

The first high-resolution gravimetric geoid for Ukraine: UGG2013

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Abstract

The first gravimetric geoid computed for Ukraine, with high-resolution, is presented in this paper. The computation of this gravimetric geoid is based on the most recent geopotential model: EGM2008 (Earth Gravity Model released in 2008). The method used in the computation of this new gravimetric geoid has been the Stokes integral in convolution form. The terrain correction has been applied to the gridded gravity anomalies, to obtain the corresponding reduced anomalies. Also the indirect effect has been taken into account. Thus, the first geoid model for Ukraine has been calculated and it is provided as a data grid in the GRS80 (Geodetic Reference System of 1980), distributed for the study area from 44 to 53 degrees of latitude and 22 to 41 degrees of longitude, on a 361x761 regular grid with a mesh size of 1.5'x1.5'. This new high-resolution geoid and the global geoid EGM2008; are compared with the geoid undulations measured for 4 points of the EUVN (EUropean Vertical Network) on Ukraine. The new geoid shows an improvement in precision and reliability, fitting the geoidal heights of these EUVN points with more accuracy than the global geoid. Moreover, this new geoid has a smaller standard deviation (10.4 cm) than any previous geoid developed for Ukraine, up to date. This new model will be useful for orthometric height determination by GPS over this study area, because it will allow orthometric height determination in the mountains and remote areas, in which levelling has many logistic problems. This new model can be also interesting for other geophysical purposes, other than geodesy and the height measurements, because it can provide a constraint for the density distribution and the thermal state of lithosphere and the viscosity in the mantle. Such details can be inferred from a geoid model and the seismic velocity structure.

Keywords: Gravity, Geoid, FFT, EUVN, Ukraine

1. Introduction

The new gravity satellite missions provide new global solutions that allow modelling the long and medium wavelengths of the Earth's gravity field. Such models, as the EGM2008 solution (Earth Gravity Model released in 2008), represent a major advance in the geodesy because they are incorporating the best quality data available for the whole Earth. However as [3] have shown, the global geoid models can fail in regions with high mountains, where a gravimetric geoid based on local data can be more reliable. For this reason, a new geoid for Ukraine has been computed to improve the geoid picture for the eastern part of Europe. This new geoid will be very desirable because it will complement some European geoids calculated with high-resolution. This new geoid and these others will give jointly a complete picture of the geoid for Europe, with more precision than the global geoid EGM2008. Also, it would be very desirable to obtain a new high-resolution geoid for Ukraine, because this geoid would be useful for orthometric height determination by GPS over this study area, allowing the orthometric height determination in the mountains and remote areas, in which levelling have many logistic prob-

lems. This new model can be also interesting for other geophysical purposes, because it can provide a constraint for the density distribution and the thermal state of lithosphere and the viscosity in the European mantle.

The new geoid will be computed as a 361x761 regular data grid in the GRS80 reference system, with a mesh size of 1.5'x1.5', completing the picture of the European geoid for Ukraine from 44 to 53 degrees of latitude and 22 to 41 degrees of longitude. This new geoid will be computed using the Stokes integral in convolution form. The necessary terrain correction will be applied to obtain the gridded reduced gravity anomalies. The corresponding indirect effect will be taken into account. After the computation of this Ukrainian Gravimetric Geoid (UGG2013), it will be compared with the global geoid EGM2008, to demonstrate the improvement in precision and reliability attained by the new geoid.

2. Data set

For the gravimetric geoid computation the necessary data sets are: (1) free-air gravity anomalies; (2) a geopotential model; (3) a high precision Digital Terrain Model (DTM); and (4) observed geoid undulations. The data

sets used for computation of the Ukrainian Gravimetric Geoid released in 2013 (UGG2013) are detailed below.

Land and marine gravity data. The land and marine gravity data used in this study has been provided by the Bureau Gravimetric International (BGI). The BGI data set has 2605 points in the study area (1213 on land and 1392 at sea). This data set of free-air gravity anomalies is distributed in the study area from 44 to 53 degrees of latitude and 22 to 41 degrees of longitude, as it is shown in Figure 1. The data window is taken so that it excludes as much as possible the zones where the gravity measurements are scarce (or there are no data). The accuracy of all these data ranges from 0.1 to 0.2 mgal. All the data were converted to the GRS80 reference system and the atmospheric correction was taken into account [7, 12].

