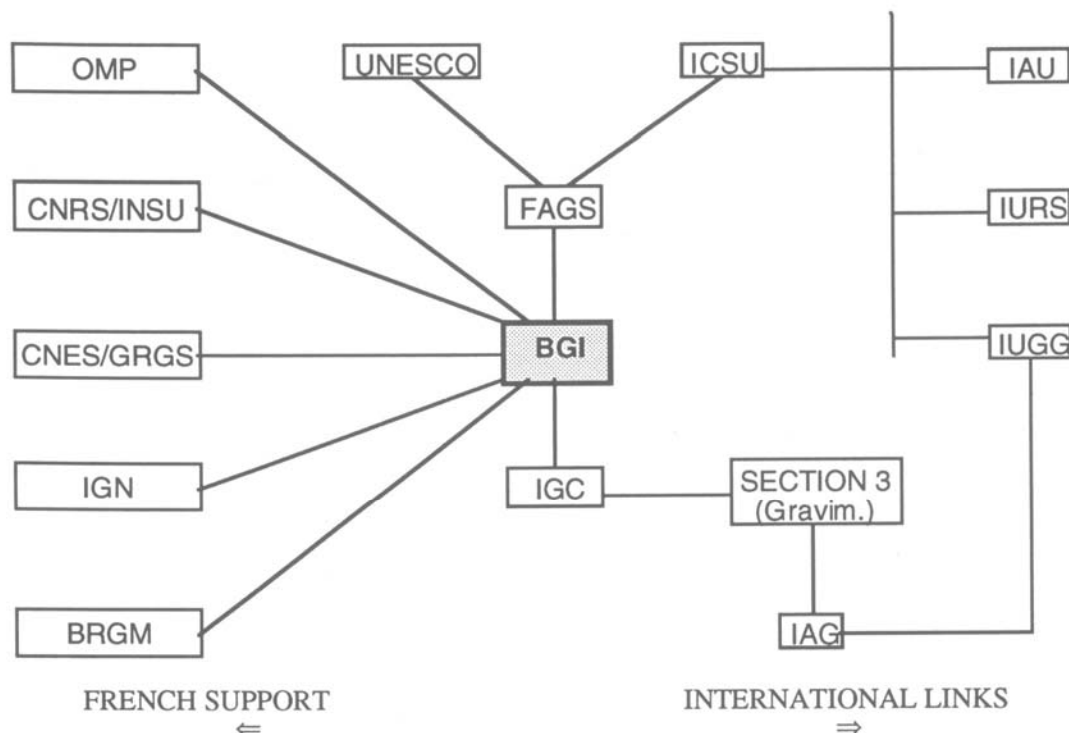
The Bureau Gravimétrique International (BGI) is one of the offices of the Federation of Astronomical and Geophysical Services (FAGS) which operates under the auspices and in part thanks to the financial support of the International Council of Scientific Unions (ICSU) and the United Nations Educational Scientific and Cultural Organisation (UNESCO). Primarily interested in the activities of these services are the International Astronomical Union (IAU), the International Union of Radio-Sciences (IURS), and of course the International Union of Geodesy and Geophysics (IUGG). - see fig. 1 and Melchior (1989). It may also be considered as an executive arm of the International Gravity Commission (IGC) within the International Association of Geodesy (IAG), one of the seven associations of which IUGG is composed.

The idea of a service for gravity data and related matters originated during the 1951 IUGG General Assembly in Brussels and BGI was created in 1953. Its offices have been located in France since the beginning, when pioneer works were being done by the first directors : Reverend Father Lejay from the Society of Jesus, Academician Tardy, and then Professor Levallois.

The central office is in Toulouse (France) since 1980, in the premises of the Observatoire Midi-Pyrénées (OMP) of which it is one of the services. The other french supporting organisations are : the Centre National d'Etudes Spatiales (CNES), the Institut Géographique National (IGN), the Centre National de la Recherche Scientifique (CNRS) - via the Institut National des Sciences de l'Univers (INSU), and the Bureau de Recherches Géologiques et Minières (BRGM). There exists a covenant between these agencies to guarantee their support to BGI.

BGI has a Directing Board composed of ten voting members (comprising the IGC president, vice-president, secretary, the section III chairman, the BGI director, plus elected members), and non voting members : the chairmen of the IGC-BGI working groups ; the secretary of the Board ; two ex-officio members (the Geoid Commission president and a FAGS representative). The Directing Board meets once every year.

Fig. 1. Links of BGI with international and national (french) bodies



List of Acronyms :

BGI	Bureau Gravimétrique International
BRGM	Bureau de Recherches Géologiques et Minières
CNES/GRGS	Centre National d'Etudes Spatiales/Groupe de Recherches en Géodésie Spatiale
CNRS/INSU	Centre National de la Recherche Scientifique/Institut National des Sciences de l'Univers
FAGS	Federation of Astronomical and Geophysical Services
IAG	International Association of Geodesy
IAU	International Astronomical Union
ICSU	International Council of Scientific Unions
IGC	International Gravimetric Commission
IGN	Institut Géographique National
IUGG	International Union of Geodesy and Geophysics
IURS	International Union of Radio Science
OMP	Observatoire Midi-Pyrénées
UNESCO	United Nations Educational, Scientific and Cultural Organisations

2. OBJECTIVES AND TERMS OF REFERENCE

The main task of BGI is to collect, on a world-wide basis, all gravity measurements and pertinent information about the gravity field of the Earth, to compile them and store them in a computerized data base in order to redistribute them on request to a large variety of users for scientific purposes. The data consist of : gravimeter observations (mainly location - three co-ordinates, gravity value, corrections, anomalies ...), mean free-air gravity values, gravity maps, reference station descriptions, publications dealing with the Earth's gravity. BGI also has access through one of his

host agencies to satellite altimetry derived geoid heights (presently from Geos 3, Seasat, Geosat, ERS1, Topex/Poseidon) ; spherical harmonic coefficients of current global geopotential models ; mean topographic heights. These data are sometimes used internally for data validation and geophysical analysis.

The data collection activities are especially conducted in the framework of large regional projects, in order to densify the world data coverage, and BGI has put emphasis on the validation of received measurements, so as to improve the quality of the delivered information.

Four working groups are presently helping BGI in different tasks :

- WG1 : Data Processing. Our Canadian colleagues, who have chaired this group from the beginning, have provided invaluable help to the Bureau.
- WG2 : World Gravity Standards. This group is now in charge, and probably for a long time, of the deployment of the International Absolute Gravity Base Station Network (IAGBN)
- WG5 : Monitoring of Non Tidal Gravity Variations. Newly established, this group has a large variety of problems to deal with, due to the growing use of superconducting gravimeters.
- WG6 : Intercomparison of Absolute Gravimeters. For a long time under the responsibility of Prof. Boulanger, the activity is now controlled by this group which continues to organise comparisons of instruments about every four years at the Bureau International des Poids et mesures (B.I.P.M.) in Sèvres, near Paris ; the next campaign is to take place in November-December of this year.

Other working groups (WG3, WG4) terminated their mandates and were naturally dismantled.

3. SERVICE ACTIVITIES

3.1. Providing Data to BGI

All kinds of gravity data can be sent to BGI, with or without restrictions of redistribution to be specified by the contributors, sometimes in the form of a protocol of usage.

Essential quantities and information for gravity data submission are :

- a) Position of the site :
 - latitude, longitude (to the best possible accuracy),
 - elevation or depth :
 - . for land data : elevation of the site (on the physical surface of the Earth)
 - . for water stations : water depth
- b) Measured (observed) gravity, corrected to eliminate the periodic gravitational effects of the Sun and the Moon, and the instrumental drift.
- c) Reference (base) station(s) used. For each reference station (a site occupied in the survey where a previously determined gravity value is available and used to help establish datum and scale for the survey), give name, reference station number (if known), brief description of location of site, and the reference gravity value used for that station. Give the datum of the reference value ; example : IGSN71.

Give supplementary elevation data for measurements made on towers, on upper floor of buildings, inside of mines or tunnels, atop glacial ice. When applicable, specify whether gravity value applied to actual measurement site or it has been reduced to the Earth's physical surface (surface topography or water surface). Also give depth of actual measurement site below the water surface

for underwater measurements.

For marine gravity stations, gravity value should be corrected to eliminate effects of ship motion, or this effect should be provided and clearly explained. Additional information are optional, but welcome.

3.2. Getting Data and Services from BGI

The most frequent service BGI can provide is data retrieval over a limited area. Data are sent on tapes or diskettes or printouts. Data coverage plots may also be provided, usually over $20^\circ \times 20^\circ$ areas. Cases of massive data retrieval requests may be considered ; they are studied and may be processed in a specific way.

Other services include :

- data screening,
- provision of gravity base station information (on micro-fiches, sometimes rushed by fax),
- data evaluation and gridding,
- computation of mean values (simple means ; computation by collocation available in the course of 1993),
- contouring,
- supply of maps, or information on existing maps (catalogue available in printed form and on tape).

The cost of the services have been established in view of the categories of users-mostly contributors of measurements and scientists, and also considering the large amount of support of our host organisations. The charging policy is explained in detail in the Bulletin d'Information. Some of the services may be provided free of charge upon request, to data contributors, individuals working in universities, such as students, and generally to any person who can contribute to our activities on a data or documentation exchange basis.

3.3. The Publications

BGI issues a Bulletin d'Information twice a year (generally in June and December). It is sent to over 300 subscribers. 71 issues have been published so far. The Bulletin contains :

- . general information in the field, news about the Bureau itself, recent additions to our holdings,
- . contributing papers in gravimetry,
- . communications at meetings dealing with gravimetry (e.g. IGC meeting).

Every four years, an issue (which may be an additional one) contains the National Reports of Activities in Gravimetry (e.g. n°70).

Besides, the full catalogue of the holdings is issued every two years. It usually comes in two volumes : one contains the coverage plots of the measured gravity points in the form of maps covering $20^\circ \times 20^\circ$ areas (fig. 2), the other is a set of tables of statistics per square degree (number of points, mean free-air value and its r.m.s.) corresponding to each map of the first volume (fig. 3).

BGI has also prepared and published a brochure publicising its role and activities (distributed mainly at the Vancouver and Vienne IUGG General Assemblies, also on other occasions), and realised (with IGN Space Company) a Spot image map with contours of Bouguer gravity anomalies.

Technical Reports are also produced, on an irregular basis, on problems, software and various applications dealt with at the Bureau.

Fig. 2. Example of data coverage plot

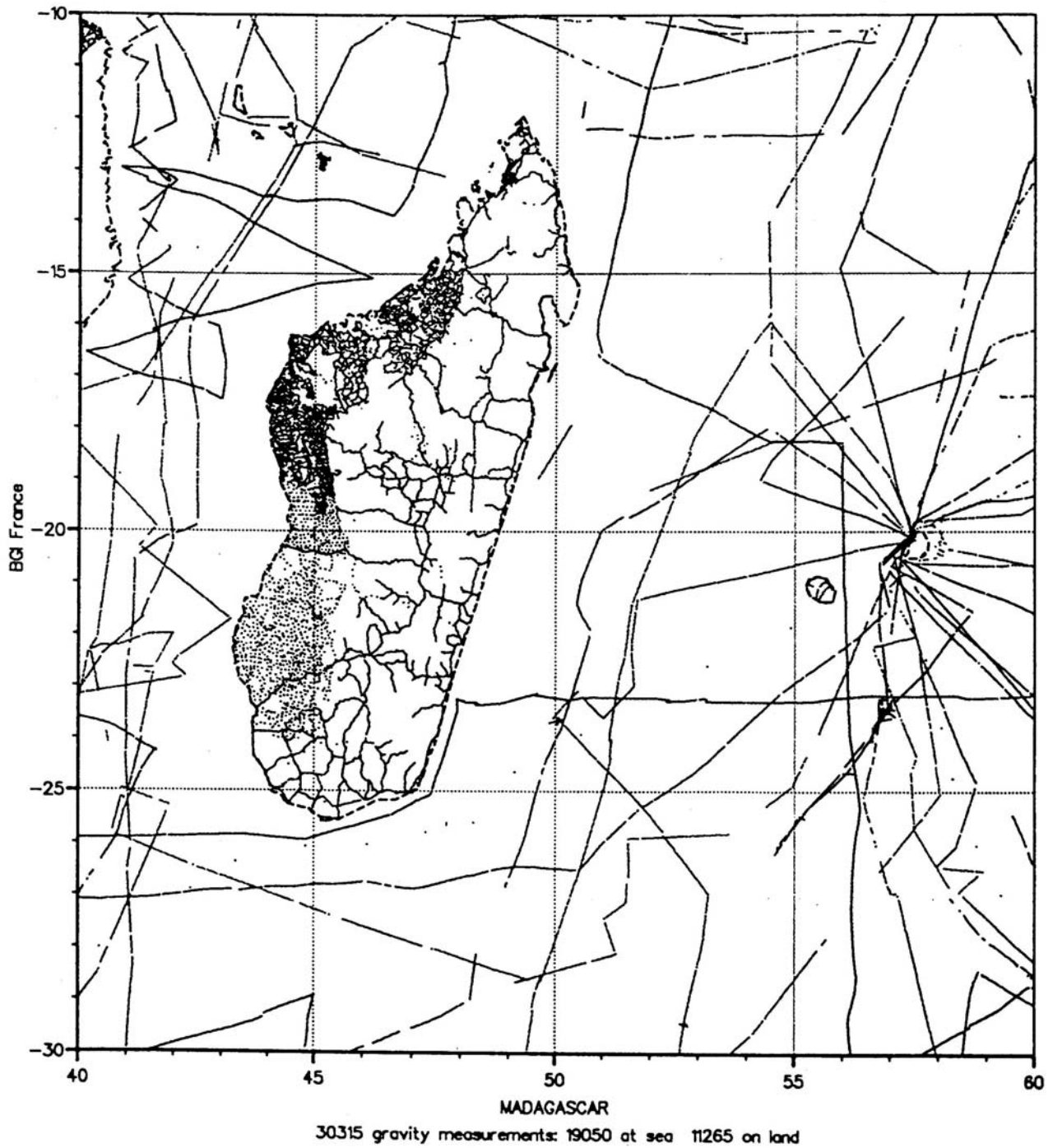


Fig. 3. Example of detailed index (Data coverage corresponding to Map 1)

BGI GRAVITY DATA: MEAN FREE AIR ANOMALY

1ST FIELD: POINTS NUMBER

2ND FIELD: MEAN VALUE (MGAL)

3RD FIELD: R.M.S (MGAL**2)

-10	213 23.4 10.1	105 -40.2 42.3	15 5.6 6.2	75 -25.6 13.0	16 -12.8 2.1	26 -18.3 4.3	29 -27.7 17.6	184 -22.5 26.3	53 -23.9 10.3	65 -27.9 26.7	26 -8.2 37.4	8 -1.2 24.0	116 -5.5 8.2	138 -13.1 11.1	52 -5.8 5.9	44 -3.8 12.2	54 -1.8 22.8	84 -3.2 9.2	66 -13.9 9.4
		151 -42.7 32.5	41 -15.2 13.2	48 129.1 132.0	64 -18.3 13.8	41 -26.4 3.9		85 -42.6 9.9		2 71.7 2.8	13 -45.1 4.8	82 -12.9 16.3	43 -1.7 8.4	29 -16.9 4.3	2 -8.1 1.3	25 -2.6 13.6	67 -14.4 10.5	40 -1.7 2.6	37 -21.3 5.9
	21 -55.9 5.6	257 -42.7 17.2	51 -63.4 12.2	27 129.1 219.8	121 21.1 166.0	72 85.1 158.0	32 -47.1 3.7	100 -57.9 6.1	15 37.2 9.1	111 54.8 17.3	15 -32.7 1.7	101 -17.2 4.5	26 -12.3 6.0	26 -20.4 5.9	35 -23.8 3.1	58 -11.0 13.1	50 -8.6 10.1	6 -6.1 1.4	16 58.7 4.8
	3 -47.8 1.6	375 -13.6 29.9	170 -40.3 11.7	223 -39.3 8.4	134 -52.4 4.7	85 -40.1 5.7	171 -38.5 8.0	34 -31.5 38.1	154 29.6 16.7	117 34.3 15.9	4 82.6 10.5	72 -5.9 3.5		23 -2 6.7	1 21.7 0.0	49 -4.5 5.5			63 11.3 54.6
		284 3.6 77.8	24 -35.7 3.3	108 -27.8 4.3	84 -36.3 7.6	97 -42.4 5.2		87 -13.1 12.8	101 1.2 32.6	44 12.3 16.4	60 47.6 20.2	71 -10.8 4.5			31 -8.6 4.7	11 11.9 4.2	62 3.7 7.3	41 -8 11.4	3 12.3 10.8
	1 -45.2 0.0	247 -45.1 42.5	575 -22.2 13.1	396 -63.3 8.2	151 -72.8 25.2	103 -63.6 32.4	322 -12.4 14.3	612 -18.5 9.7	145 -4.9 10.3	38 1.3 6.8	47 -27.3 28.1	35 -27.3 2.4			42 28.9 53.0	6 1.4 1.6	32 8.8 10.7	9 -7.4 3.3	68 -17.7 17.1
	102 -20.1 14.1	490 -47.1 41.9	159 -40.4 15.9	175 -25.5 10.5	346 12.6 19.8	414 -5.1 15.2	407 -26.0 8.3	240 -3.1 12.8	53 50.4 19.5	117 3 20.4	45 -15.8 12.2	51 -14.9 11.7		30 -16.8 3.8	84 -14.0 14.9	28 -9.5 16.2	73 4.9 9.3	6 -18.4 2.5	95 -0 19.4
	44 -7.9 13.0	105 -42.2 35.5	98 -4.4 28.1	136 -18.1 12.5	774 6.0 24.4	384 8.1 17.6	83 -10.4 22.3	76 50.3 33.1	110 35.0 20.6	66 15.9 19.4	3 -43.9 2.1	27 -16.8 4.3	79 -2.1 6.9	133 -2.2 5.9	14 3.4 5.2	146 -10.3 11.5	68 -8.6 19.3	28 -19.8 10.5	98 -2.5 41.4
	67 -40.4 6.8	46 -26.6 29.7	32 21.1 12.5		720 -7.5 11.7	380 -9.3 34.2	154 46.3 12.9	202 62.1 16.1	137 23.2 25.1	90 18.5 32.6	13 -47.8 3.0		74 -5.0 6.9	71 -8.1 6.1	167 -3 11.8	198 -5.1 14.1	114 -34.1 14.2	59 50.0 59.0	23 36.9 59.0
	47 -42.2 7.9	92 -44.9 15.2	38 16.8 19.8		176 -20.1 19.8	330 -23.5 19.8	115 40.8 20.0	91 67.2 26.5	2 56.5 2.1	37 -9.8 1.1	100 -13.3 3.7				26 -13.0 5.3	97 -25.9 8.3	114 -59.6 24.4	252 69.9 102.5	105 -14.7 72.2
-20	24 -22.6 7.4	152 -23.3 14.5	17 -33.2 15.0	6 4.3 2.3	151 -15.2 28.1	142 49.4 27.5	49 49.6 22.1	81 47.0 39.1		43 -21.3 7.3	24 -2.9 8.8	24 -8 15.1	23 -3.7 15.9	24 8 15.9	1 13.0 0.0	65 281.3 61.4	202 -6.8 50.2	210 -29.2 24.2	170 -2.4 16.1
	26 -24.4 8.6	105 -11.3 9.0	58 -15.3 19.9	97 12.4 12.2	166 -2.7 14.8	81 -4.2 20.1	146 26.4 16.7	99 46.9 39.3		52 -24.8 5.7	77 5.1 5.9	24 -5.5 1.2	8 -18.5 4.5	1 13.0 0.0	65 281.3 61.4	202 -6.8 50.2	210 -29.2 24.2	170 -2.4 16.1	44 16.5 9.0
	184 7.6 29.5	147 3.4 11.4	60 -20.0 11.0	113 30.0 12.9	200 17.6 16.0	165 41.9 30.3	150 29.2 19.1	203 7.3 34.4	13 75.7 3.6		45 -14.0 1.7	73 -5.2 11.8		1 1.6 0.0	5 -2.8 1.0	51 -14.3 4.4	186 -9.7 11.9	100 -15.0 24.7	108 -8 14.4
	155 -3.1 10.5	59 5.2 7.9		76 27.0 12.3	236 11.5 23.4	118 31.8 14.8	46 36.0 17.4	157 32.3 29.4	145 -7.5 6.2	116 -2.8 7.5	214 -25.0 13.6	185 7.5 10.3	105 21.2 16.0	76 5.2 3.5	97 11.1 7.2	104 6.4 30.2	235 9.1 9.6	165 -8.7 26.1	87 2.6 10.0
	33 -3.4 5.9	126 6.2 17.3		28 39.4 10.6	132 50.4 10.8	151 30.1 9.7	138 10.8 34.4	131 27.0 42.3		34 -7.5 4.0	17 -16.5 3.6	76 4.3 5.6	27 3.7 3.8	27 1.6 9.3	12 43.2 3.2	61 7.9 16.7	173 5.9 25.3	41 -21.1 17.2	29 -12.5 17.2
	134 -7.5 9.6	167 -5 10.0	58 3.7 7.0	58 1.2 14.4	161 19.5 32.7	122 11.4 28.4	31 66.7 41.0	1 -24.9 19.1	45 -12.2 6.2	76 -1.2 8.1	76 6.4 7.9	26 14.3 3.2	26 8.5 26.3	53 14.3 26.3	70 -8.9 3.7	100 6.4 18.7	47 -3.7 10.7	26 -8.1 2.9	24 -6.5 9.6
	37 -27.9 4.9	126 16.8 22.3	69 4.8 10.3	71 -8.9 22.0	37 -22.3 20.2	37 -7.4 6.9	30 -6.7 10.4	35 -7.5 5.9	48 -20.5 7.6	71 -16.2 4.7	93 -10.1 6.3	32 -6.6 3.7	21 -11.9 5.8	9 -9.7 1.1	15 -17.9 4.5		105 2.1 7.7	27 9.8 22.6	56 -8.2 9.4
	75 -9.5 14.8	81 2.7 18.8	3 -5.7 .5		35 11.6 20.6	40 42.6 10.4	60 39.5 22.8	14 36.7 10.5		6 2.5 1.1	42 -1.4 4.2	56 1.1 10.2		25 -11.6 4.2		4 -8.9 2.6	78 6.7 3.3	24 1.5 3.2	34 -23.5 21.5
	63 -24.0 7.8	34 -14.1 4.9			12 10.7 4.8	1 6.2 0.0		28 39.8 6.4	79 38.0 15.9	134 14.6 6.7	38 -3.0 3.9	20 -6.6 11.8	17 8.6 4.4	6 -3.2 1.9	16 -12.0 5.3	18 -8 3.1	115 29 10.0	29 -3.6 19.9	29 1.2 15.2
-30		55 -13.2 8.3	31 3.9 3.9	33 -6.1 16.4	64 16.1 17.5	9 47.1 22.8	21 20.3 17.2	40 11.7 4.6	3 7.7 .4	48 23.0 12.0			37 16.7 8.0	11 -6.2 4.8	23 -5.6 3.8		88 7.2 20.7	57 .6 17.5	111 1.4 17.5
40																			
50																			
60																			

4.1. Data Base :

The first computerised data base was established in 1976 with the help of BRGM, of IGN and of the Institut de Physique du Globe in Paris (where BGI was located at that time). A great deal of effort was put, between 1980 and 1982, at building up an entirely new data bank and management system on one of the CNES main frames (CDC Cybers). This resulted in a quite sophisticated system, entirely specific to gravity data, but very difficult to maintain. Along the years, after having suffered from computer changes and staff turn-over, also facing difficulties in upgrading the software (for instance to speed up the merging operations which had to be done more and more frequently with the increase of the data volume), B.G.I. decided to change its strategy in data base maintenance.

