On the quasigeoid solutions for the Ukraine and Moldova area

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Abstract. The UQG2006 and UQG2007 regional quasigeoids with accuracy < 5 – 10 cm were accepted for the adjustment of the Ukrainian geodetic network. These solutions for the Ukraine and Moldova area were computed by means of the regularization method. The gravimetric quasigeoid UQG2006 was constructed from digitized gravimetry in the land area and gravity anomalies in the Black Sea region derived from BGI gravimetry and altimetry data of six satellite missions. The combined UQG2007 solution was estimated from the heterogeneous data set of GPS/leveling quasigeoid heights and the same gravity anomalies. Comparisons of the UQG2006 and UQG2007 quasigeoids with independent GPS/leveling data show a good accordance with Std.Dev. < 8 cm and Std.Dev. < 4 cm, respectively.

Key words. Regional quasigeoid, gravimetry, altimetry, GPS/leveling

1 Introduction

Since 1997 the gravimetric quasigeoid EGG97 (Denker and Torge, 1998) became a most representative solution for the Moldova and Ukraine area. MOLDGEO2004 (Marchenko and Monin, 2004) and MOLDGEO2005 quasigeoids with accuracy < 10 cm were based on the fit of EGG97 to the Moldavian dense set of GPS/leveling data given in the Baltic 1977 height system. In this case the EGG97 solution has transformed preliminary to the Baltic height system and used as additional information. Similar approach in the Ukraine area led to pessimistic results due to a low accuracy of the EGG97 quasigeoid in few Ukrainian regions (such as Crimea), which is probably caused by applied in this area (see, Denker and Torge, 1998) digital terrain model (DTM) with the resolution above 5000 m.

This paper focuses on new quasigeoid solutions with accuracy < 5 – 10 cm for the conversion of ellipsoidal heights into normal heights and the adjustment of the Ukrainian geodetic network. The UQG2006 and UQG2007 solutions for the Ukraine and Moldova area were constructed on the basis of the collocation method with regularization (Neyman, 1979; Moritz, 1980). The gravimetry-only quasigeoid UQG2006 (Marchenko et al., 2007) was based on the digitized gravity anomalies (land area) and gravity anomalies in the Black Sea area derived basically from altimetry data of six satellite missions via the regularization method. Terrain reductions were computed from the $1' \times 1'$ digital terrain model GEBCO. The UQG2007 combined solution was derived from the heterogeneous data set of GPS/leveling and the above-mentioned gravimetry data. In this case terrain reductions were based on the $3'' \times 3''$ digital terrain model SRTM3 (Jarvis A. et al., 2006) having (in comparison with leveling data) over one order better accuracy than DTM GEBCO.

Thus, both gravimetric UQG2006 and combined UQG2007 solutions are constructed by the application of the regularization method. Since kernel functions corresponding to radial multipoles were applied (Marchenko, 1998), we use only singular point harmonic functions for regional quasigeoid solutions on the basis of gravimetry, satellite altimetry, and GPS/leveling data.

2 Initial data. Preprocessing

All initial free air gravity anomalies Δg were used as two individual sets of the land and marine (Black Sea region) Δg. We start from the brief description of the marine Δg derived basically from the following altimetry data in the Black Sea area:

- subset 1 represents 643128 TOPEX/POSEIDON, ERS-1, ERS-2, JASON-1, ENVISAT, and GFO corrected Sea Surface Heights (SSH) taken for the period from 1992 to 2005 year and corrected by CSL AVISO for different geophysical phenomena and instrumental effects.

As a result of data gaps in the corrected SSH (Black Sea region) the additional set of point gravimetry data is used to support the SSH-only solution:

- subset 2 represents 4774 values of BGI marine point gravimetry data in addition to land gravimetry surrounding the Black Sea area.

Fig. 1 illustrates AVISO corrected SSH (transformed to the GRS80 ellipsoid) from six satellite missions in the Black Sea area. It has to be pointed out, that the surface in Fig. 1 differs from the geoid due to Sea Surface Topography and various remain effects averaged in time. Fig. 2 demonstrates results of processing of these two subsets by regularization method in the form of gravity anomalies at the grid points $2' \times 2'$, which were obtained by Marchenko and Yarema (2006) and called in the following as the Set 1 (marine Δg).

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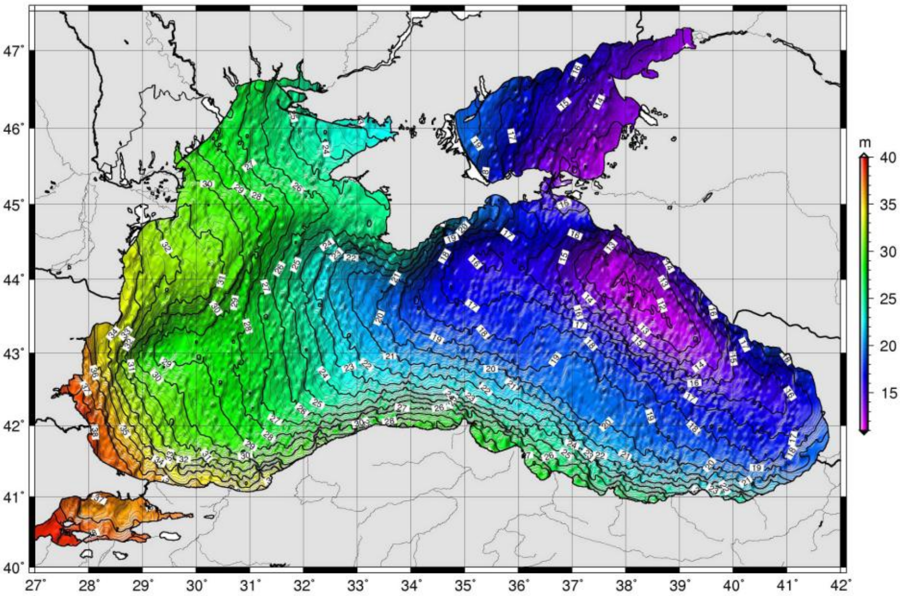