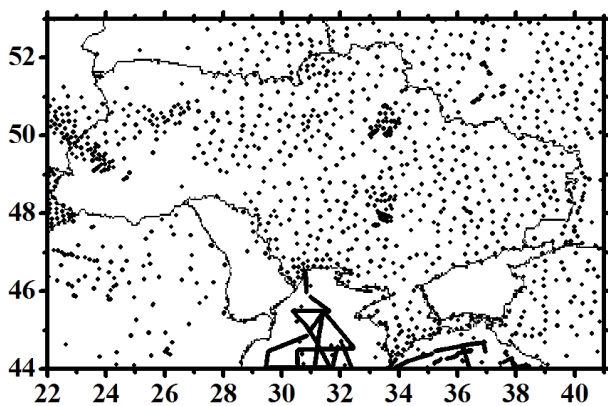


Figure 1. Geographical distribution of the gravity data over the study area (2605 free-air gravity anomalies).

Geopotential model. The EGM2008 geopotential model represents a major advance in modelling the Earth's gravity and geoid [9]. Therefore, this global model is the geopotential model that must be used for the computation of the long-wavelength contribution to the geoid and the gravity anomaly, to obtain a high-precision geoid in the study area.

Digital terrain model (DTM). Any gravimetric geoid computation based on Stokes' integral must use anomalies that have been reduced to the geoid, usually by means of Helmert's second method of condensation [2, 5]. This involves the computation of the terrain correction and the indirect effect on the geoid, which are computed from a DTM. A DTM is also necessary to compute the Residual Terrain Model reduction (RTM reduction or also called RTM correction) for the point anomalies, in order to obtain smooth gravity anomalies, which are more easily gridded. For the present study, a new elevation model for the whole study area, with a 3''x3'' spacing, has been obtained from the Shuttle Radar Topography Mission (SRTM) elevation data and the ETOPO1 bathymetry data, following the process described by [2]. To minimize the loss of accuracy associated to the low resolution of the ETOPO1 bathymetry, the data window

was selected so that they include the marine data as small as possible (Figure 2).

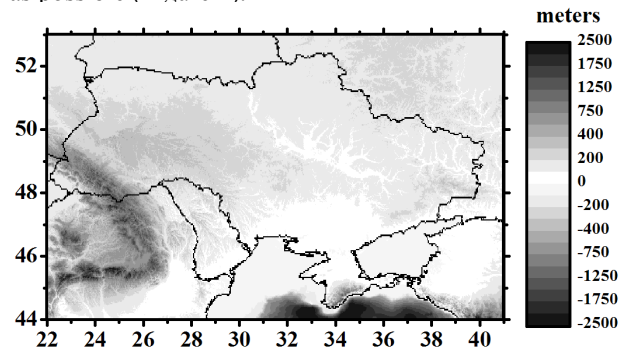


Figure 2. Topographic digital model used in this study (3'' x 3'' mesh size).

EUVN points used as a control data set. The height data of 4 points of the EUropean Vertical Network (EUVN) existing for the study area have been used as a control data set, to check the computed geoid. Figure 3 shows the geographical distribution of these points and Table 1 their coordinates and heights. The orthometric heights have been computed from the normal heights through a well-known formula [2, 5].

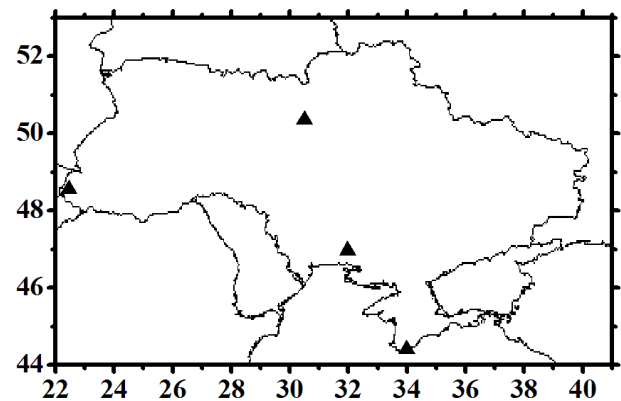


Figure 3. Geographical distribution of the EUVN points used as control data set (triangles).