Instead of putting more efforts from BGI staff and following the availability of the ORACLE software on a main frame (IBM 4381) at CNES, it was decided in early 1991 and after extensive satisfactory testing to discontinue the usage of the old software (which was running on CDC Cybers) and to switch to ORACLE (level 6). A first version was operational in the fall of 1991. Attention was exerted to ensure no interruption in the services ; for this purpose, the two software with two different data bases have been run and used in parallel up to early 1992 until not a single failure appears with the new system. This is described by Toustou (1992a).

4.2. Data Collection

The data base content, as concerns actual measurements, is regularly increasing. It contains a little more than 4.5 million point values in about 3000 sources (to be for instance compared with 800 000 measurements in 1979). There still remain several sets of land data to be added. This has been a very slow process due to the characteristics of the old CDC software, until the new system was perfectly working. Large data sets of marine data were received from NGDC* in the context of the European Geoid Project and, recently, BGI acquired the totality of the NGDC data on CD-Rom. These will be processed and merged in the course of 1993.

New catalogues, available on request, will be produced in 1993,

- . General coverage of gravity data per 20 x 20 degrees area,
- . Index catalogue of data distribution : statistics per degree square.

In addition, new efforts were exerted in trying to get data from the ex-Eastern countries due to the important geopolitical changes. In most cases, gridded values of free-air gravity and topography were obtained such as in Poland (5' x 5'), Hungary (5' x 7.5'), Rumania (5' x 5').

As it was pointed out many times, and especially on the occasion of international meetings (e.g. Balmino, 1983), some countries still do not provide any gravity data to BGI. Whether this is due to military regulations or a policy which intends to protect some national economic interests, we feel that it does much harm to these countries which, in turn, cannot pretend receiving data from the bureau or from similar organisations, and with which scientists will be discouraged to cooperate in the long run.

4.3. Data Validation

A great deal of effort have been directed at validating data for several years. Firstly, several land data validation software were intercompared (on the occasion of a dedicated workshop in 1989) some of them developed by BGI - such as SYSTEVAL and DIVA/VERSET (Toustou et al., 1989),

* National Geophysical Data Center, Boulder, USA

Fig. 4. Bouguer anomaly map with six measurements predicted "doubtful"



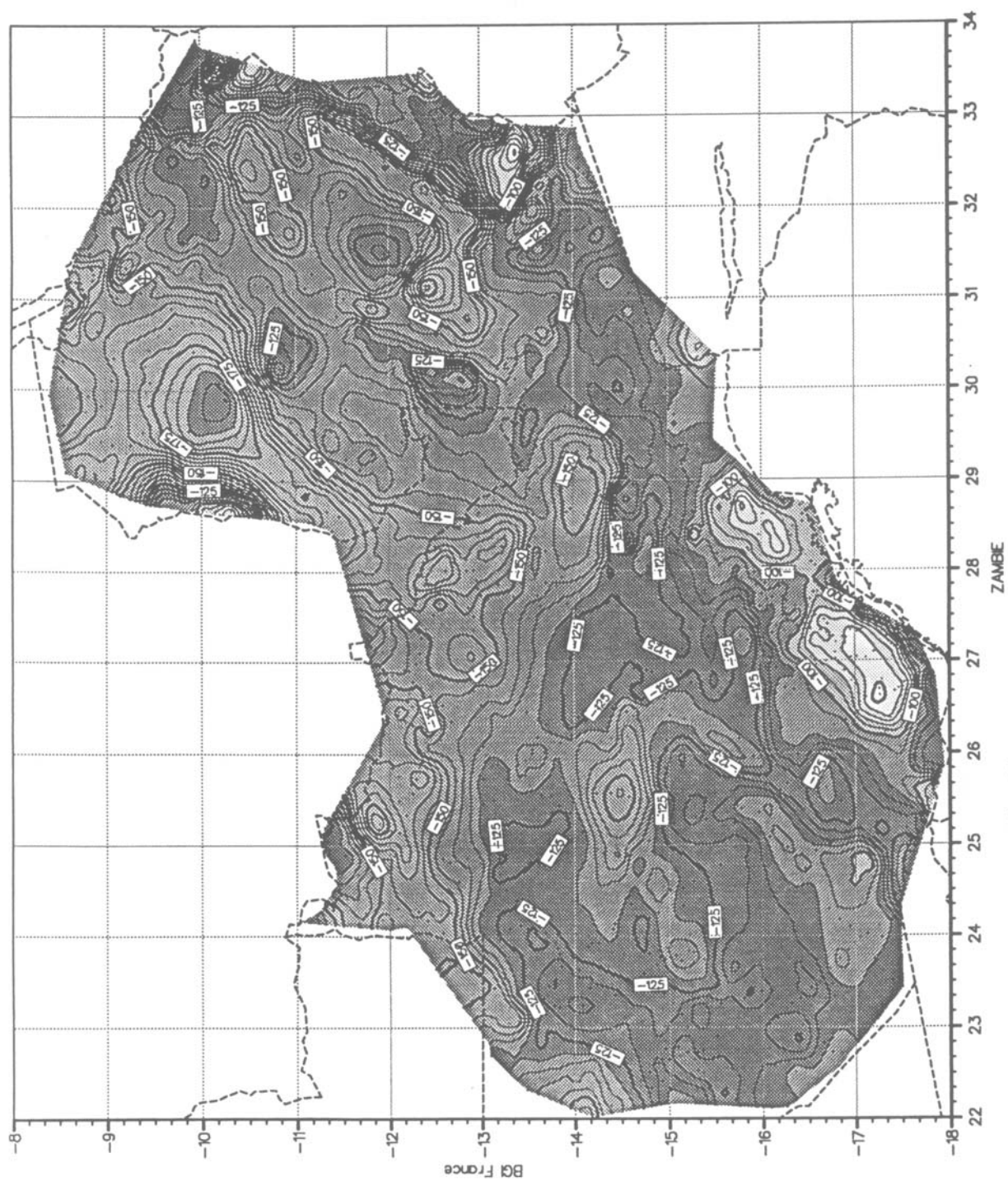


Fig. 5. Bouguer Anomalies - 1817 gravity measurements

others having been provided by working group members or associates. The in-house software was then used to validate all BGI land data on a one by one source basis (example on fig. 4) ; inter-source comparisons (adjustments using overlapping parts) still have to be done. Then was undertaken the conversion of DIVA/VERSET on a Sun-Sparc 2 workstation using the SUNPHIGS library, to allow future portability and (possibly) easier upgrading.

Plans are now made to install similar software for the validation of marine data especially to solve for cross-over minimisation parameters. A program (SEAGRA) for performing this task, was received from H.G. Wenzell. It was installed, upgraded ; in particular, the decomposition of each cruise into legs has been implemented in an automatic mode in late 1991. A complete tool in its first version is available and was presented at another workshop organised on these topics (Oct. 1992).

The final product of a validation session is the set of data with erroneous measurements being flagged, and maps of free-air and/or Bouguer anomalies (such as on fig. 5).

4.4. Requests

The Bureau has satisfied almost one thousand requests for data (of any type) in the last ten years, with sharp increases in 1986 and 1991 (fig. 6). This activity presently employs one person more than half-time.

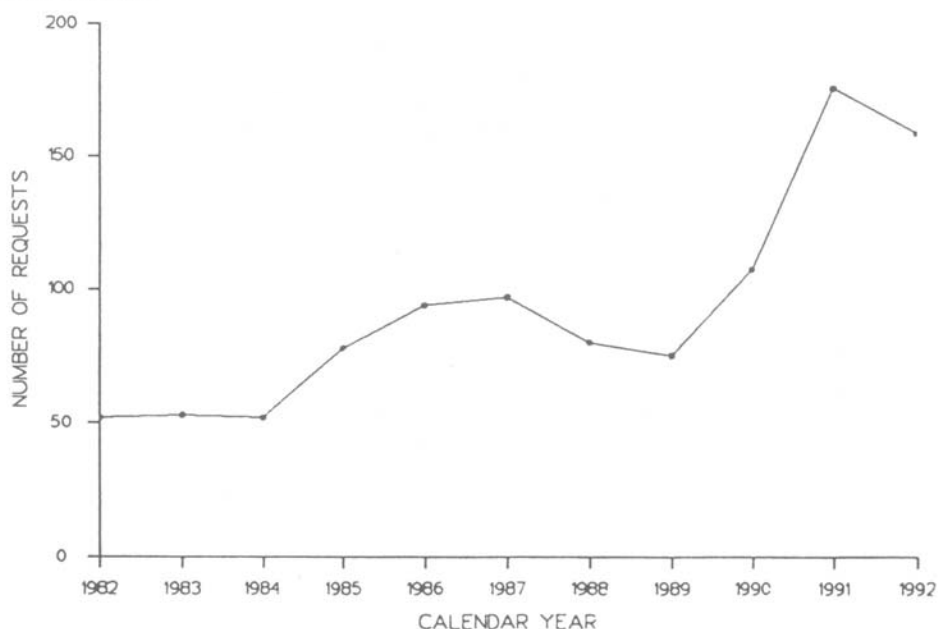


Fig. 6.

4.5. Bibliography

Compiling bibliographical references of all publications received at the Bureau and dealing with gravimetry or closely related subjects has been an historical task. The digitisation of these references started with the current material in 1980 when the office was moved from Paris to Toulouse.

The digitization of the old bibliography (prior to 1980) was undertaken in 1990. This was a huge work which was performed by the BGI secretary, with an additional temporary help. It was completed in 1991. A file is available on floppy disk.

The corresponding data base is resident on hard disk on a P.C., also on the main frame IBM 4381 (where resides the gravity base), and is managed by means of the ORACLE software, too.

4.6. Miscellaneous

- training of students : data validation procedures, computational methods (determination of gravity, geoid, etc ... from spherical harmonic models ; numerical works with the integral equations of physical geodesy), graphics.

- compilation of absolute measurements : still difficult notwithstanding the help of Working Group 2 (agencies do not answer to our request for data and facts). This point was debated by the Directing Board in its last meeting (1992).

- status of IGSN 71* reference gravity stations : a situation was established in 1989, with the help of some sub-commissions of IGC. About 1200 stations (out of 1850) still exist, the others having been destroyed (generally when reconstructing buildings ...). The other reference stations (e.g. the ACIC* file) are updated from time to time. Their description sketches are on microfiches and numerical values (code number, site co-ordinates, g-value, microfiche number) are in a computer file (like for the IGSN71 network) ; there are about 8540 reference stations as of January 1993 (1220 in IGSN71*, 5200 in ACIC, 320 in WHOI*, 800 in LAGSN77*, 1000 in various national networks).

- BGI also archives gravity maps of various types and scales, from different origins. A map file and a catalogue (print-out and computer form) exist, which contain references of ~ 3000 maps. Software to extract information from the file exists. Nowadays, only a few new maps are added each year.

5. SPECIAL PROJECTS OR EVENTS

5.1. Contributions to Gravity Maps

- 10° x 10° maps displaying marine gravity data, cruises location and gravity values themselves were made at BGI for the following areas :

 - . North Atlantic Ocean (in 1984).

 - . Whole Pacific Ocean (in 1985).

This important work was done at the request of the Soviet Geophysical Committee to complete their Geological-Geophysical Atlas (finally published with the help of UNESCO in 1992). Subsequent interpretation work was performed in cooperation with Russian scientists (Kogan et al., 1985).

- Gravity Maps over the Mediterranean Area :

At the request of the UNESCO/IBCM project, BGI compiled a first series of 10 maps of Bouguer Gravity anomalies (scale 1:1 000 000) from existing measurements plus additional values which were gridded from maps, and a second series which is mostly based on the maps by Morelli.

A 3' x 5' grid was produced as an intermediate product.

- 5' x 5' Gravity Map of the World :

The Bureau and WG1 members (at the Geological Survey of Canada, GSC) prepared a 5' x 5' gravity map of the whole world. BGI produced the part of the basic grid over land areas (Bouguer anomalies) while GSC prepared the oceanic part (free air). It was published in 1991.

* IGSN : International Gravity Standardization Net 1971, ACIC : Aeronautical Chart and Information Center
WHOI : Woods Hole Oceanographic Institution, LAGSN77 : Latin American Gravity Standardization Net 1977

5.2. Use of Satellite Altimetry

Satellite altimetry derived geoid heights have been used on several occasions in various research activities in co-operation with several groups in the BGI supporting french agencies.

- A combine Geos 3 and Seasat altimetric geoid (derived from an adjusted mean sea surface and a model of the mean ocean circulation) was especially produced and used to derive a quasi-global oceanic set of 15' x 15' free-air gravity anomalies (Balmino et al., 1987).

Attempts at combining satellite derived data with measured gravity were also done over the North-Algerian Marge, the Aegeen trench, and around the Reunion Island. Deviations of the vertical have also been computed on request from these data.

- Simultaneous use of GEOSAT, ERS1 and TOPEX POSEIDON altimetry derived geoid heights to compute mean gravity anomalies in the North Atlantic, North Sea, Arctic Ocean and in the part of the Pacific and Indian Oceans in the context of large regional projects (cf. 6).

5.3. Geoid Computations

At the time the International Service for the Geoid did not exist, a number of requests were made to BGI to help in the computation of the geoid over limited areas (Balmino, 1986 b). These determinations were made using the classical Stokes integral (regularised at origin and with respect to a high degree spherical harmonic model of the geopotential), sometimes also by the collocation method (Balmino, 1982 and 1986 c). For example :

- the geoid over and around Madagascar was computed from a combination of gravity and Seasat altimetry data, and from a 180 x 180 spherical harmonics reference field (Rakotoary, 1986).

- the gravimetric geoid over the straight of Gibraltar and over most Morocco was determined in the context of the fixed link project and at the request of the Morocco government (Balmino et al., 1989). Geoid heights were in good agreement with satellite Doppler and survey derived quantities at control points.

- in the framework of a geophysical project supported by ICSU, BGI computed a gravimetric geoid over Jordania, based on a 360 x 360 reference field provided by the Ohio State University and on gravity measurements specifically available for the project.

- a 3' x 5' gravimetric geoid over France has been in preparation for many years. The project involves about 400 000 gravity measurements over France (provided by BRGM) and about 100 000 in neighbouring countries and oceanic areas. After an attempt (Deloménie, 1987) which showed some defects in the methodology (no terrain corrections had been applied) but also in the data (there were still large gaps and data were poorly validated), the activity was frozen for some time due to manpower constraints and other priorities. It started again in 1992 (Balma et al., 1992) with a complete analysis of the various stages and of the operational questions (including existing software) and continues in collaboration with IGN and BRGM, which expressed their interest of continuing the support to BGI in the realisation of this project.

5.4. Participation in the ICL/CC5 Activities

The Co-ordinating Committee Number 5 of the International Committee for the Lithosphere asked the help of BGI to compile an index of all centers archiving gravity, topographic, magnetic, seismic, and other data. This was achieved in 1988 and resulted in a catalogue published by CC5. The Director of the Bureau continues to represent the International Gravity Commission on CC5.

5.5. Participation in RGIA

The Bureau contributes to the activities in the project : "Réseau Géodésique Intégré sur l'Afrique", in which the establishment of new gravity networks, the making of absolute measurements, and questions of data densification are discussed.

5.6. Activities Related to Digital Terrain Models

- Digitisation of the worldwide bathymetry :

BGI was engaged in the digitisation of the GEBCO 5th Edition Bathymetric Charts between 1982 and 1991, with the help of the GEBCO Sub-Committee on Digital Bathymetry, the Canadian Hydrographic Service and the Institut Géographique National.

The main steps involved in such a work were :

- (a) automatic numerization of the contours (by a scanner) - performed at IGN, France.
- (b) interactive correction of the digitised level curves.
- (c) constitution of a data base for future updating of the GEBCO maps.
- (d) computation of analytical terrain models and production of grid values.

Step (b) was the BGI responsibility and was by far the most difficult and demanding in manpower and software. It was completely reanalyzed in 1984 after it was discovered that the previously developed package was very incomplete and inadequate. A new and quite sophisticated software was then developed and proved to be very efficient operationally.

Five maps were digitised (5-13, 5-14, 5-15, 5-16, 5-18) in 1986 and 1987. One person was assigned in 1987 by IGN to work full time on it. BGI completed the Northern Europe sheet (5-01) in 1987, the North Atlantic (5-04), Central Atlantic (5-08), and North Polar sheet (5-17) in 1988. The course of this effort was interrupted in 1989 for priority reasons and restarted in October 1990. Then the two sheets for the North and Central Indian oceans (5-05 and 5-09) were produced in 1991. The contribution of the Bureau and IGN to the project therefore consists in eleven files (out of eighteen) and is considered to be terminated. There remains the software which could be used for other similar applications (Toustou, 1985).

- A new DTM project :

The International Service for the Geoid and BGI have in mind to build up a data base of DTMs for a variety of uses, but obviously with major applications in geodesy and geophysics.

After a short period of excitement on the french space agency side (one of the supporting organisations of BGI), the enthusiasm cooled down for it appears now that the agency interest for the project is not so great or at least too diluted in something else (links with IGBP*) of which the future is not so certain.

Nevertheless, it was decided to go ahead, though slowly, and to first set up a limited DTM base around the Western Mediterranean, in relationship with other projects over this area. Main activities will consist in :

- . collecting existing (gridded) DTMs, probably with variable resolution.
- . creating DTMs from the BGI gravity data base (height information).

* International Geosphere Biosphere Program

- . comparing these two types of DTMs.
- . providing the best possible grids over limited areas.

6. PARTICIPATION OF BGI IN RECENT REGIONAL GRAVITY PROJECTS

6.1. African Gravity Project (AGP)

This project was formulated in 1985 by geophysicists D. Fairhead of the University of Leeds (U.K.) and A. Watts of Lamont Doherty Geological Observatory. The goal was the compilation of all available private and public domain gravity data for Africa to derive a map of the gravity field of the African continent and its continental margins. The project was managed by the University of Leeds Industrial Services (ULIS) and sponsored by 16 oil companies. The Bureau participated in 1986-87 by bringing its data base over that region. It is now also responsible for the archival of all the AGP data and received in addition the files of the produced grid values of free-air, Bouguer anomalies and elevations at 5' resolution both in longitude and latitude, for internal use in validating future acquisitions of data from Africa. Maps are being sold since early 1991 and the grids should be in the public domain in 1988.

6.2. South American Gravity Project (SAGP)

BGI was involved with the same group at the Leeds University in their South American gravity compilation project on the same basis as the African project in 1985-88. In addition, BGI brought its expertise and validated the initial data set (about 70 000 gravity observations) over this continent. The project terminated in June 1991, but final products were produced later, in the course of 1992 : 5' x 5' of free air and Bouguer anomalies, and of topography ; atlas of maps. BGI is a depository of these products which are used internally, but are not freely available - except over local areas in the context of special studies or for lower resolution data sets (e.g. 30' x 30' obtained by the Ohio State University).

6.3. Other Regional Gravity Projects (SEAGP, WEEGP)

ULIS made new plans in 1990 for projects similar to the ones above mentioned, in South-East Asia (SEAGP) and in Europe, including the ex-Eastern countries and ex-USSR, up to Ural (WEEGP). BGI is also involved in these activities. Both projects started in mid 1991. Of special interest is WEEGP since it is in some way combined with the efforts of the Sub-Commission for the Geoid in Europe (of the International Commission for the Geoid). Great emphasis is put on WEEGP due to the new situation in this part of the world ; specifically, Russia is going to provide gridded data at the resolution of 4 km x 4 km. SEAGP is facing insuperable problems with India and China ; Indian authorities are keeping all recent gravity data and Chinese authorities refuse to provide any kind of gravity information (apart from a poor resolution map with contour lines at 25 mgal interval).

7. MEETINGS

One of BGI's role is obviously to foster all programs aimed at improving our knowledge of the geopotential (Balmino, 1988 and 1989).

Besides organising the meetings of IGC, which take place every four years (1986 and 1990 in Toulouse, France ; 1994 in Graz, Austria - jointly with the International Commission for the Geoid), BGI therefore tries to be present on the world geodetical and geophysical scene : IUGG and IAG general assemblies , symposia and workshops. For instance in the past year, BGI participated in the following :

- European Geoid Workshop (Prague, May 1992), with one paper.
- WEEGP-SEAGP Workshop (Toulouse, Oct. 1992), with four papers.
- Workshop on Marine Gravity Data validation (Toulouse, Oct. 1992) with 3 technical papers by BGI staff, including a report on a test case distributed for comparison to participants (proceedings appeared in Bulletin d'Information n°71).

8. PROGRAM OF ACTIVITIES FOR THE NEXT YEARS

- Continue data collection, archiving and distribution : emphasis will be on those countries which have not, or seldom, contributed to the BGI data bank. First priority will be given to careful data evaluation ;
- Continue the publication of the Bulletin d'Information ;
- Assist IGC in setting up the International Absolute Gravity Base Station Network (IAGBN), and assist in the inter comparisons of instruments ;
- Support projects aimed at acquiring new absolute gravimeters in the context of WG1 (IAGBN) and WG5 activities ; for instance the joint venture between Belgium, France and Luxembourg to obtain the financing of such an instrument from European and national sources ;
- Establish simple procedures for the collection and archiving of absolute measurements ;
- Link with the Commission for the Geoid in data preparation in view of geoid computations and evaluations to be performed by the International Service for the Geoid ;
- Assist in promoting satellites techniques to improve our global knowledge of the Earth's gravity field : satellite-to-satellite tracking, satellite gradiometry, etc ...

9. THE BGI STAFF

The staff of the Bureau is composed of the following, as of January 1993 :

Position	Supporting Institute	Percentage of time spent in BGI activities (%)	Name
Director	CNES	30	G. Balmino
Secretary	CNRS	30	N. Rommens
Secretary	CNRS (under contract)	30	M. Barriot
Engineer	CNES	100	M. Sarrailh
Engineer	CNES	20	B. Moynot
Analyst/Prog.	IGN	100	D. Toustou
Technician	IGN	100	G. Balma

Acronyms : CNES : Centre National d'Etudes Spatiales
 CNRS : Centre National de la Recherche Scientifique
 IGN : Institut Géographique National

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THE EUROPEAN GEOID PROJECT

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1. INTRODUCTION

Regional geoid resp. quasigeoid determinations are nowadays required with an accuracy of ± 1 to 10 cm over distances from 100 to some 1000 km in order to meet the demands of geodesy, geophysics, oceanography and engineering. Especially the combination of GPS heighting with classical levelling is one of the primary drivers for precise geoid computations. As a consequence, the IAG International Geoid Commission decided at its meeting in Milan, 1990, to establish a subcommission for Europe in order to support the derivation of a new geoid model for Europe, which should be significantly improved in spatial resolution and accuracy as compared with presently available models. The Institut für Erdmessung (IfE), University of Hannover, was asked to do the necessary calculations and serves as a computing center in this project.

In the following, we first review some of the more important geoid (quasigeoid) calculations performed for Europe from the 1950's to 1980's. We then discuss the possibilities for improved geoid determinations and give a summary of the work performed by IfE since 1990. This discussion includes the most recent quasigeoid calculation performed by IfE in 1991, which was presented at the XX IUGG General Assembly, Vienna, August 1991, as well as some of the more recent developments of the European Geoid Project with respect to data collection and evaluation. A completely updated European quasigeoid solution is planned to be presented at the IAG General Meeting in Beijing, 1993. Here it should be noted that in all our computations we deal with the quasigeoid because this has the advantage that only gravity field data observed at the earth's surface and in the exterior of the earth enter into the calculations, and no assumptions about the gravity field in the earth's interior have to be made. A subsequent transformation to the geoid introducing a density model can easily be performed.

2. GEOID/QUASIGEOID DETERMINATIONS AVAILABLE FOR EUROPE

At the end of the 1940's attempts for a geoid calculation in Europe were made. The first one uses astrogeodetic vertical deflections and the astronomic levelling technique in order to reduce the Central European Triangulation Net to the Hayford ellipsoid (Wolf 1949). The gravimetric solution of Tanni (1949) is based on $1^\circ \times 1^\circ$ and $5^\circ \times 5^\circ$ isostatic anomalies. Both determinations give relative geoid heights with accuracies of a few meters only. Work then concentrated on the astrogeodetic solution with strong efforts undertaken within the framework of IAG. These attempts led since 1954 to the continuously improved Bomford geoid (Bomford 1972). The last version of this astrogeodetic geoid includes some 1000 vertical deflections, resulting in a relative accuracy at the one meter level in well surveyed areas like Finland, France, Federal Republic of Germany and others (Levallois and Monge 1978).

A major step forward then was the gravimetric quasigeoid calculation EGG1 performed at IfE, Hannover, in the beginning of the 1980's (*Torge et al. 1982*). This solution is characterized by the following features:

- establishment of a high resolution gravity data base ($1^\circ \times 1^\circ$ and $6' \times 10'$ mean gravity anomalies) with careful analysis and screening of the data;
- statistical analysis of the data and derivation of signal and error covariance functions, needed for anomaly prediction and for error propagation in the transformation process;
- least squares spectral combination with closed integral formulas, using a global satellite derived model (GEM 9), 12000 $1^\circ \times 1^\circ$ anomalies (integration radius 20°), and 104000 $6' \times 10'$ anomalies (integration radius 3°), yielding quasigeoid values in a $12' \times 20'$ grid;
- straightforward error calculation with a priori error variances and covariances (for the anomalies), resulting in an absolute error estimate of ± 0.9 m, and relative errors of $\pm 0.3...1.1$ m/100...1000 km, for the quasigeoid derived.

By including about 5000 vertical deflections, this solution was extended to the EAGG1 quasigeoid (*Brennecke et al. 1983*), where the astrogeodetic data improved the gravimetric solution mainly in areas with lacking or insufficient gravity data.

An independent accuracy control of the quasigeoid is provided by the central and northern part of the European GPS traverse, established by IfE in cooperation with many other institutions in 1986 and 1987 (*Torge et al. 1989*). This traverse follows first order levelling lines of the United European Levelling Network (UELN), thus providing about 70 geometrically (GPS/levelling) derived quasigeoid heights, covering a range of 3000 km with an average station distance of 50 km (see Figure 1). From loop misclosures of the GPS traverse and UELN, and from the results of comparisons with more recent quasigeoid calculations (see section 4), we estimate the relative accuracy of the quasigeoid heights thus derived to be a few centimeters to one or two decimeters for distances between 50 and 3000 km.

Table 1 contains the number of control points used for the comparison, the RMS discrepancies and the maximum and minimum deviations (after constant bias subtraction) for those older European geoid solutions and for some recent global models. The comparisons with the astrogeodetic geoid (*Levallois and Monge 1978*), the gravimetric quasigeoid EGG1 (*Torge et al. 1982*), the astro-gravimetric quasigeoid EAGG1 (*Brennecke et al. 1983*), the tailored spherical harmonic model IFE88E2 (*Bašić et al. 1989*) and the global model OSU89B (*Rapp and Pavlis 1990*) are given in Figure 1. The large scatter of the pure astrogeodetic geoid reveals the deficiencies of this solution, with large station distances (especially in Norway) and not homogeneously distributed data, as well as systematic errors of the astronomic positions. The gravimetric geoid EGG1 is distorted by a strong slope of 1.5 m in Denmark, Sweden and Norway, which is due to the bad quality of the data used in this region (*Bašić et al. 1989*). This slope has completely disappeared in the more recent solutions for Scandinavia (see *Forsberg 1990* and section 4) as well as in the astrogeodetic solution EAGG1, demonstrating how even scarce vertical deflection data may improve a regional calculation.

Solution	Authors	No. of Stations	RMS [m]	Min. [m]	Max. [m]
Astrogeodetic	Levallois/Monge 1978	60	± 1.17	-2.73	+2.38
Gravimetric (EGG1)	Torge et al. 1982	60	± 0.63	-1.33	+1.13
Astrogravimetric (EAGG1)	Brennecke et al. 1983	60	± 0.24	-0.59	+0.60
Global Model (GPM2)	Wenzel 1985	60	± 0.70	-1.45	+1.26
Global Model (OSU86F)	Rapp and Cruz 1986	60	± 0.77	-2.04	+1.09
Tailored Model (IFE88E2)	Bašić et al. 1989	60	± 0.32	-0.89	+0.94
Global Model (OSU89B)	Rapp and Pavlis 1990	60	± 0.31	-0.81	+0.69
Global Model (OSU91A)	Rapp et al. 1991	60	± 0.32	-0.90	+0.76

Table 1: Statistics for the comparison of different geoid/quasigeoid solutions with the European GPS traverse results.

3. POSSIBILITIES FOR IMPROVED GEOID/QUASIGEOID DETERMINATIONS

In the 1980's, new global, regional and local gravity field data sets became available, as well as digital terrain models (DTM's). In connection with progress in gravity field modelling procedures and computing facilities, the possibility of proceeding from regional dm...m accuracies to cm...dm accuracies became visible.

The progress in data collection may be summarized as follows:

- availability of improved satellite-only models including the associated covariance matrices, e.g. GEM-T1 (*Marsh et al. 1987*) and GEM-T2 (*Marsh et al. 1989*);
- availability of new satellite altimeter data from the Geosat Exact Repeat Mission, ERS-1, and TOPEX, including precise orbits;
- availability of global high resolution geopotential models, as GPM-2 (*Wenzel 1985*), with a spherical harmonic development to degree and order 200, corresponding to a spatial resolution of 1° , and OSU86E/F (*Rapp and Cruz 1986*) or OSU89A/B (*Rapp and Pavlis 1990*) resp. OSU91A (*Rapp et al. 1991*) complete to degree and order 360, corresponding to a resolution of $30'$; the overall accuracy of these models is estimated to be ± 0.3 to 0.7 m (see also the comparison with the GPS traverse in Table 1) including the omission error of ± 0.4 m and ± 0.2 m respectively;
- calculation of regional geopotential models, by tailoring global models to the gravity field in Europe. We mention the 360 models IFE88E1 (*Bašić 1989*) and IFE88E2 (*Bašić et al. 1989*). Both use OSU86F, and tailor the harmonic coefficients in the medium spectral range either to a $12' \times 20'$ or to an updated $0.5^\circ \times 0.5^\circ$ mean gravity anomaly data set for Europe; the estimated accuracy is ± 0.3 m, as verified by comparison with the GPS traverse (see Table 1);

- availability of point gravity anomalies, at least for larger parts of Europe and its surroundings, with $\pm 2 \dots 20 \mu\text{ms}^{-2}$ accuracy and station distances of 2 to 5 km (see Figure 2);
- availability of high resolution digital terrain models, as the global $5' \times 5'$ model, and regional models with a block size of $1 \text{ km} \times 1 \text{ km}$ or smaller;
- increasing availability of point geoid/quasigeoid heights from Doppler/levelling ($\pm 0.2 \dots 0.5 \text{ m}/100 \dots 1000 \text{ km}$) and GPS/levelling ($\pm 0.01 \dots 0.1 \text{ m}/1 \dots 1000 \text{ km}$) control points.

This led to the development of a strategy for improving the existing European geoid solutions by applying the "remove-restore technique" (Denker et al. 1986). Here the gravity field information is splitted up into three different parts:

- the long wave part up to about 200 km wavelength is taken from a global or regionally tailored geopotential (spherical harmonic) model;
- the medium wave part (200 km to $5 \dots 20 \text{ km}$ wavelength) is taken from terrestrial point gravity field data, as gravity anomalies and astrogeodetic vertical deflections;
- the short wave part is derived from a high resolution digital terrain model.

Removing the long wave and the short wave part from the point data (which corresponds to a spectral filtering) leads to a residual gravity field data set, on which gravity field transformation algorithms for deriving the quasigeoid are applied. As shown, among others, by Denker (1988) and Bašić (1989), integral formulas and least squares collocation give comparable results at this transformation. By restitution of the long and short wavelength gravity field part to this residual quasigeoid solution, the final quasigeoid is derived. Simulation studies revealed that by applying this technique, the data collection area in integral formulas and collocation may be strongly reduced, leading to more economic calculations.

Theoretical and numerical investigations proved the great potential of updated quasigeoid calculations based on the concepts described above, with a few cm accuracy over distances of some 10 to some 100 km. As two remarkable examples, we mention here the new quasigeoid for the western states of the Federal Republic of Germany (Denker 1989) and the new Scandinavian quasigeoid (Forsberg 1990). Both calculations were done using Stokes formula in connection with the remove-restore technique, where the numerical evaluation was performed by means of FFT. In the 1989 solution for the Federal Republic of Germany, the tailored spherical harmonic model IFE88E1, 46000 point gravity anomalies ($2 \dots 5 \text{ km}$, $\pm 3 \mu\text{ms}^{-2}$), $6' \times 10'$ mean anomalies in the adjacent area (distance $2^\circ \dots 3^\circ$, $\pm 10 \dots 50 \mu\text{ms}^{-2}$), and a $30'' \times 50''$ DTM ($\pm 5 \dots 50 \text{ m}$) were used. While the long wave model part of the quasigeoid (calculated in a $60'' \times 100''$ grid) provides values between 39.2 and 50.5 m, the medium wave part adds only a signal of $\pm 0.18 \text{ m}$ (maximum 1.2 m), while the terrain model part is $\pm 0.02 \text{ m}$ (maximum 0.17 m). GPS/levelling quasigeoid heights again provide a reliable control for the gravimetric quasigeoid solutions. This was possible in local areas (Hannover test area), where a $\pm 0.02 \text{ m}$ RMS discrepancy was found, as well as for the entire computation area, where the comparison with the European GPS traverse gave a RMS discrepancy of $\pm 0.05 \text{ m}$, while for the DÖNAV campaign (preliminary results obtained at the computing center IfE, Hannover; Prof. Seeber, personal communication) a RMS discrepancy of $\pm 0.06 \text{ m}$ was obtained (Denker 1990). Forsberg (1990) applied very similar procedures for Scandinavia and obtained a RMS difference of $\pm 0.10 \text{ m}$ versus GPS/levelling results from the European GPS traverse over a range of about 2000 km.

4. THE 1991 QUASIGEOID CALCULATION

From the above discussion it is evident, that new gravity field data sets of high quality are available today and will be extended in the near future. This includes new global models, new satellite altimeter data from the ERS-1 and TOPEX missions, further gravity data not yet released, more GPS/levelling quasigeoid heights, as well as regional high resolution digital terrain models. Taking these possibilities into account, and with the experiences already obtained, IfE, Hannover, proposed at the First International Geoid Commission Symposium in Milan, 1990, to attack the determination of an improved quasigeoid for Europe and offered to do the calculations (*Torge and Denker* 1990). The Geoid Commission, recognizing the urgent need for such an improved solution, decided to establish a Subcommittee for the Geoid in Europe in order to support the geoid project and elected Dr. Vermeer as its chairman. IfE was then officially requested by the chairman of the subcommittee to serve as a computing center in this project. The procedure for the computation of an improved quasigeoid for Europe to be followed at IfE, will be - at the present state of discussion - the following one (see also *Torge and Denker* 1990):

- development of improved global geopotential models, with inclusion of improved "satellite-only" models, and straightforward error calculation, with eventual inclusion of global topographic-isostatic models;
- extension of the existing IfE gravity field data base by including point gravity data (or high resolution mean anomalies), regional DTM's, satellite altimetry and sea surface topography models, as well as GPS/levelling quasigeoid heights;
- careful screening of these data with respect to gross and systematic errors, transformation to the same reference (height system, tidal reductions), and error assessment of the data;
- improvement of the existing software for quasigeoid calculations, including error propagation;
- development and testing of new gravity field solutions for Europe.

The strategies for the calculation may be characterized by the following items:

- quasigeoid determination using improved global models, observed high resolution (eventually gridded) gravity data and digital terrain models;
- quasigeoid determination as above with inclusion of satellite altimeter data and sea surface topography models;
- quasigeoid determination as above with inclusion of GPS/levelling point data;
- quasigeoid determination using locally derived gravity field data, especially national quasigeoid solutions, as a pragmatic solution, to be followed if high resolution (point) gravity anomalies are not provided for large areas. This strategy requires more investigations about the combination of national solutions with special emphasis on the transformation to a common reference, and the control of long wavelength error propagation.

As a first step, the existing IfE gravity data base was extended by the gravity data stored at Bureau Gravimétrique International (BGI), Toulouse, which is by far the largest data source. Furthermore, since the initiation of the European geoid project in 1990, the subcommittee and IfE have approached a number of national agencies and individuals to release high resolution terrain data (block size 250...1000 m) as well as

additional gravity data not stored at BGI. This led to the release of a significant amount of new data, which is summarized in the following (status as of August 1991):

- gridded $3' \times 5'$ mean gravity and elevation data for Scandinavia;
- point and mean gravity and elevation data for the Netherlands;
- point gravity data for France (90000), Belgium, Luxembourg and Hungary;
- $5' \times 5'$ mean gravity and elevation data for Poland;
- point gravity values (80000) and terrain data (100000) for East Germany (cooperation with Institut für Angewandte Geodäsie, Frankfurt/Leipzig);
- high resolution terrain models for Austria (1.3 million data), Switzerland (1.1 million data) and Italy (12 million data);
- positive answer from Dr. Fairhead, Manager of the West-East European Gravity Project (WEEGP), indicating that gravity data from this project might be released for the computation of an improved European geoid.

The status of the IfE gravity field data base may be characterized by the following figures. In the mid of 1991 the data base contained about 1.5 million point gravity data, with the majority of data coming from BGI, and about 15 million terrain data. About 800000 gravity values for central, northern and western Europe have been validated so far using batch and interactive procedures developed at IfE. Figure 2 shows the distribution of the available gravity data for central and northern Europe.

At the present state of evaluation, IfE is concentrating on a pure gravimetric solution, using residual gravity anomalies reduced for the effect of a global spherical harmonic model as well as for the effect of the topography using the residual terrain reduction (RTM) technique (see *Forsberg and Tscherning* 1981). Due to the large amounts of data which have to be employed, FFT is used for the field transformation from residual gravity anomalies to corresponding height anomalies. The effect of the global model and of the topography are added back subsequently. As at present high resolution terrain data are lacking for large parts of Europe, some test calculations were performed for Scandinavia using the global $5' \times 5'$ terrain model. However, the internal comparisons as well as an independent control with GPS/levelling results from the European GPS traverse (*Torge et al.* 1989) showed clearly, that the global $5' \times 5'$ DTM can not be used for this purpose due to the poor resolution and quality of this model. Large aliasing errors were found, which disturb the solution in a systematic manner. This can be explained by the fact that the classical terrain corrections, entering directly or indirectly in the computations, are always too small when using terrain models with an inadequate resolution. The test solution using the $5' \times 5'$ DTM did not show any improvement as compared to the existing solution EGG1 (*Torge et al.* 1982). Thus, it is clear that high resolution terrain data play a crucial role for the development of an improved quasigeoid for Europe.