3. Methodology and processing

The computation method for the calculation of a gravimetric geoid detailed by [2] was followed. In this paper only a brief review of this computation process is presented.

Gravity data gridding. Since the gravity data set consists of point anomalies distributed randomly, an interpolation process must be applied to obtain a regular data grid. Before this interpolation, it is very suitable to remove the short-wavelength and the long-wavelength effects applying the well-known relationship (the RTM correction):

Table 1. The 4 EUVN points used as a control data set (validation points), the geoid heights predicted by the available geoids over the study area and the differences between the geoid heights predicted by the available geoids and the geoid heights.

Point (n.)	Latitude (°N)	Longitude (°E)	h (m)	H* (m)	H (m)	N = h-H (m)	EGM2008 (m)	UGG2013 (m)	EGM2008 - N	UGG2013 - N
1	50.364172466667	30.496775433333	224.880	199.520	199.522	25.358	25.542	25.484	0.184	0.126
2	48.562879091667	22.452619527778	273.803	235.414	235.414	38.389	38.557	38.450	0.168	0.061
3	46.971520211111	31.973399736111	78.242	52.668	52.668	25.574	25.636	25.486	0.062	-0.088
4	44.415355600000	33.991604838889	386.475	362.114	362.097	24.378	24.739	24.305	0.361	-0.073

(h = ellipsoidal height, H* = normal height, H = orthometric height, N = geoid height, EGM2008 = Pavlis et al. (2008), UGG2013 = This paper)

$$\Delta g_{\text{red}}^{\text{pts}} = \Delta g_{\text{free}}^{\text{pts}} - 2\pi k \rho (h - h_{\text{ref}})^{\text{pts}} + c^{\text{pts}} - \Delta g_{\text{GM}}^{\text{pts}} \quad (1)$$

where the superscript *pts* denotes each point randomly distributed over the study area, Δg_{free} is the free-air gravity anomaly, k is Newton's gravitational constant, ρ is the density of the topography (2.67 g/cm^3), h is the elevation (shown in Figure 2), h_{ref} denotes the elevation of the reference surface (this reference surface is obtained by applying a 2D low-pass filter with a resolution of $60'$, to the elevation field), c is the terrain correction computed at each point, and Δg_{GM} is the gravity anomaly computed from the geopotential model EGM2008, considering 360 as maximum degree in the spherical harmonic expansion (i.e. subtracting only the long-wavelength effect in the point data).

When the smooth anomalies have been obtained by (1), it can be observed that some points have gravity anomalies with high values. These erroneous values are associated to bad gravity data points. To avoid the inclusion of these bad data in the computation process, the gravity anomalies given by (1) greater than 30 mgal have been removed. Thus, 54 points have been removed from the total data set (2605 points), leaving 2551 points for the interpolation on a regular grid. This regular grid has been obtained by using Kriging-based routines which are a part of OriginLab software package (© 1991-2003 OriginLab Corporation, Northampton, MA 01060 USA). The gridded data are distributed over the study area from 44 to 53 degrees of latitude and 22 to 41 degrees of longitude, on a 361×761 regular grid with a mesh size of $1.5' \times 1.5'$.

Finally, RTM must be restored in the gridded anomalies to obtain the true free-air anomalies relative to EGM2008. This RTM effect can be restored by

$$\Delta g_{\text{free}}^{\text{grid}} = \Delta g_{\text{red}}^{\text{grid}} + 2\pi k \rho (h - h_{\text{ref}})^{\text{grid}} - c^{\text{grid}} \quad (2)$$

where the superscript *grid* denotes each point of the regular grid considered ($361 \times 761 = 274721$ points), Δg_{free} is the free-air gravity anomaly, Δg_{red} is the gravity anomaly reduced by (1) and gridded.