Therefore, in a first iteration, a new quasigeoid solution was computed for central and northern Europe, as only here we had reliable terrain and gravity data of sufficient resolution as well as GPS/levelling data for an independent control available. This new solution was presented at the XX IUGG General Assembly, Vienna, August 1991 (*Denker and Torge* 1992). For the calculations, the spherical harmonic model OSU89B complete to degree and order 360 (*Rapp and Pavlis* 1990) was used. Terrain reductions were computed using the RTM technique in connection with a $15' \times 15'$ moving average filter for the construction of the reference topography. The residual gravity data were gridded on a $3' \times 5'$ grid for northern Europe and a $60'' \times 100''$ grid for central

Europe, depending on the density of the available terrain and gravity data. The field transformation was carried out by planar FFT (investigations based on spherical procedures are under way with very encouraging results). Finally the effect of the global model and of the topography were added back. A contour line map of the computed quasigeoid for central Europe is displayed in Figure 3.

Description	No. of Stat.	Bias Fit			Bias + Tilt Fit		
		Rms [m]	Min. [m]	Max. [m]	Rms [m]	Min. [m]	Max. [m]
DÖNAV	35	0.086	-0.214	+0.144	0.048	-0.074	+0.096
ALGESTAR (CH)	37	0.116	-0.262	+0.260	0.084	-0.162	+0.186
3D Traverse (CH)	28	0.082	-0.177	+0.133	0.043	-0.110	+0.088
GPS Traverse	67	0.169	-0.364	+0.274	0.158	-0.333	+0.277
	32	0.136	-0.343	+0.167	0.071	-0.147	+0.126
	27	0.063	-0.131	+0.115	0.049	-0.093	+0.102

Table 2: Statistics for the comparison of the 1991 quasigeoid solution for central and northern Europe with different GPS/levelling results.

The quality of the computed solution was evaluated by GPS/levelling data from four different campaigns and is summarized in Table 2. Here it has to be noted that strict normal heights were only available for the European GPS traverse (*Torge et al. 1989*) and for the 3D traverse in Switzerland (*Wirth 1990*), while so-called normal-orthometric heights (approximation to normal heights) and orthometric heights had to be used for the DÖNAV (*Seeber 1988*) and the ALGESTAR campaign (*Marti 1990*) respectively. The comparisons were always done using a bias as well as a bias and tilt fit to account for possibly existing long wavelength gravity model error and inaccuracies in the GPS/levelling results. Using the bias fit only, the comparison of the 1991 quasigeoid with preliminary results for the DÖNAV campaign (computing center IfE, Hannover; Prof. Seeber, personal communication) yields the differences shown in Figure 4. Altogether, 35 stations located in the Federal Republic of Germany were used, as at present levelled heights are not available for the other DÖNAV stations. It can be seen from Figure 4 that a long wavelength discrepancy in north-south direction is existing between the three data sets involved, which may be an indication for long wavelength gravity model errors. The RMS discrepancy is ± 0.086 m for the bias fit and reduces significantly to ± 0.048 m for the bias and tilt fit. A second comparison of the 1991 quasigeoid was possible using GPS/levelling results from the ALGESTAR campaign (*Marti 1990*) covering entire Switzerland. Again we observe a significant reduction of the RMS difference from ± 0.116 m to ± 0.084 m for the bias versus the bias and tilt fit (see Table 2). However, here one must also consider that strict normal heights were not available, and that the gravimetric quasigeoid may suffer from lacking high resolution terrain data in Italy, which were not at our disposal when the solution was done. For the 30 km long 3D traverse in Switzerland (*Wirth 1990*), located in an extremely rugged area of the Alps with elevations above 4000 m, we get a RMS difference of ± 0.082 m and ± 0.043 m for the bias and bias and tilt fit respectively. The comparison of the 1991 quasigeoid with the European GPS traverse is shown in Figure 5. Here it has to be mentioned that high

resolution terrain and gravity data were only available for the southern part, while 3' × 5' mean gravity data had to be employed for Scandinavia. Therefore, as was to be expected, we get the best results in the southern traverse section with RMS differences for the bias fit of ± 0.136 m (32 stations) resp. ± 0.063 m (27 stations; 5 southernmost stations eliminated due to possibly existing problems in this traverse part). For the entire traverse with a length of about 3000 km the RMS difference is ± 0.169 m. This is a significant improvement as compared to the existing solutions EGG1 and EAGG1 (see also *Denker and Torge 1992*). From Figure 5 we can also observe a very long wavelength discrepancy, especially in Scandinavia. This may come from long wavelength gravity model errors, but also from possibly existing small but systematic errors in the 3' × 5' mean gravity data for Scandinavia. A further improvement in the RMS discrepancies is expected by using more high resolution terrain and gravity data in and around the computation area, and by converting the levelling heights to strict normal heights before doing the comparisons.

5. RECENT DEVELOPMENTS IN DATA COLLECTION

Since the XX IUGG General Assembly in Vienna in August 1991, about 30 new gravity data sources were received, validated, and then added to the data base. These new sources include new data supplied by the German state survey agencies as well as new data for Hungary (1.5' × 2.5' grid), Poland (2 × 2 km² grid), Romania (5' × 7.5' grid), Norway, Sweden, Albania and Iceland.

Country	Block Size of Available Digital Terrain Model
Austria	11.25" × 18.75"
France	4.5" × 6.0"
Germany	30" × 50" 1" × 1"
Hungary	1.5' × 2.5'
Italy	7.5" × 10.0"
Netherlands	3' × 5'
Poland	2 km × 2 km
Norway	1 km × 1 km
Romania	5.0' × 7.5'
Spain	200 m × 200 m
Sweden	1 km × 1 km
Switzerland	250 m × 250 m
Global Model ETOPO5	5' × 5'

Table 3: List of digital terrain models stored in the IfE data base (status February 1993).

The amount of terrain data has substantially increased since August 1991. New models were received for France, Germany, Poland, Spain, Norway, Sweden, Romania and Hungary. A list of the presently available digital terrain models (DTMs) is presented in Table 3. As the individual models for each country do not have the same block size and usually refer to a local horizontal datum, it was necessary to convert all models to a standard block size and horizontal datum. We decided to use 7.5" x 7.5" and multiples of this (e.g. 15" x 15", 30" x 30") as a standard block size. For the horizontal datum a geocentric system based on the GRS80 ellipsoid was selected. Because initial tests with the individual DTMs showed clearly that some gross errors are existing in most data sets, it was decided to do a rough check by comparing each elevation with adjacent values. Through this simple procedure a number of errors were found (e.g. 1000 m errors, intermixed map labels etc.). After correction of the gross errors detected as well as the transformation to a geocentric system, the elevations were regridded to the standard block size of 7.5" x 7.5" using a spline interpolation; multiples of this grid size were then derived by simple averaging. All computations were done on a super computer because of the large amounts of data which must be handled (magnitude GBytes). However, as one can also see from Table 3, high resolution DTMs are still lacking for some parts of Europe with a rough topography, but here some promising negotiations are under way.

6. CONCLUSIONS

Since the development of the most recent European quasigeoid solutions EGG1 (*Torge et al. 1982*) and EAGG1 (*Brennecke et al. 1983*) in 1982 and 1983 respectively, significant new gravity field data sets of high quality have become available. This includes new satellite-only models, new satellite altimeter data from GEOSAT, ERS1 and TOPEX, point gravity data, digital terrain models, as well as an increasing number of GPS/levelling control points. Test computations in central and northern Europe promise a significant improvement of the existing European quasigeoid solutions by almost one order of magnitude using these new data sets. The RMS discrepancies versus different GPS/levelling data sets range from ± 0.04 to 0.16 m over distances from a few 100 km to 3000 km. IfE will continue its work as the computing center within the European geoid project. Emphasis will be put on the establishment of a high resolution terrain model covering Europe, as these data play a crucial role for the success of the entire project. The next completely updated quasigeoid calculations for entire Europe is planned to be presented at the IAG General Meeting in Beijing in August 1993.

Acknowledgements

The authors are grateful to Bureau Gravimétrique International, Toulouse, Prof. R.H. Rapp, The Ohio State University, and NASA Goddard Space Flight Center for providing regional and global gravity field data sets. Special thanks are due to the numerous agencies and individuals who released high resolution gravity and terrain data for the geoid project. The computing center thanks Dr. Vermeer for his cooperation within the frame of the IAG Subcommission for the Geoid in Europe. The calculations were carried out at Regionales Rechenzentrum für Niedersachsen (RRZN), Hannover.

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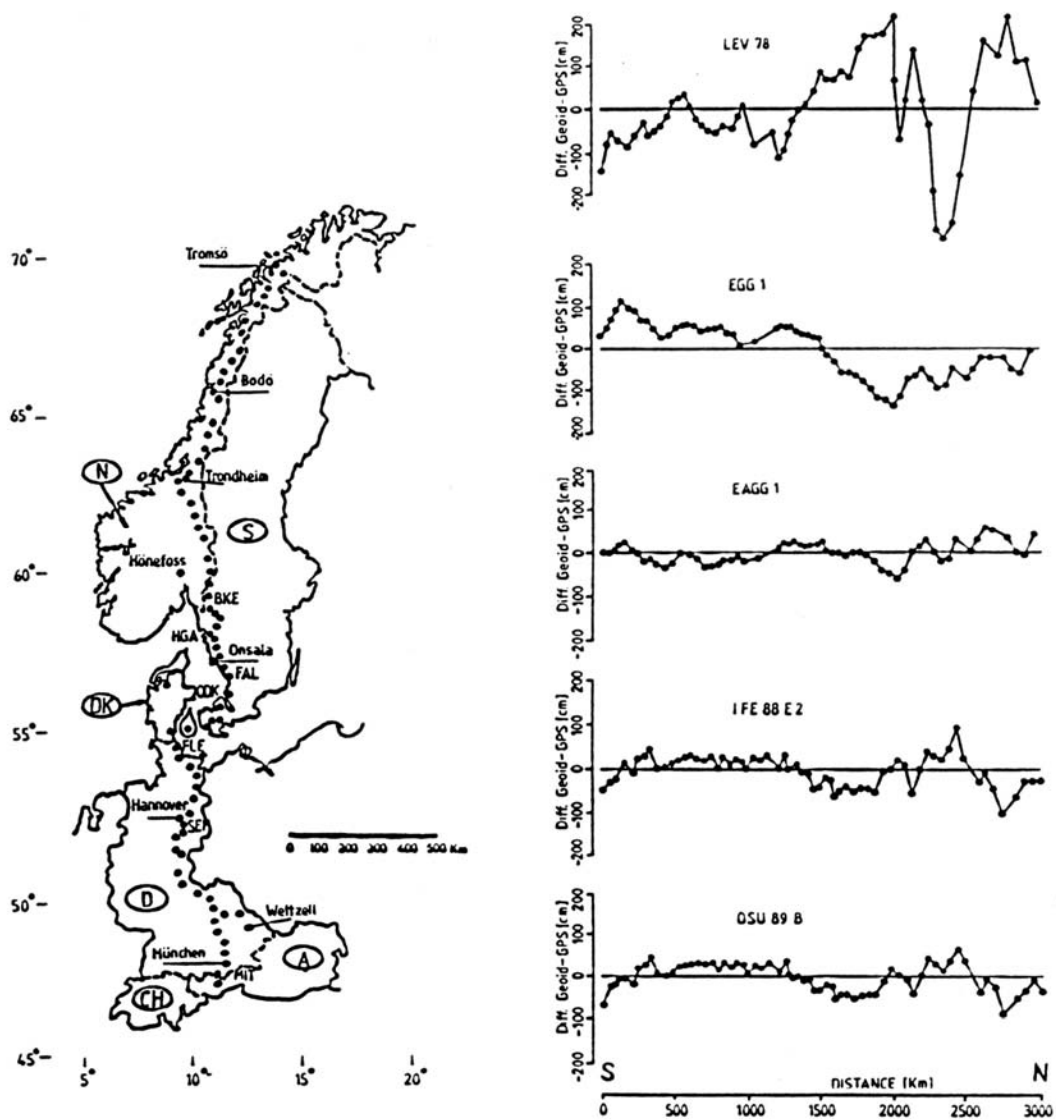


Figure 1: European north-south GPS traverse (left part) and comparison of results from gravity field modelling for Europe with GPS/levelling quasigeoid heights along this traverse (right part).

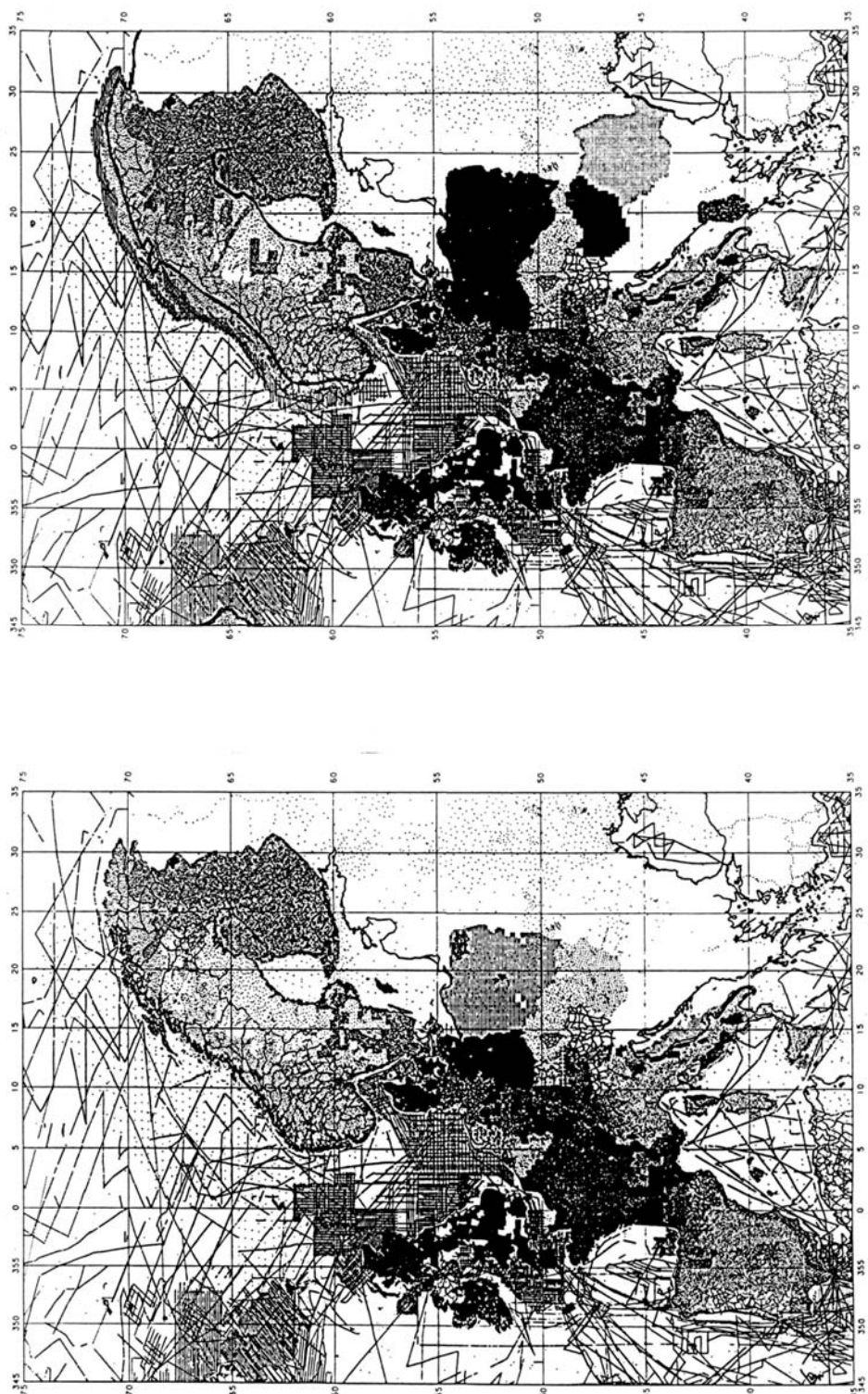


Figure 2: Locations of point gravity data stored in the IFE data base (status of August 1991 shown in the left part; status of February 1993 shown in the right part).

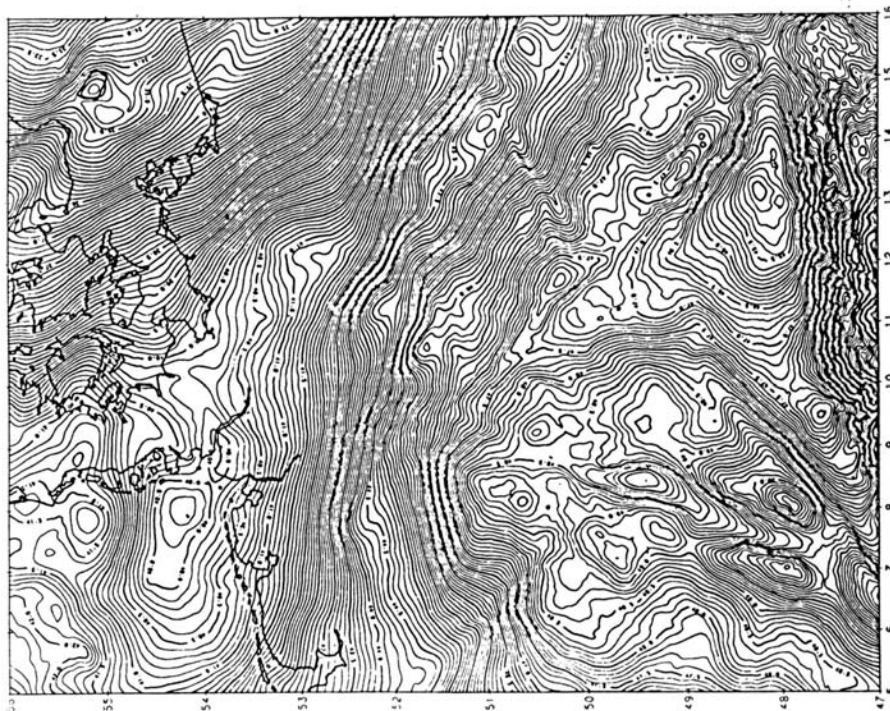


Figure 3: 1991 Quasigeoid solution for Central Europe. Contour interval is 0.1 m.

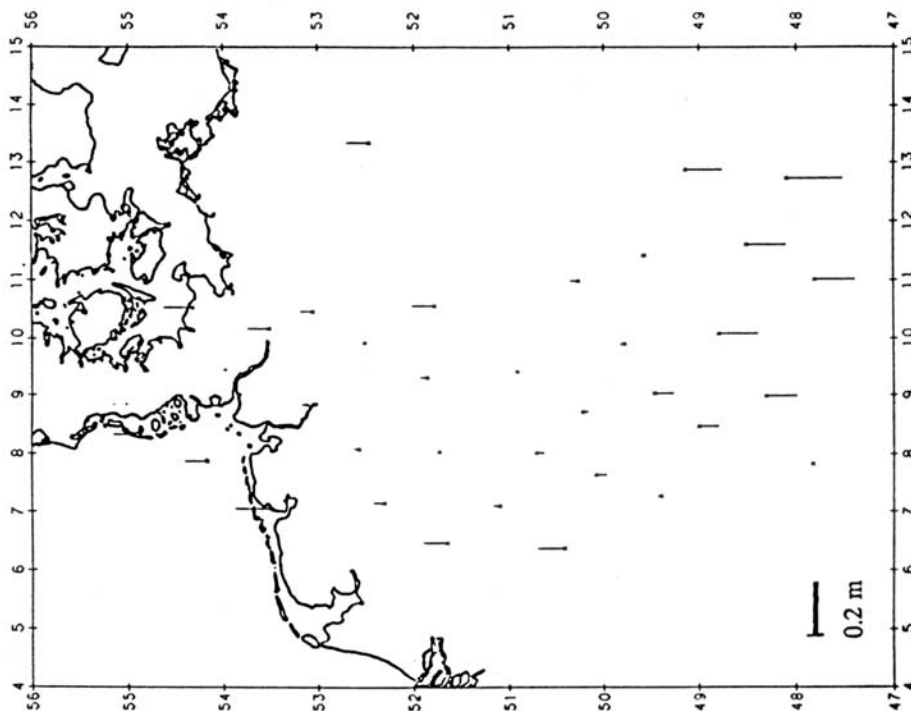


Figure 4: Comparison of the 1991 quasigeoid solution for central and northern Europe with GPS/levelling results from the DÖNAV campaign (constant bias subtracted).

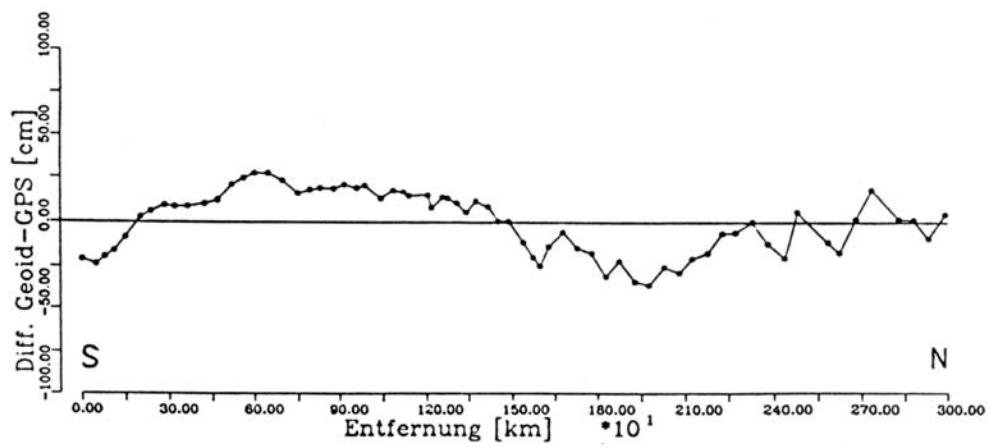


Figure 5: Comparison of the 1991 quasigeoid solution for central and northern Europe with GPS/levelling results from the European GPS traverse (constant bias subtracted).



Validation of the global digital height model *ETOPO5U* referring to the Italian height model (*Italian DEM*)

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Abstract

The existing set of high resolution local digital terrain models gives the possibility to combine this detailed information available with a global $5' \times 5'$ model to get a proper reference topography in the Mediterranean area. Within the merging process the global model is validated referred to the local one in the subarea common to both. In particular the validation and merging steps are presented in the case of one local data set.

1 Introduction

In this paper a comparison between two different $5' \times 5'$ digital height models will be described. The one is ETOPO5U confined to the area of Italy, the other one a $5' \times 5'$ digital elevation model derived from a high resolution elevation model for Italy.

The validation of the ETOPO5U values is done by computing various statistics of the height differences of the two models. They comprise a direct height comparison and an investigation of the height differences of the whole data set, separated into positive and negative heights, and finally subdividing the area into 1° blocks, a more detailed analysis is carried out. Performing longitudinal and latitudinal shifts of the ETOPO5U grid a

best agreement of the two models is achieved and the presence of a 5' longitude shift is detected.

Several tables and figures are collected in the third section showing histograms of height distributions and height differences over the total data set as well as two local 1° blocks. A discussion of the results of the block comparison is summarized afterwards and final conclusions sum up the presented topic.

2 Data Description

The two data sources contain mean height values referring to quadratic elements of 5' × 5' size (in the following these elements are called 'cells' or 'pixels').

Rectangular subarea for data comparison:

$$\begin{array}{c} 36^\circ \leq \varphi \leq 48^\circ \\ 5^\circ \leq \lambda \leq 19^\circ \end{array}$$

Total number of pixels $144 \times 168 = 24192$

2.1 Italian DEM

The original data source extracted from the ITALIAN DEM and provided by the Politecnico di Milano included a total of 9 ITDEM files containing blocks of 30' × 20' with grid steps of 7.5'' and 10''.

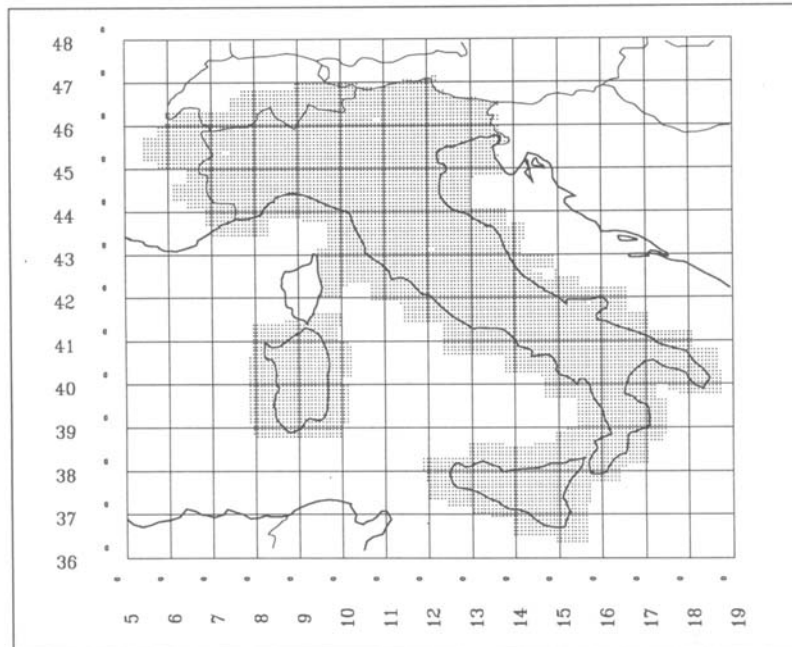


Figure 1: Data coverage of the computed 5' × 5' Italian DEM

The transformation from the local Italian DEM grid to ellipsoidal GRS80 coordinates and the generation of $5' \times 5'$ mean height values by computing weighted means of the original data set has been performed by H. Scharler. If more than 50% of the original values were unknown no new grid values have been computed (these unknown or not surveyed elements have been coded ... 9999). The resulting $5' \times 5'$ grid contains a total of 8644 cells and their coverage can be seen in Fig. 1.

2.2 ETOPO5U model

ETOPO5U as used for the GEOMED project is a $5' \times 5'$ digital elevation model covering the Mediterranean area in the geographical window

$$\begin{array}{rcl} 26^{\circ} 5' & \leq & \varphi \leq 49^{\circ} \\ -11^{\circ} & \leq & \lambda \leq 40^{\circ} 55' \end{array}$$

The coordinate latitude, longitude of the representative point refer, as in TUG87, to the left upper corner of the grid element. The mean height values are stored parallelwise with decreasing latitude (e.g. $49^{\circ} 00'$ first belt, $48^{\circ} 55'$ second belt, etc.) within each latitude belt the longitude is increasing from -11° to $40^{\circ} 55'$ (e.g. $-11^{\circ} 00'$ first block, $-10^{\circ} 55'$ second block, etc.).

This makes a total of 172224 cells of mean height values over the whole area. No flags for unknown or excluded values had been used.

A first study of the ETOPO5U model in the rectangular subarea referenced by the local Italian model showed the presence of some $1^{\circ} \times 1^{\circ}$ blocks near coastal zones, which contain obviously inaccurate positive height values. see also Fig. 10 as an example). The bad blocks detected are depicted in Fig. 2.

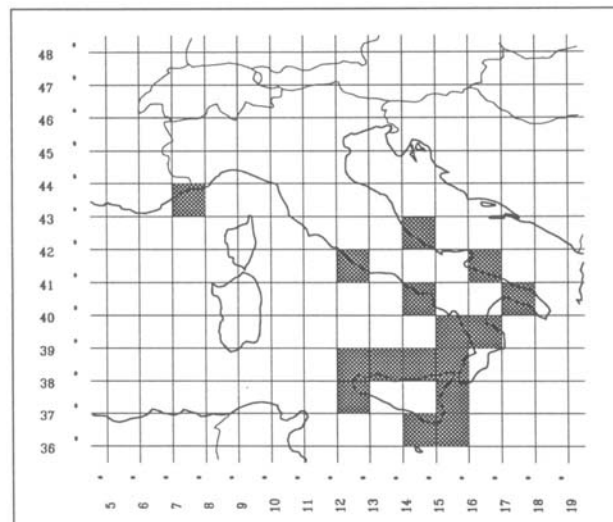


Figure 2: Bad $1^{\circ} \times 1^{\circ}$ blocks in the ETOPO5U model

3 Validation

3.1 Height statistics

The comparison of the two height models has been performed at several stages, using the coverage of the Italian data set as location reference.

At first the distributions of the heights of the two models have been compared. For that purpose the heights have been classified with the resulting histograms shown in Figure 3 & 4. The positive heights differ significantly at both ends of the height spectrum - especially smaller heights are dominating so one can expect the ETOPO5U model to be more flat. The agreement in the mid range is satisfactory with discrepancies beyond 3000 meters – again the ETOPO5U values are dominating. The negative heights differ over the whole spectrum, once again showing a large peak near zero heights.

Description of histograms:

The height values and the computed height differences have been classified for graphical representation purposes. The percentage of elements per class has been plotted, whereas the absolute number of elements has been added as data tables.

The intervals according to the class description are (for some examples of height differences) :

class description	{	<	$(-\infty, -3000)$	}	corresponding intervals (all values in meters [m])
		-3000	$[-3000, -2500)$		
		-2500			
		\vdots			
		-100	$[-100, -50)$		
		-50	$[-50, 0)$		
		50	$[0, 50]$		
		100	$(50, 100]$		
		\vdots			
		2500			
		3000	$(2500, 3000]$		
		>	$(3000, \infty)$		

Height distribution of the Italian DEM and ETOPO5U of the total area

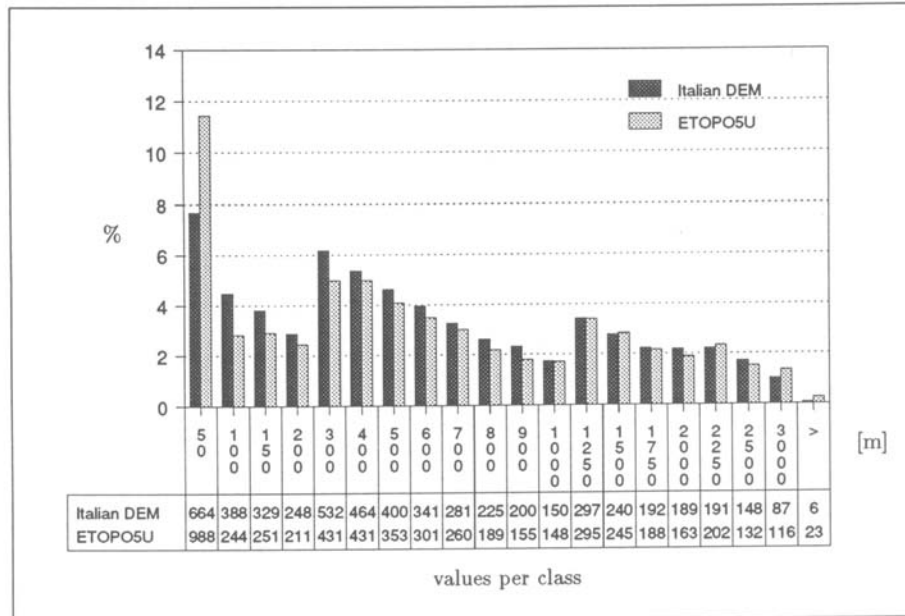


Figure 3: Positive topography

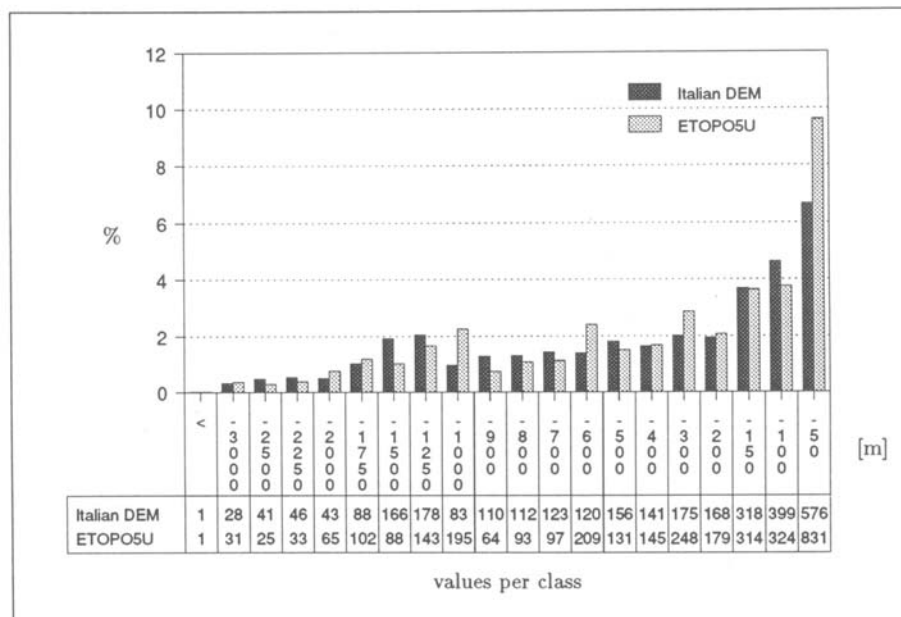


Figure 4: Negative topography

3.2 Statistics of height differences

The next step computes the distribution of height differences of the two models in the total subarea. In the sequel a distinction between positive and negative topography is made throughout this paper.

The comparison process comprises several stages. The distribution of height discrepancies over the total area is computed. Then a reduced data set (disregarding obviously bad 1° blocks in the ETOPO5U model) is used for further investigations and finally the total area is subdivided into nonoverlapping $1^\circ \times 1^\circ$ blocks and a statistics of local differences is carried out.

$$\text{Height difference : } \Delta h_i = h_i^{\text{ItalianDEM}} - h_i^{\text{ETOPO5U}}$$

The following statistical parameters have been evaluated:

- (i) Arithmetic mean : $\text{MEAN}(\Delta h) = \overline{\Delta h} = \frac{1}{n} \sum_{i=1}^n \Delta h_i$
- (ii) Root Mean Square : $\text{RMS}(\Delta h) = \sqrt{\frac{1}{n} \sum_{i=1}^n (\Delta h_i - \overline{\Delta h})^2}$
- (iii) Maximum difference : $\text{MAX}(\Delta h) = \text{sgn}(\Delta h_i) \max_i |\Delta h_i|$, $i = 1 \dots n$

with $n \dots$ number of cells.

Performing longitudinal and latitudinal shifts of the ETOPO5U grid in order to get a best fit between the two height models, the obvious presence of a **5' longitudinal shift** of the ETOPO5U grid to the west had been detected.

The results comprising the new height differences after applying the shifting process have been included in the figures and tables for a direct comparison.

The next pages contain the following information:

- Fig. 5 & 6 Height differences over the total area comparing the discrepancies before and after the grid shift
- Fig. 7 & 8 Height differences after eliminating bad blocks
- Fig. 9 A $1^\circ \times 1^\circ$ block with a proper fit after a 5' longitudinal grid translation of the ETOPO5U model to the east
- Fig. 10 An example of a bad ETOPO5U block

Description of the total area differences (Fig. 5 & 6):

The percentage of the original height differences in the range from -50 to 50 meters increases from 35% to 60% after the grid shift, but there still remain large discrepancies at the upper end of the spectrum.

The area description and the evaluated statistical parameters are collected in Table 1. The RMS of the positive topography reduces significantly – on the contrary the RMS of the negative topography gets even worse (note that a global grid shift had been applied to get a best fit for positive heights).

Description of the total area differences without bad blocks (Fig. 7&8):

Comparing the reduced data set (16 blocks containing 600 positive heights and 1115 negative values had been eliminated after a detailed block statistics) with the original one, one detects that the number of large positive differences reduces drastically. Since the eliminated blocks are located near coastal zones this effect is due to inaccurate flat height information in the ETOPO5U model (see also Fig. 10 as a bad block example). The RMS reduces to nearly 100 meters for the remaining 4972 cells of the positive topography.

The bathymetric heights remain of the same quality as of the total data set – still no significant improvement visible.

Distribution of height differences between the Italian DEM and ETOPO5U

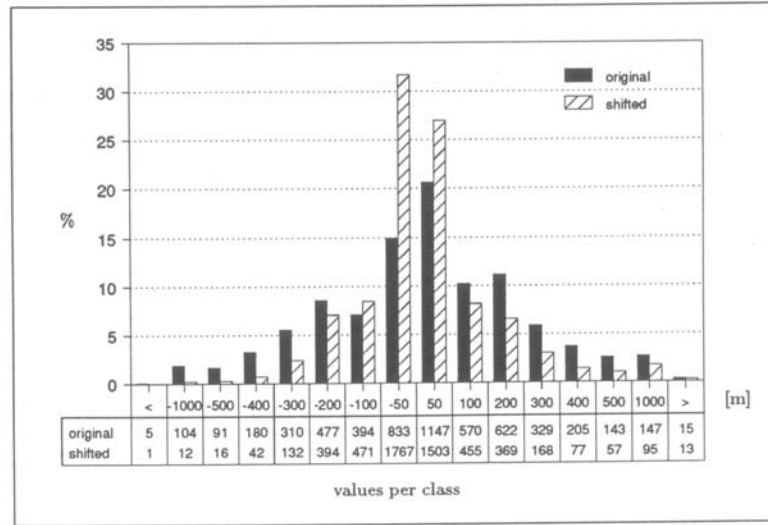


Figure 5: Height differences of the positive topography

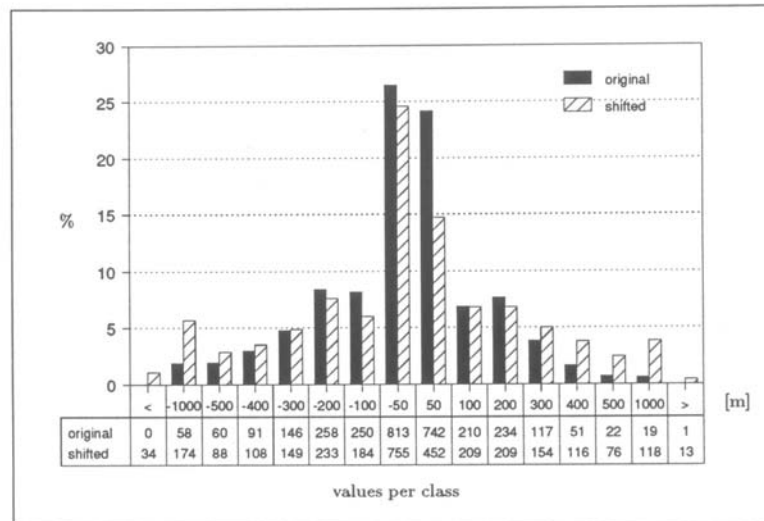


Figure 6: Height differences of the bathymetry

Topography	$36^{\circ} \leq \varphi \leq 48^{\circ}$ $5^{\circ} \leq \lambda \leq 19^{\circ}$	points	MEAN(Δh)	RMS(Δh)	MAX(Δh)
positive	original	5572	22.0	235.0	1699.0
	shifted		19.0	160.0	1632.0
negative	original	3072	-23.0	178.0	1464.0
	shifted		-25.0	318.0	-1986.0

Table 1: Total area height differences of the two models

Distribution of height differences between the Italian DEM and ETOPO5U after elimination of $1^\circ \times 1^\circ$ bad blocks of the ETOPO5U - model

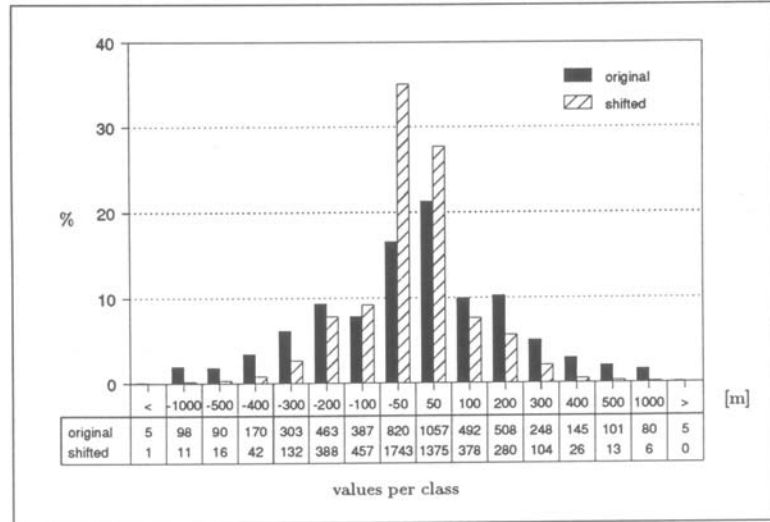


Figure 7: Histogram of height differences of the positive topography

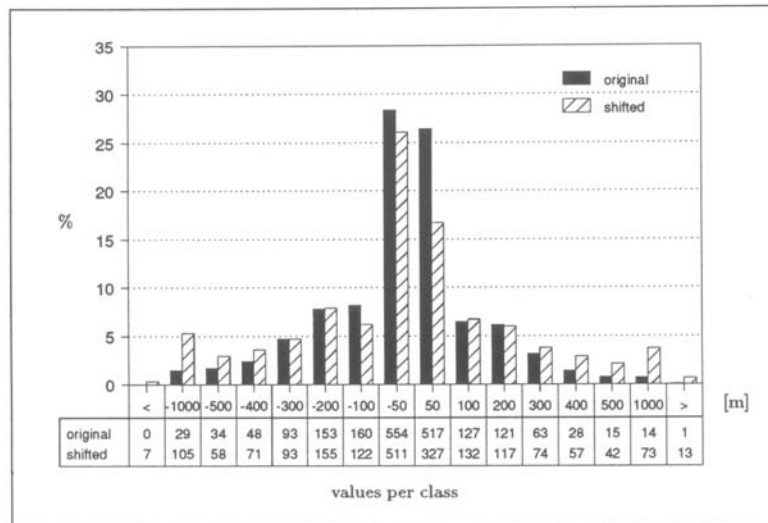


Figure 8: Histogram of height differences of the negative topography

Topography	$36^\circ \leq \varphi \leq 48^\circ$ $5^\circ \leq \lambda \leq 19^\circ$	points	MEAN(Δh)	RMS(Δh)	MAX(Δh)
positive	original	4972	-2.0	214.0	1699.0
	shifted		-9.0	105.0	-1208.0
negative	original	1957	-20.0	167.0	1464.0
	shifted		-19.0	298.0	1762.0

Table 2: Height differences of the reduced data set (bad blocks eliminated)

Height differences of the Italian DEM and ETOPO5U in two local $1^\circ \times 1^\circ$ blocks

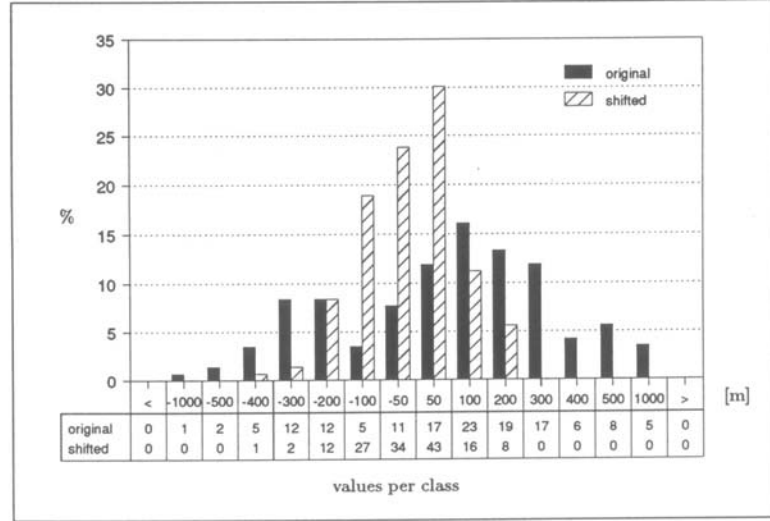


Figure 9: Differences of positive heights with a good fit

Topography	$42^\circ \leq \varphi \leq 43^\circ$ $13^\circ \leq \lambda \leq 14^\circ$	points	MEAN(Δh)	RMS(Δh)	MAX(Δh)
positive	original	143	65.0	248.0	969.0
	shifted		-14.0	79.0	-364.0
negative	original	1	-9.0	9.0	-9.0
	shifted		-10.0	10.0	-10.0

Table 3: Height differences of a selected $1^\circ \times 1^\circ$ block with reliable fit after a 5' longitudinal shift

Description (Fig. 9):

The block has been taken from central Italy as an example for a locally good correlation of the two models. The height discrepancies of the original data sets (only positive topography shown) can be seen at once having a large RMS of 248 meters. Applying the grid transformation the RMS declines to 79 meters and the distribution is changing accordingly, eliminating nearly all large differences.