Geoid computation. This new geoid has been computed by the classical remove-restore technique. Following this method, the geoid model for the study area is obtained by the sum of three terms

$$N = N_1 + N_2 + N_3 \quad (3)$$

The first term N_1 is the geopotential model contribution to the geoid undulation. This term has been computed from the geopotential model EGM2008, considering 360 as maximum degree in the spherical harmonic expansion (i.e. only considering the long-wavelength contribution to the geoid height). The second term N_2 is the indirect effect of Helmert's second method of condensation reduction on the geoid. The third term N_3 is the contribution of the residual gravity. Figure 4 shows the values of each term in formula (3). The geoid solution for the study area is shown in Figure 5 and it is obtained summing all previously computed terms (Figure 4) according to equation (3). This Ukrainian Gravimetric Geoid (UGG2013) with a mesh size of $1.5' \times 1.5'$ is extended 9×19 degrees over the study area. As it can be seen in Figure 5, an important indirect effect appears clearly in some contours, making these contour lines more rough in areas with high mountains. [3] have demonstrated that a geopotential geoid model (like EGM2008) can fail in these regions, being the gravimetric geoid a more reliable model in areas with high mountains.

This new model and a simple FORTRAN program for PC can be obtained from the internet address: <http://airy.ual.es/www/UGG2013.htm>. This computer program allows the computation of the geoid height (using the UGG2013 model) in any point over the study area shown in Figure 5.

Geoid validation. The new geoid UGG2013 has been checked by comparison with the geoid undulations measured for the 4 validation points located in the study area (the EUVN points shown in Figure 3). Table 1 shows the results of this comparison. The geoid height predicted by the EGM2008 model, for any point on the earth, can be easily obtained by means of useful software available from the internet address:

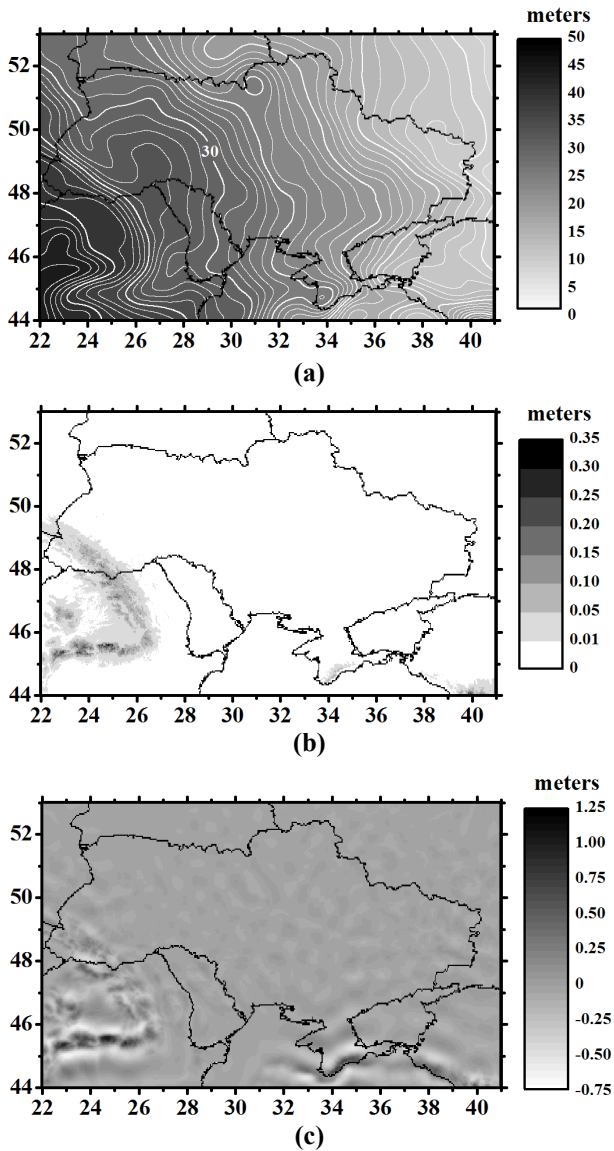


Figure 4. (a) The EGM2008 geoid model computed for the study area, considering the maximum degree of the harmonic expansion equals to 360. Contour interval: 1 m. (b) The indirect effect on the geoid (plotted positive). (c) The residual geoid undulation. Contour interval: 0.05 m,