Description (Fig. 10):

As mentioned before there exist some $1^\circ \times 1^\circ$ blocks containing very bad height information especially located near coastal zones (e.g. the Sicily border or the territory around San Remo which have extremely unrealistic height values in the ETOPO5U-model). To get an impression of the unreliable data an example from central Italy has been chosen.

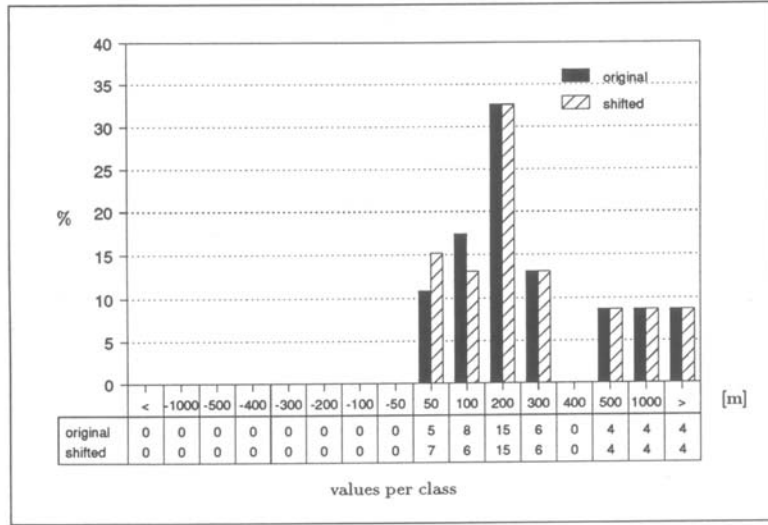


Figure 10: An example of a bad block in the ETOPO5U model

Topography	$42^{\circ} \leq \varphi \leq 43^{\circ}$ $14^{\circ} \leq \lambda \leq 15^{\circ}$	points	MEAN(Δh)	RMS(Δh)	MAX(Δh)
positive	original	46	328.0	518.0	1545.0
	shifted		324.0	516.0	1511.0
negative	original	82	1.0	13.0	33.0
	shifted		-16.0	21.0	-47.0

Table 4: Statistics of height differences over a 1° bad ETOPO5U block

3.3 Discussion

The positive topography agrees very well in central Italy ($1^{\circ} \times 1^{\circ}$ blocks nonoverlapping with coastal zones having an RMS of approx. 50 to 100 meters after performing a longitudinal shift of the ETOPO5U grid one pixel eastwards). The height discrepancies around the northern Italian border, due to its rough topography, are about 100 to 150 meters RMS. The southern part of Italy has some deviations in the range of 50 - 100 meters RMS compared to the ETOPO5U values. The Adriatic Sea fits quite well, as expected. The island of Sicily has some 50 - 100 RMS (without north and east border 1° blocks) and finally Sardinia about 50 - 150 meters RMS.

The situation is more troublesome when the comparison is restricted to the bathymetry. The obvious longitudinal shift present in the land area is changing to a rather indifferent situation here. The optimal block shift values found do not show some tendency and

REGION	TOPOGRAPHY			
	positive		negative	
	original	shifted	original	shifted
Italy North	150 - 300	50 - 200	100 - 350	200 - 500
Italy Central	100 - 250	50 - 100	20 - 200	50 - 250
Italy South	100 - 250	50 - 100	150 - 300	150 - 400
Adriatic Sea	20 - 100	0 - 50	0 - 50	0 - 100
Sicily	100 - 300	50 - 100	50 - 300	100 - 600
Sardinia	100 - 300	50 - 200	50 - 500	150 - 600

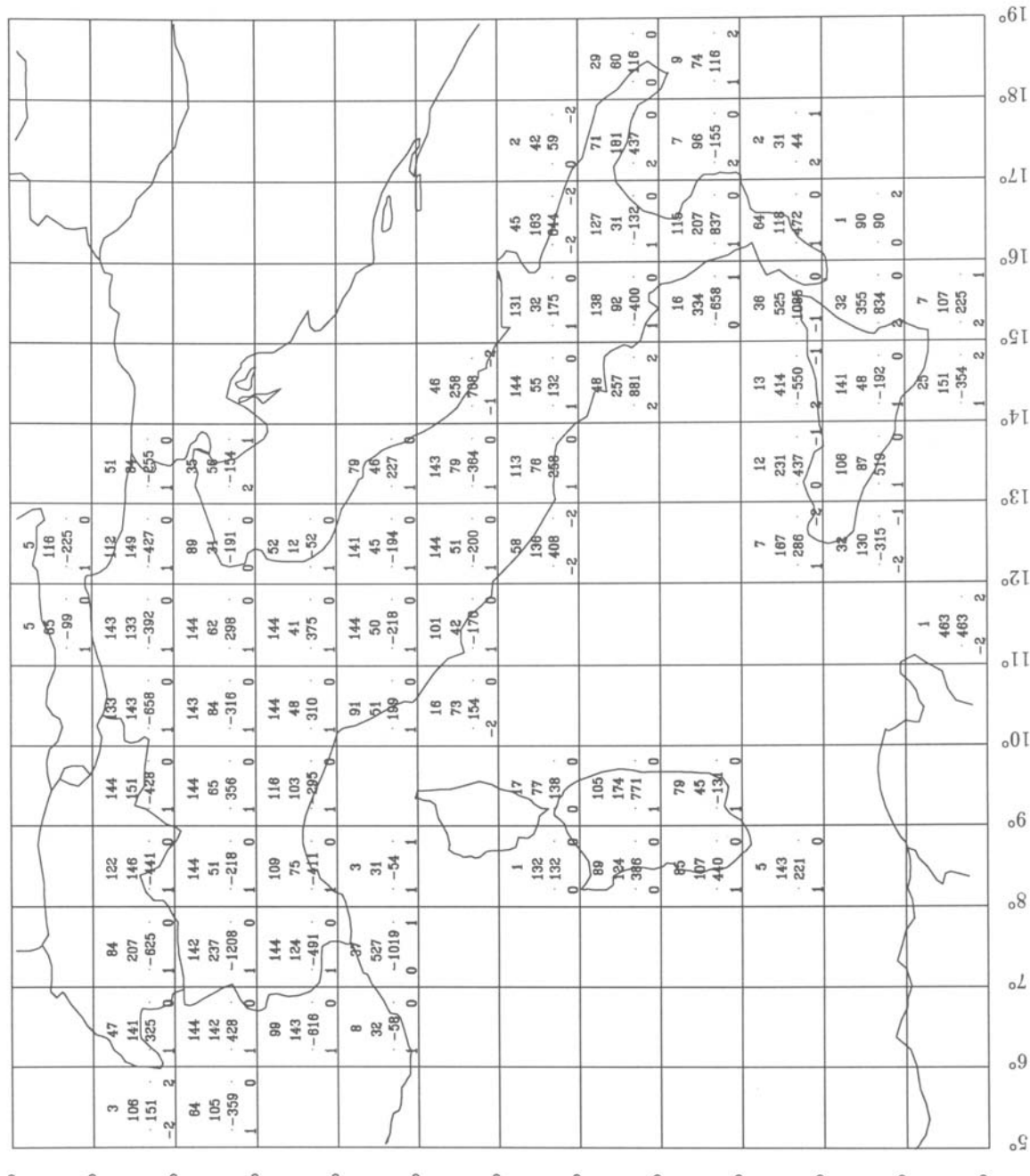
Table 5: Height differences ($\text{RMS}(\Delta h)$ [m]) according to $1^\circ \times 1^\circ$ blocks

they don't agree at all with the one pixel land shift theory. Due to the fact that the bathymetric height information is usually less accurate than overland height values one can expect the large discrepancies to result from inaccuracies rather than shift deviations present. So we are still convinced of the $5'$ longitudinal shift to the east, taking the even poorer fit of the bathymetry into account.

The fit of the negative topography (bathymetry) is acceptable in the Adriatic Sea (10 - 100 meters RMS), and near the coastline of central Italy (50 - 250 RMS). In the open sea and around Sicily we find between 100 and 600 meters RMS, with up to 1000 meters RMS in one local $1^\circ \times 1^\circ$ block at the west coast of Sardinia.

4 Conclusions

The global $5' \times 5'$ digital height model ETOPO5U has been compared with the data set of the local Italian DEM in the area around Italy. A detailed analysis of the height discrepancies has been performed and computing longitude and latitude grid shifts a $5'$ longitudinal shift of the ETOPO5U model eastwards has been applied for best agreement, at least as far as positive heights are concerned. The resulting fit of the two models is of varying quality – generally one can state that there do occur rather large discrepancies caused e.g. by $1^\circ \times 1^\circ$ bad blocks in the ETOPO5U model or by rough topography. Bathymetric height information is even more troublesome, since it does not at all confirm the $5'$ land shift theory.



ITALIAN - DTM
↑
ETOPO5U - DTM

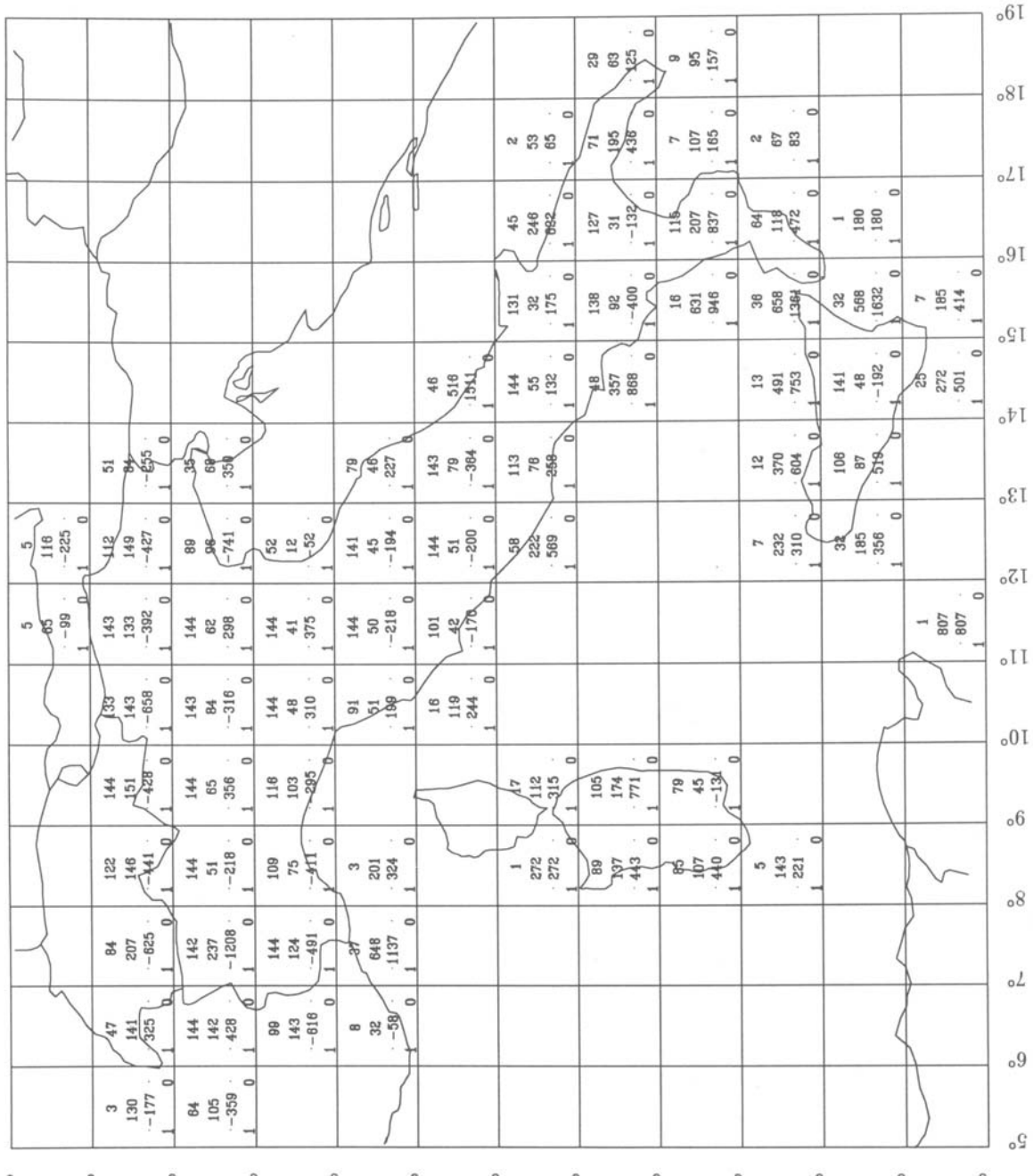
positive
topography
(shifted ETOPO5U-model)
(local 1° blocks)

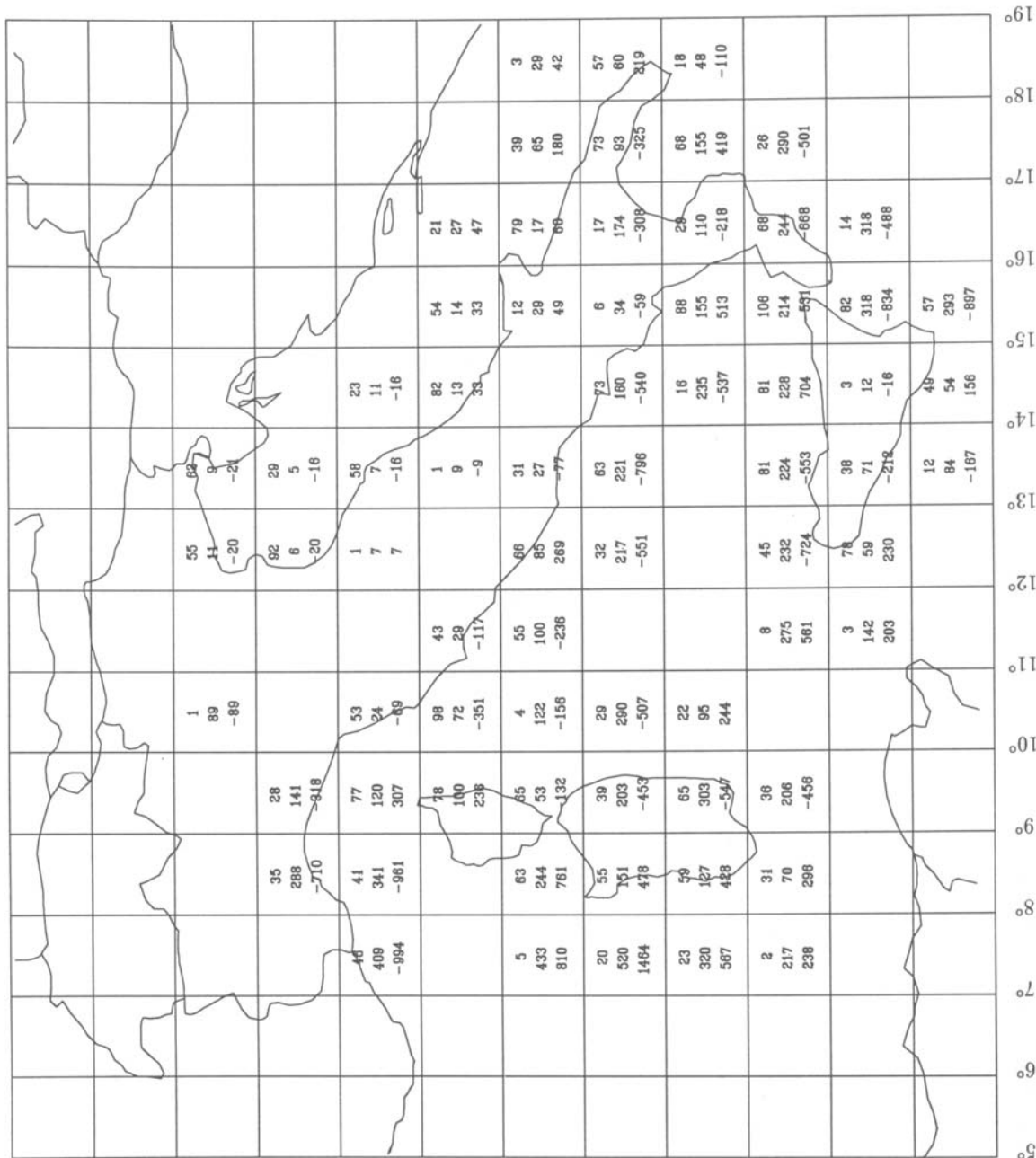
cells
RMS(Δh)
MAX(Δh)
↔ shift values ↑

1° × 1°

$$\Delta h = h_{Italian} - h_{ETOPO5U} [m]$$

+ - + - +
shift values [5']





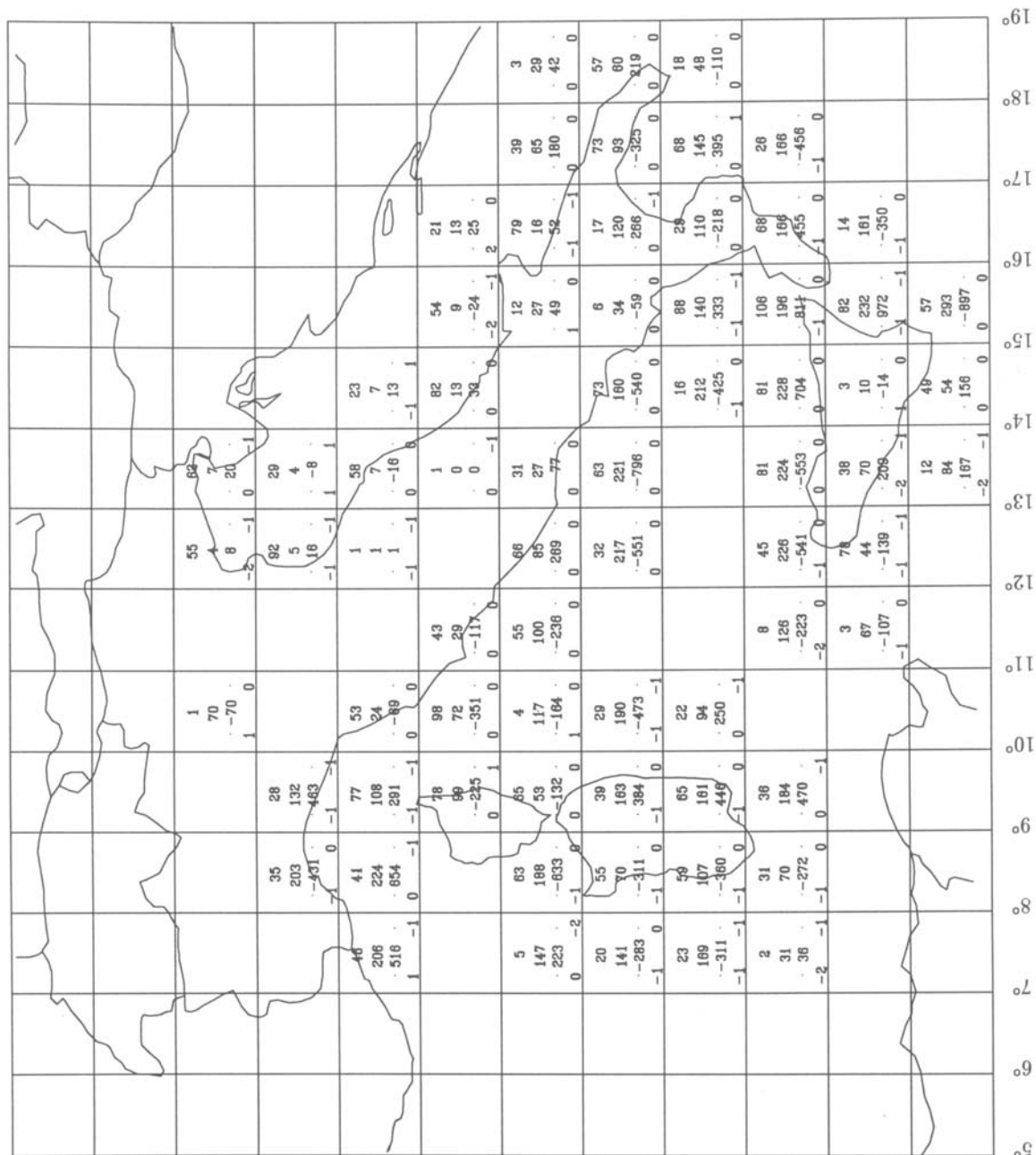
ITALIAN - DTM
↑
ETOPO5U - DTM

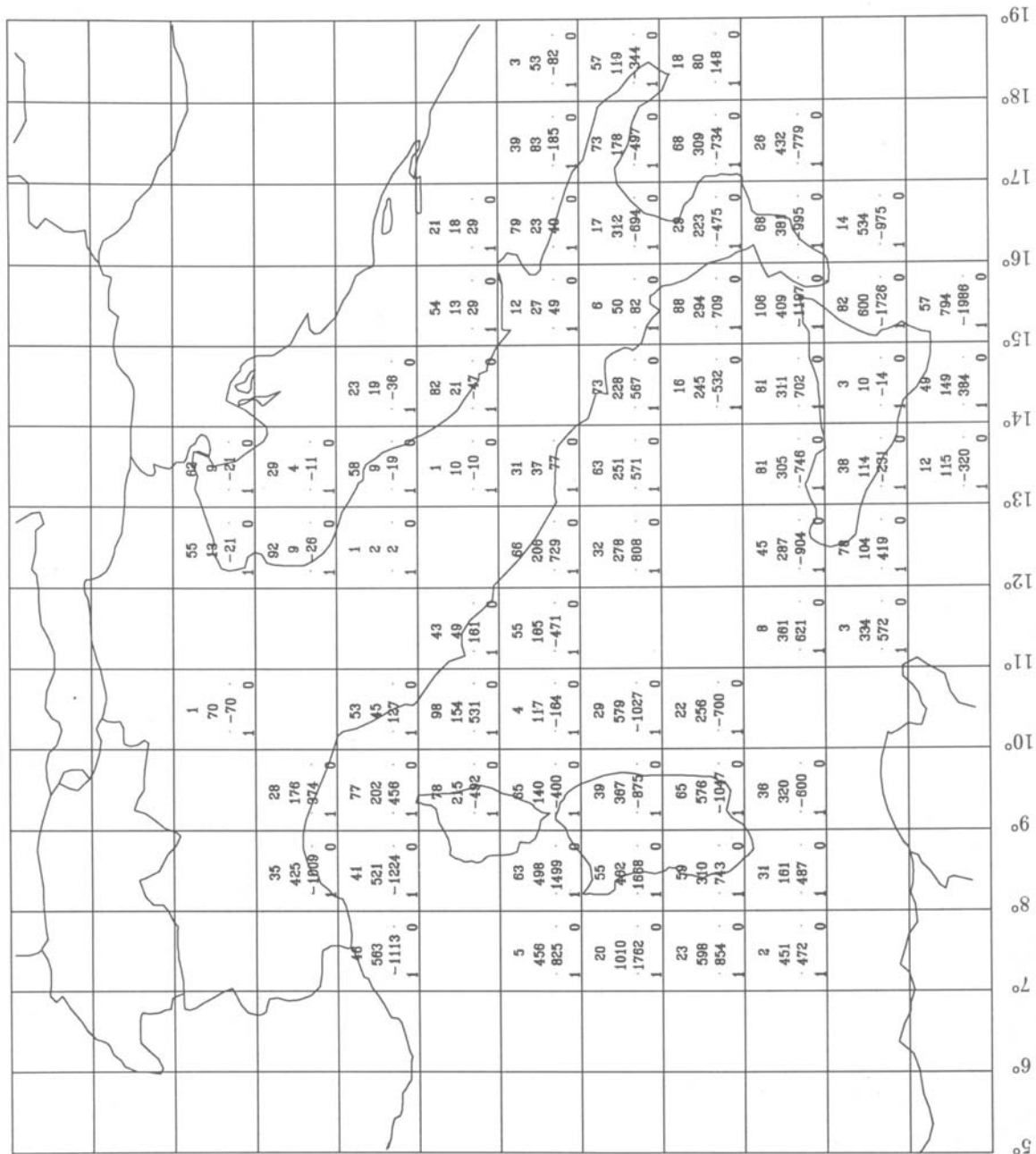
negative
topography

cells
RMS(Δh)
MAX(Δh)

$1^\circ \times 1^\circ$

$\Delta h = h_{Italian} - h_{Etopo5U} \quad [m]$





ITALIAN - DTM
↑
ETOPO5U - DTM

negative
topography
(shifted ETOPO5U-model)
(total area)

cells
RMS(Δh)
MAX(Δh)
↔ shift values ↑

$1^\circ \times 1^\circ$

$\Delta h = h_{Italian} - h_{Etopo5U} \quad [m]$
+ ↔ - shift values [5']

A strategy for geoid repairing

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1. Introduction

The presence of undetected outliers in database introduces errors in the derived estimates. If, in an "a posteriori" processing, a small number of outliers is found, the problem of correcting properly the estimates arises. Particularly, in geodetic applications, gravity outliers can affect geoid estimates. In this paper, we try to define a procedure to repair the distortions in geoid estimates caused by a small number of outliers in the gravity database without recomputing the solution globally.

Numerical tests have been carried out to analyze the practical application of the devised method.

2. Gravity outliers and geoid

Suppose you have computed a residual gravimetric geoid via collocation (B. Benciolini et al., 1991) and that some outliers were not removed from gravity data.

If you apply "a posteriori" a more robust procedure to detect the outliers, able to find them, you must face the problem of correcting the previous estimates.

The simpler procedure, at least from a conceptual point of view, is to recompute the solution with the cleaned data set.

However, this operation is a waste of time especially if you computed the geoid on a large area or with a large gravity database. Another possible approach to the solution of the problem is to quantify the effect caused by the outliers and to remove it from the estimated geoid. As we shall prove in the following this

can be done on a local base, i.e. by using only a small portion of the data, namely those surrounding the outliers.

With no loss of generality, we can assume that only one outlier is still present in the database used in the computation.

In this case we can split the vector containing the data in two parts

$$\underline{\Delta g} = \begin{bmatrix} \Delta g(P_1) \\ \vdots \\ \Delta g(P_1) \\ \vdots \\ \Delta g(P_n) \end{bmatrix} = \begin{bmatrix} \tilde{\Delta g}(P_1) \\ \vdots \\ \tilde{\Delta g}(P_1) \\ \vdots \\ \tilde{\Delta g}(P_n) \end{bmatrix} + \begin{bmatrix} 0 \\ \vdots \\ \varepsilon_1 \\ \vdots \\ 0 \end{bmatrix} = \underline{\tilde{\Delta g}} + \underline{\varepsilon} \quad (1)$$

where ε_1 indicates the presence of the outlier at point P_1 , $\tilde{\Delta g}(P_1)$ is the "correct" gravity value at the same point and

$$\tilde{\Delta g}(P_k) = \Delta g(P_k) \quad \forall k = 1, \dots, n \quad k \neq 1$$

If we substitute (1) in the collocation solution (Moritz, 1980) for the geoid we have

$$\begin{aligned} N(P) &= \sum_{i,j=1}^n C_{N\Delta g}(P, P_i) \left[C_{\Delta g \Delta g}(P_k, P_t) + \sigma_v^2 \delta_{kt} \right]_{ij}^{-1} \Delta g_j = \\ &= \sum_{i,j=1}^n C_{N\Delta g}(P, P_i) \left[C_{\Delta g \Delta g}(P_k, P_t) + \sigma_v^2 \delta_{kt} \right]_{ij}^{-1} \tilde{\Delta g}_j + \\ &+ \sum_{i,j=1}^n C_{N\Delta g}(P, P_i) \left[C_{\Delta g \Delta g}(P_k, P_t) + \sigma_v^2 \delta_{kt} \right]_{ij}^{-1} \varepsilon_j = \\ &= \hat{N}(P) + \sum_{i=1}^n C_{N\Delta g}(P, P_i) \left[C_{\Delta g \Delta g}(P_k, P_t) + \sigma_v^2 \delta_{kt} \right]_{i1}^{-1} \varepsilon_1 = \\ &= \hat{N}(P) + \delta N(P) \quad k, t = 1, \dots, n \end{aligned} \quad (2)$$

The disturbing term due to the outlier depends on the i -th column of the inverse of the matrix

$$D = \left[C_{\Delta g \Delta g} + \sigma_v^2 I \right] \quad (3)$$

and its computation it still time consuming. A first initial approximation of $\delta N(P)$ could be the one obtained using only one data, i.e. ε_1 in P_1 . The resulting formula is:

$$\hat{\delta N}(P) = \frac{C_{N\Delta g}(P, P_1)}{C_{\Delta g \Delta g}^2(0) + \sigma_n^2} \varepsilon_1 \quad (4)$$

which is very simple and straightforward. Unfortunately, we shall see in a numerical test that equation (4) gives a poor estimate of $\hat{\delta N}(P)$ and cannot be used.

Another way of solving the problem is to compute the correction to the solution only using a reduced vector $\tilde{\underline{\varepsilon}}$, that is a vector containing m elements, $m \ll n$.

The reduced vector $\tilde{\underline{\varepsilon}}$ is an approximation of $\underline{\varepsilon}$, contains ε_1 and has $m-1$ zero elements in $m-1$ points surrounding P_1 .

This implies that we can compute an estimate of $\delta N(P)$ in (2) of the type

$$\hat{\delta N}(P) = \sum_{i,j=1}^m C_{N\Delta g}(P, P_i) \left[C_{\Delta g \Delta g}(P_k, P_t) + \sigma_v^2 \delta_{kt} \right]_{i,j}^{-1} \tilde{\underline{\varepsilon}}_j \quad (5)$$

In doing so, we define, in a fast way, a $\delta N(P)$ which is between the insufficient approximation given by equation (4) and the rigorous evaluation of $\delta N(P)$ by (2).

Obviously, to define a proper estimation procedure of $\delta N(P)$ we must select m i.e. decide how far we have to move from P_1 .

A reliable criterion is based on the correlation length L of the autocovariance function $C_{\Delta g \Delta g}$.

The numerical tests we did proved that this quantity plays a determinant role in defining the dimension of the vector $\tilde{\underline{\varepsilon}}$; if a window of size L is set around the outliers in P_1 , we obtain, using (5), $\hat{\delta N}(P)$ which is a very good estimate of $\delta N(P)$ in (2).

Still, a final problem must be solved to define completely this geoid repair procedure, i.e. the way of estimating ε_1 .

Once we have realized that in P_1 there is an outliers, we need an estimate $\hat{\varepsilon}_1$ of ε_1 to compute $\delta\hat{N}(P)$ in (5).

This can be done by using collocation, following a scheme which is homogeneous (in the covariance sense) to the one adopted in (5).

If we estimate $\hat{\Delta g}(P)$ using $\underline{\Delta g}$ in (1) we get

$$\begin{aligned}
\hat{\Delta g}(P) &= \sum_{i,j=1}^n C_{\Delta g \Delta g}(P, P_i) \left[C_{\Delta g \Delta g}(P_k, P_t) + \sigma_v^2 \delta_{kt} \right]_{ij}^{-1} \Delta g_j = \\
&= \sum_{i,j=1}^n C_{\Delta g \Delta g}(P, P_i) \left[C_{\Delta g \Delta g}(P_k, P_t) + \sigma_v^2 \delta_{kt} \right]_{ij}^{-1} \tilde{\Delta g}_j + \\
&+ \sum_{i,j=1}^n C_{\Delta g \Delta g}(P, P_i) \left[C_{\Delta g \Delta g}(P_k, P_t) + \sigma_v^2 \delta_{kt} \right]_{ij}^{-1} \varepsilon_j = \quad (6) \\
&= \hat{\tilde{\Delta g}}(P) + \sum_{i=1}^n C_{\Delta g \Delta g}(P, P_i) \left[C_{\Delta g \Delta g}(P_k, P_t) + \sigma_v^2 \delta_{kt} \right]_{i1}^{-1} \varepsilon_1 =
\end{aligned}$$

$$k, t = 1, \dots, n$$

In $P = P_1$ we have

$$\begin{aligned}
\hat{g}(P_1) &= \hat{\tilde{g}}(P_1) + \\
&+ \sum_{i=1}^n \left(C_{\Delta g \Delta g}(P_1, P_i) + \sigma_v^2 \delta_{i1} \right)_{ij} \left[C_{\Delta g \Delta g}(P_k, P_t) + \sigma_v^2 \delta_{kt} \right]_{i1}^{-1} \varepsilon_1 + \\
&- \sum_{i=1}^n \sigma_v^2 \delta_{i1} \left[C_{\Delta g \Delta g}(P_k, P_t) + \sigma_v^2 \delta_{kt} \right]_{i1}^{-1} \varepsilon_1 = \quad (7) \\
&= \hat{\tilde{g}}(P_1) + \varepsilon_1 - \sigma_v^2 \left[C_{\Delta g \Delta g}(P_k, P_t) + \sigma_v^2 \delta_{kt} \right]_{i1}^{-1} \varepsilon_1
\end{aligned}$$

Very roughly speaking, the element of the inverse matrix is of the order of ⁽¹⁾

(1) This is true since we consider covariance matrix of residual gravity data and hence the covariance matrix is close to a diagonal

$$\frac{1}{C_{\Delta g \Delta g}(P_1, P_1) + \sigma_v^2} \quad (8)$$

This implies that the last term in (7) can be neglected since, in general, $C_{\Delta g \Delta g}(0) \gg \sigma_v^2$. In this hypothesis (7) becomes

$$\hat{\Delta g}(P_1) \cong \hat{\tilde{\Delta g}}(P_1) + \varepsilon_1 \quad (9)$$

As expected the filtered values $\hat{\Delta g}(P_1)$ is an estimate of $\Delta g(P_1)$ and is affected by the presence of the outlier.

However, if we remove from the data set the $\Delta g(P_1)$ value we can obtain a proper estimate of $\hat{\tilde{\Delta g}}(P_1)$ and then, from equation (9), ε_1 . Furthermore, we can restrict the data set for computing $\hat{\tilde{\Delta g}}(P_1)$ to m values surrounding P_1 ; this will save a lot of computing time. The basic idea is to apply the same criterion that we used for defining a proper estimate of $\hat{\delta N}(P)$ in (5).

This ends the geoid repairing procedure since we are now able to estimate ε_1 and compute and remove $\hat{\delta N}(P)$ from $\hat{N}(P)$.

3. Numerical tests

We selected an area in the Western Mediterranean with the following boundaries

$$\begin{aligned} 38^\circ &\leq \phi \leq 41^\circ \\ 0^\circ &\leq \lambda \leq 3^\circ \end{aligned}$$

In this zone we have gridded gravity data with $\Delta\phi = \Delta\lambda = 5'$ (R. Barzaghi et al., 1992), hence for this test we used 1369 gravity values. To analyze the influence of an outlier on geoid estimate we added an error $\varepsilon_{\Delta g} = 35$ mGal to the gravity value in the point $(\phi, \lambda) = (39.5, 1.5)$ which is in the center of the area under investigation.

matrix.

The model covariance function of the gravity data has a correlation length of 15' and is plotted in fig. 1.

We started computing the rigorous δN effect of equation (2), induced by the outlier, in each point of the data grid. This constitutes a reference field of values to be compared with the approximated formulas derived previously; the first problem to solve is in fact to define a proper estimator for $\delta N(P)$.

To this aim we used equation (4) and equation (5) with different values of m ; the mean, the standard deviation and the correlation of the obtained values are summarized in table 1

	δN	$\hat{\delta N}$ (eq. (4))	$\hat{\delta N}$ (eq. (5))		
			$m = 9$	$m = 25$	$m = 49$
ϵ (m)	0.0035	0.3751	-0.0821	0.0108	-0.0014
σ (m)	0.0058	0.2545	0.0489	0.0096	0.0053
$\rho(\delta N/\hat{\delta N})$	--	0.6300	-0.4600	0.8700	0.9600

Table 1

The plots of δN , $\hat{\delta N}$ from equation (4), $\hat{\delta N}$ from equation (5) for the different values of m are in fig 2, fig.3, fig. 4, fig. 5 and fig. 6 respectively.

The results prove that the approximation of $\delta N(P)$ obtained using equation (4) is too poor and cannot be used while the one derived with equation (5) gives a reliable estimate of δN if a proper window of values is selected around the outlier.

In this case the size of this area has a radius of 15' which is exactly the correlation length of the autocovariance of Δg .

Furthermore, we made another test with a high outlier, $\epsilon_{\Delta g} = 180$ mGal, to investigate the relationship between the magnitude of the outlier and the size of the data window.

It comes out that this parameter is not affected significantly from the magnitude of $\epsilon_{\Delta g}$; the optimal window is again the one with $m = 49$.

So a rule of thumb for this geoid repair procedure is to select Δg

data in an area with radius equal to the correlation length L of $C_{\Delta g \Delta g}$ and to estimate the correcting term $\hat{\delta N}(P)$, using equation (5), in an area of radius $2L$, as the tests performed proved also that the effect of the outlier is relevant up to this distance.

Reference

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Benciolini B., Manzino A., Sansò F., Sguerso D. (1991): ITALGEO'90: Progress Report June '90. In "International Association of Geodesy Symposia 106: Determination of the Geoid, present and future". R.H. Rapp and F. Sansò (Eds), Springer-Verlag.

Moritz H. (1980): Advanced Physical Geodesy. H. Wichmann-Verlag, Karlsruhe.

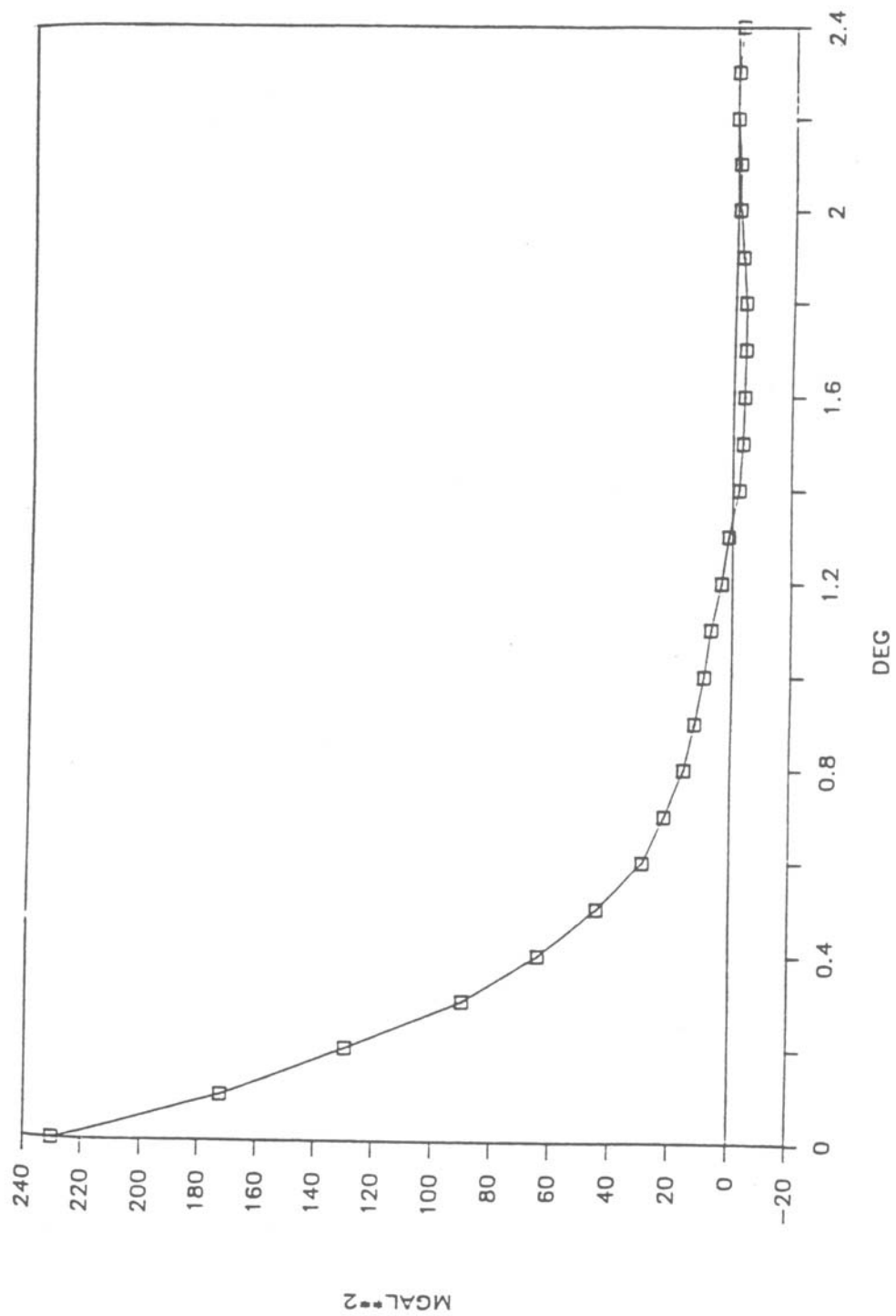


Fig. 1 - Covariance function of residual gravity.