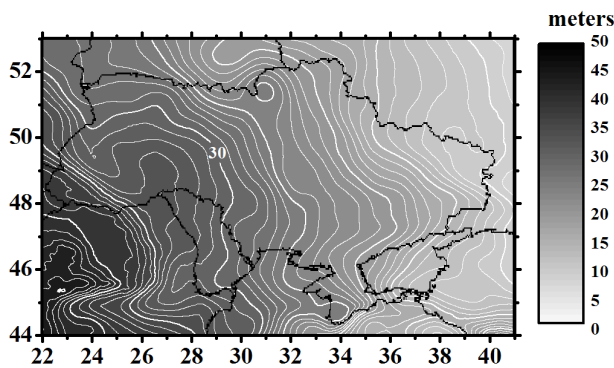


Figure 5. The Ukrainian Gravimetric Geoid released in 2013 (UGG2013) obtained as a sum of the terms given by the equation (3). Contour interval: 1 m.

<http://earth-info.nga.mil/GandG/wgs84/gravitymod/egm2008/index.html>. This software allows the computation of the spherical harmonic expansion up to the maximum degree and order, defined for this global model (degree 2190 and order 2159). The geoid heights obtained from the EGM2008 model for the validation points have been computed using this software (Table 1). In Table 1, the differences between the geoid heights predicted by the available models (EGM2008 and UGG2013) and the geoid heights measured for the validation points also are shown. The statistics of these differences are shown in Table 2. In this Table, it should be noted that the new geoid UGG2013 shows an improvement in precision and reliability, fitting the geoidal heights measured for the validation points better than EGM2008.

Table 2. Statistics of the differences listed in Table 1.

Differences	Mean (m)	Std. dev. (m)
EGM2008 - N	0.194	0.124
UGG2013 - N	0.007	0.104

4. Conclusions

The computation methods based on the FFT have allowed the calculation of a precise geoid for Ukraine, which is a major advance in the modelling of the geoid for the eastern part of Europe. The gravimetric geoid determination has been carried out by means of the Stokes integral in convolution form. This method, which has previously been shown to be an efficient method to compute a high-resolution geoid [2], yielded a regular gridded geoid of 361x761 points (274721 points) in the GRS80 reference system, with a mesh size of 1.5'x1.5', distributed from 44 to 53 degrees of latitude and 22 to 41 degrees of longitude. The new geoid shows less discrepancy with the geoid undulations measured for the validation points (4 EUVN points available on the study area), than the most recent global geoid (EGM2008). The new geoid has a standard deviation of 10.4 cm (Table 2). This is the smallest error obtained by any geoid solution for Ukraine, up to date. Nevertheless, an important problem arises from the scarcity of the gravity data for the study area considered (Figure 1). For this reason, the computation of a gravimetric geoid with a centimetre precision, for this study area, is not possible with the present gravity data. This centimetre precision in the geoid model can be obtained for this area, when the data coverage can be increased with new gravity data. Updating of the international compilations (like BGI databank) is needed to supply new gravity data, measured for many areas in the world in which the scarcity of gravity data, is a severe problem for the development of new geophysical mod-

els. In spite of this, a new high-resolution geoid has been obtained for Ukraine and it will be useful for the orthometric height determination by GPS over this area, because it will allow the orthometric height determination in mountains and remote zones, where levelling has many logistic problems. On the other hand, this Ukrainian geoid will complement others precise geoids computed for some European countries, giving joint to those precise models a geoid picture for Europe, with more precision than EGM2008, that can be very useful for geophysical purposes different to the geodesy, because they can provide a constraint for the density distribution and the thermal state of European lithosphere [4]. It is well known that there are some elementary relations between mass distributions inside Earth and the geoid [10]. The geoid also can be used to infer details of the viscosity structure in the top 1000 km of the mantle, in combination with the tomographic structure beneath the study area [1, 6, 8]. In those studies, a combination of temperature profiles and viscosity in the mantle can be inferred from the geoid and the seismic structure of the study area. Moreover, previous studies developed in different areas of Earth have shown the relation between the plate motions and the undulations in the geoid [11]. In these studies, it was shown that the geoid exhibits linear anomalies, trending in the direction of the absolute plate motion. Thus, the new Ukrainian geoid is a major contribution for the knowledge of geophysics in the eastern part of Europe.

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and the gravity anomaly). The author is also grateful to Dr. Martina Sacher (Bundesamt für Kartographie und Geodäsie, Leipzig, Germany) who provided the EUVN data used for validation of the computed geoid.

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