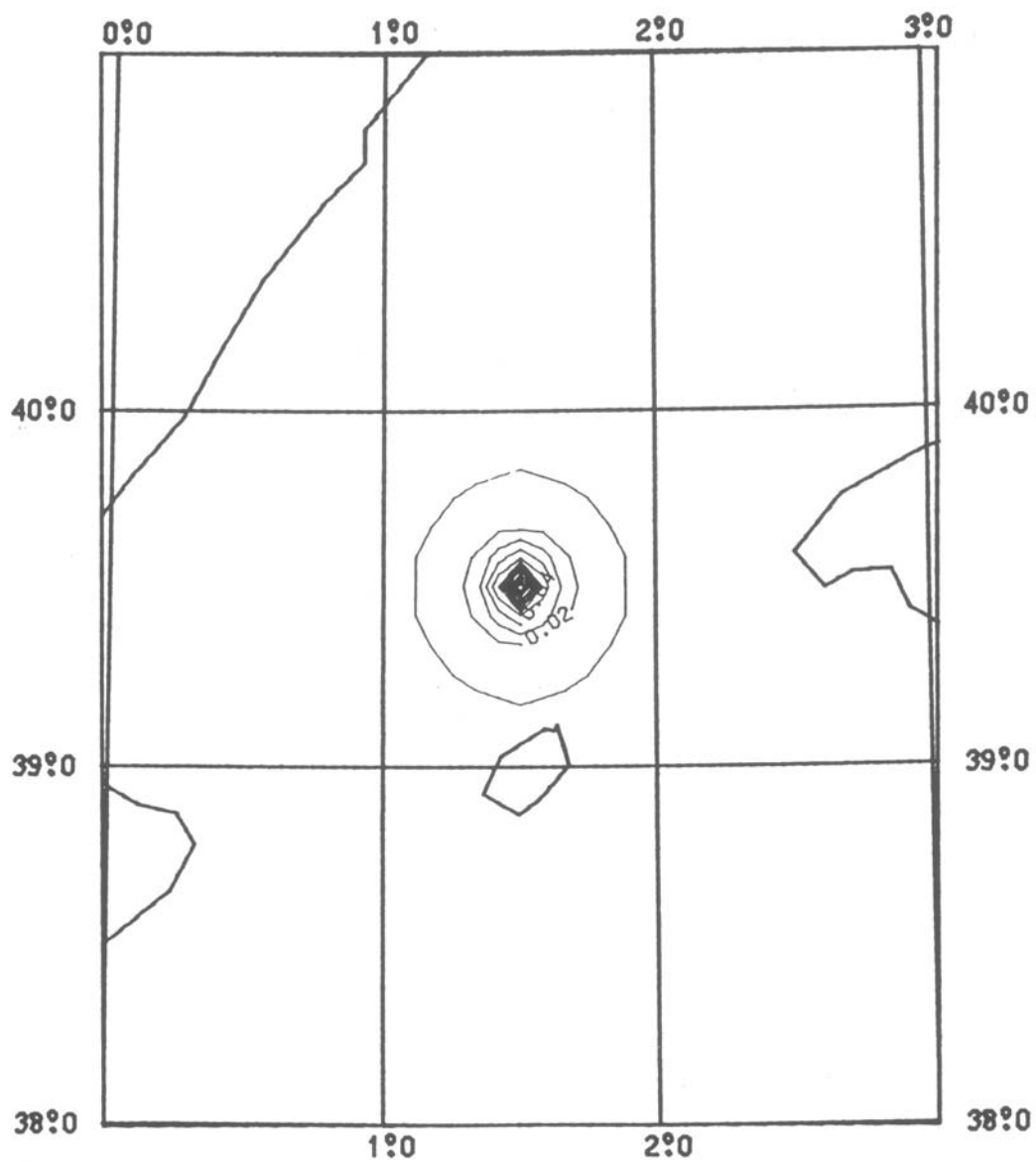


Fig. 2 - Data δN computed using formula (2).

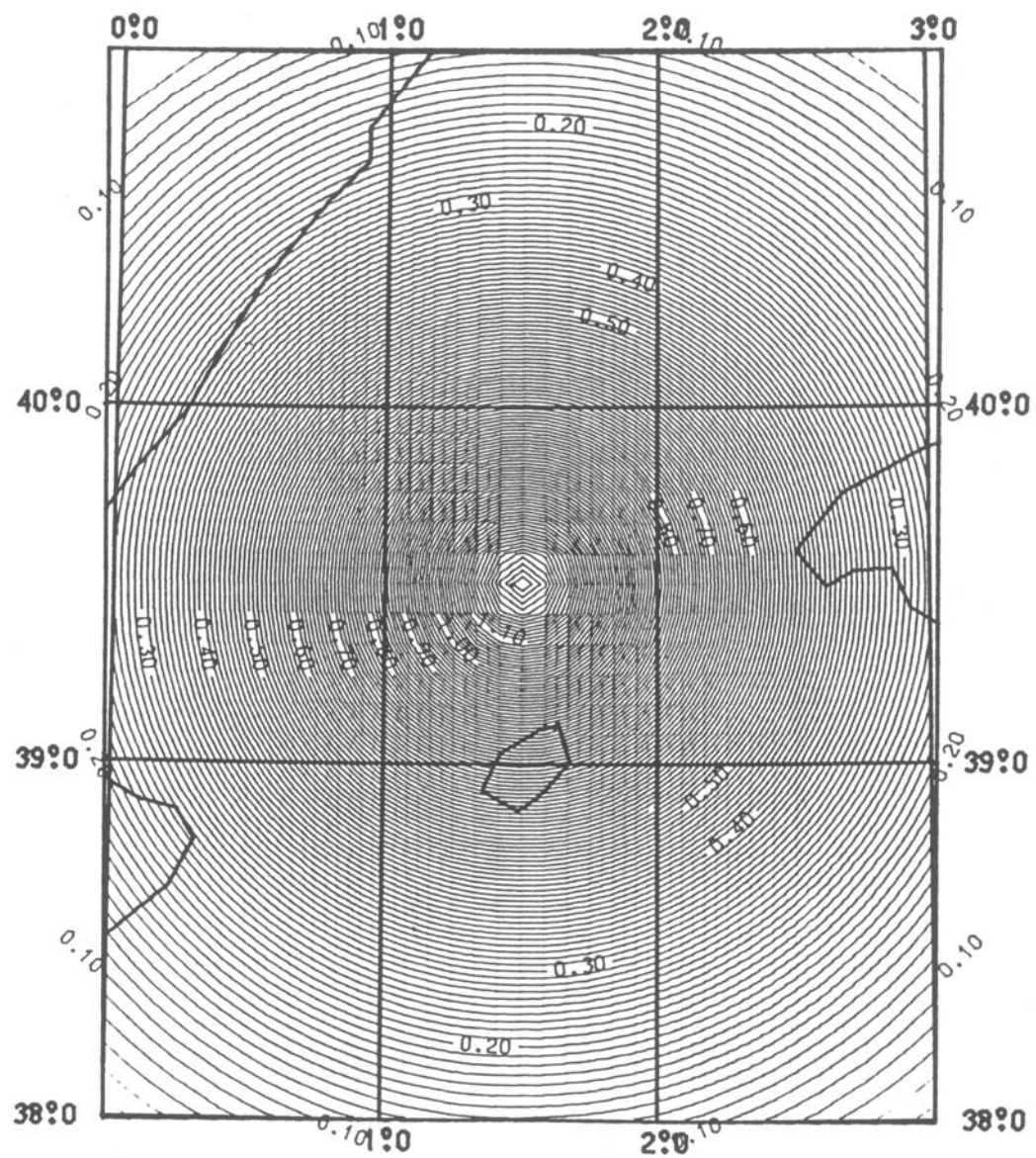


Fig. 3 - $\delta \hat{N}$ computed using formula (4).

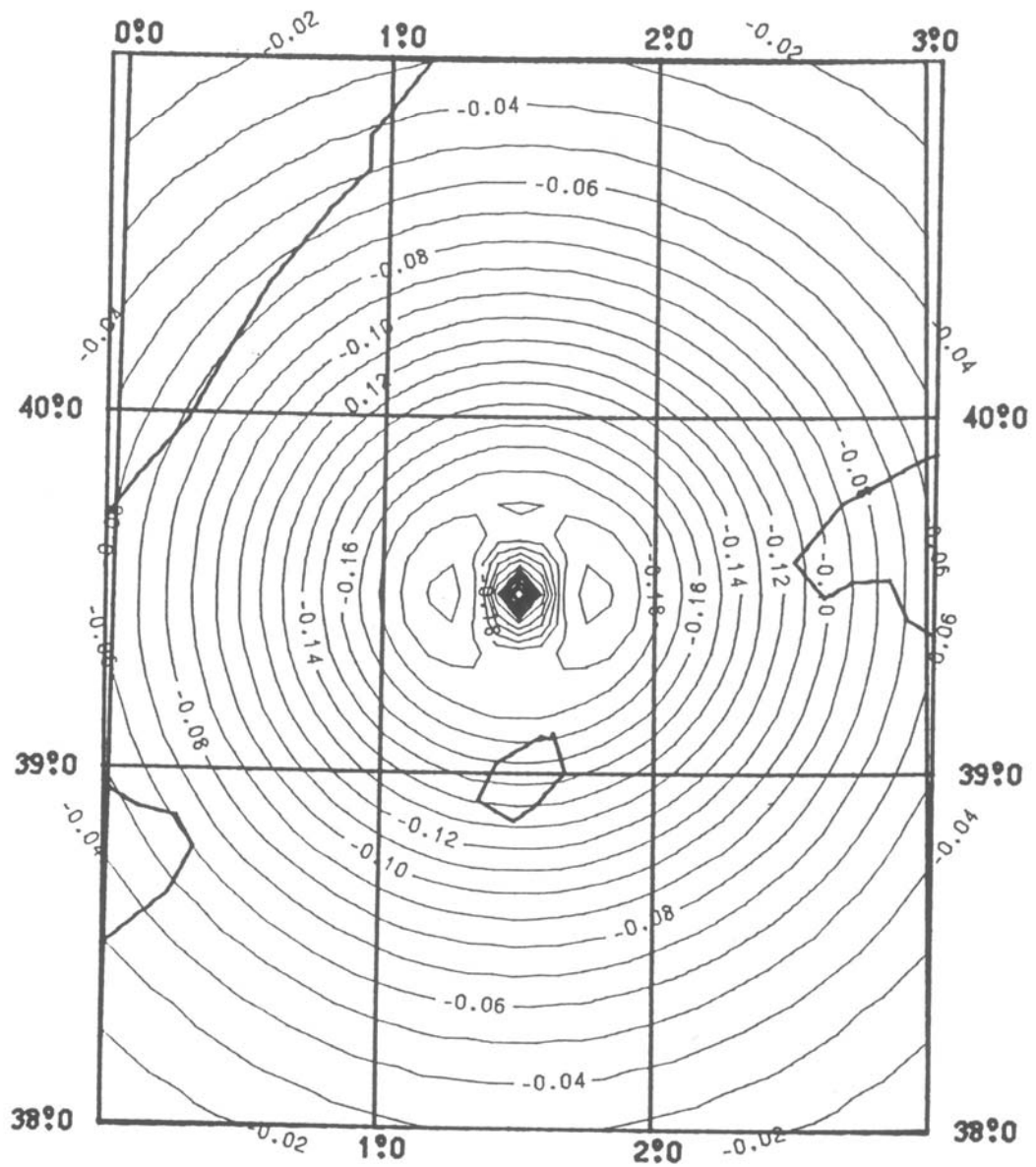


Fig. 4 - $\delta \hat{N}$ computed using formula (5) with $m = 9$.

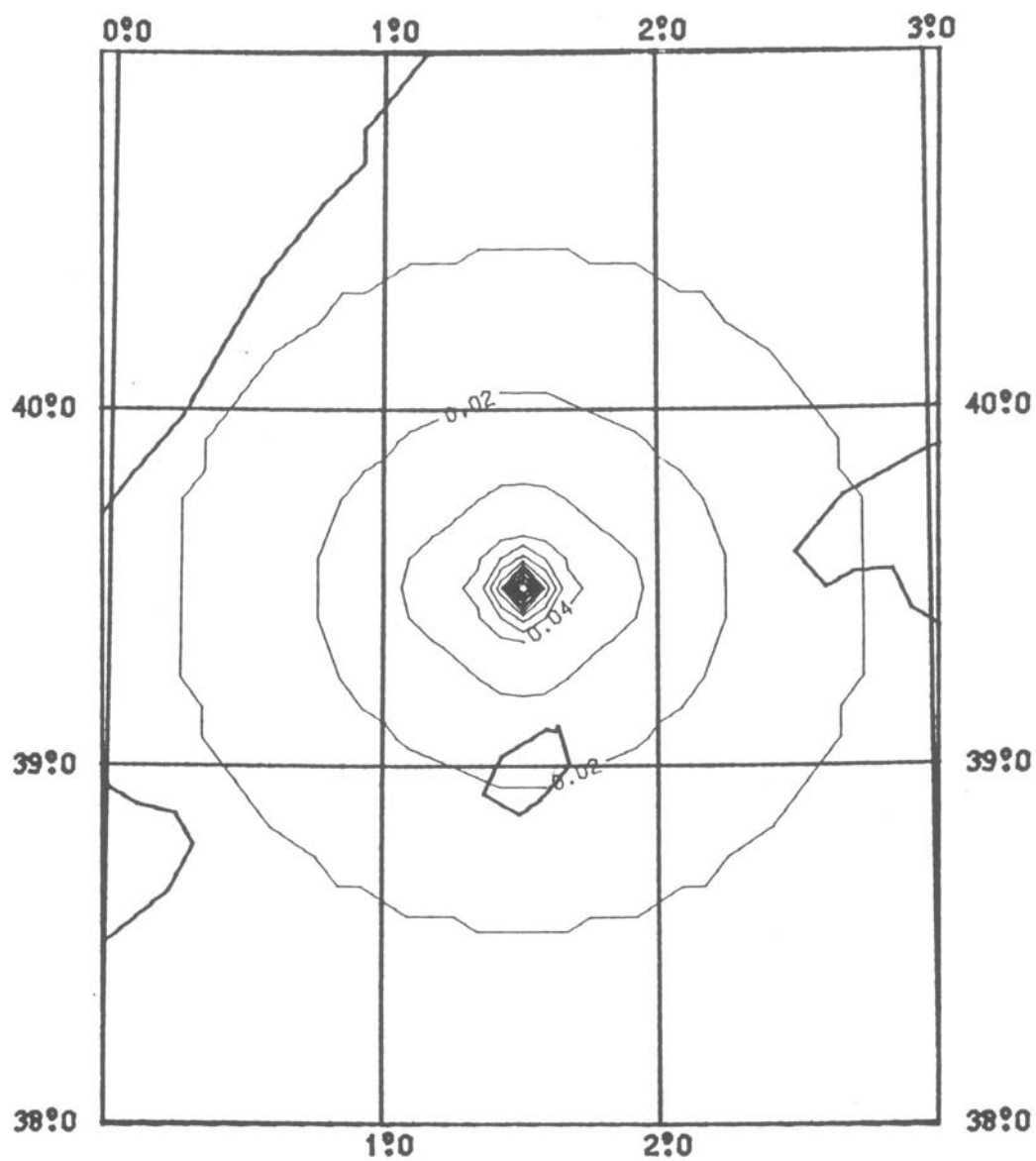


Fig. 5 - $\delta\hat{N}$ computed using formula (5) with $m = 25$.

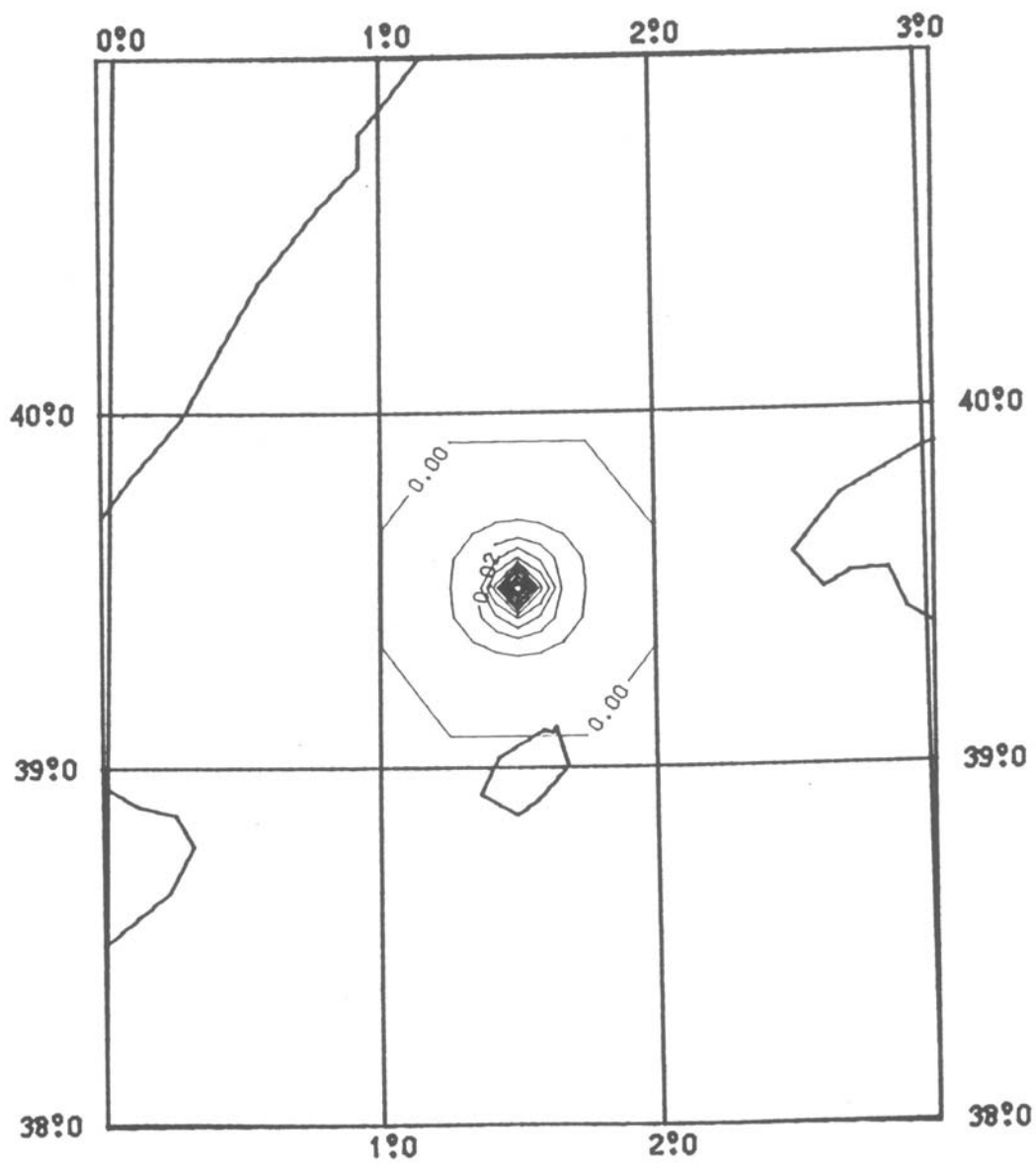


Fig. 6 - $\hat{\delta N}$ computed using formula (5) with $m = 49$.