Fig. 1. AVISO corrected SSH (m) from TOPEX/POSEIDON, ERS1, ERS2, GFO, JASON-1, and ENVISAT altimetry

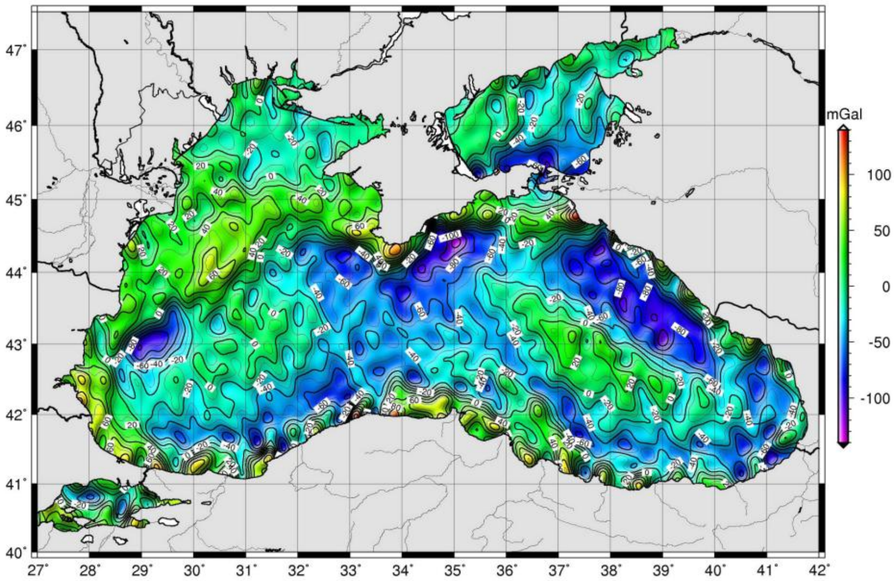

Fig. 2. Gravity anomalies (mGal) from six satellites missions (1992 – 2005) and BGI gravimetry in the Black Sea area

The second Set 2 of free air anomalies at the grid points 2′×2′ was derived from the digitized Bouguer anomalies covering the Ukraine and Moldova area. Then classical terrain reduction was applied for the conversion of the free air anomalies Δg to the Faye anomalies Δg_F. This reduction was based on the (a) DTM GEBCO given for the grid 1′×1′ and (b) DTM SRTM3 given for the grid 3″×3″. It should be mentioned, that the Faye anomalies Δg_F were adopted as basic initial information for quasigeoid computations. In the case of UQG2006 Δg_F were produced by means of the DTM GEBCO using the numerical integration with the radius 167 km. In the case of the UQG2007 quasigeoid classical terrain reduction was derived from the DTM SRTM3 with the same radius of integration and shown in Fig. 3.

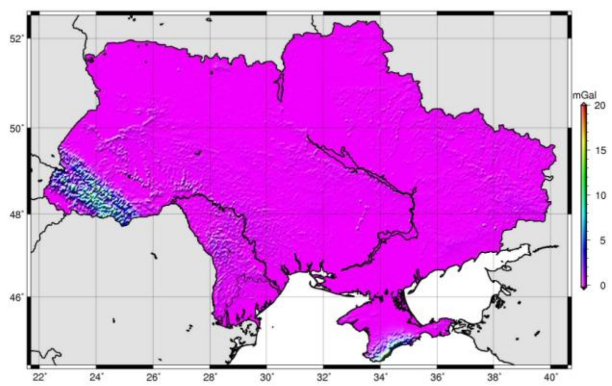

Fig. 3. Classical terrain reduction (mGal) in the Ukraine and Moldova area based on the DTM SRTM3

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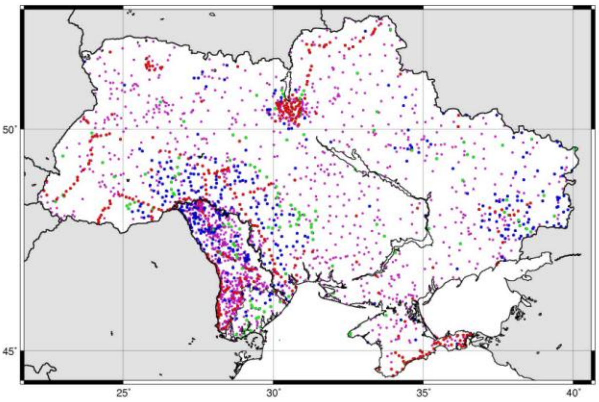

Fig. 4. GPS/leveling data of different orders:
● – 1st order; ● – 2nd order; ● – 3rd order; ● – 4th order

Fig. 4 illustrates the Set 3 containing 3000 points of GPS/leveling data of different orders covering the Ukraine and Moldova area. This set was applied (a) for the evaluation of the UQG2006 quasigeoid and (b) for the construction of the combined UQG2007 solution. Finally Set 3 was formed after detection of gross errors in GPS/leveling data given in the Baltic 1977 height system.

3 Gravimetric and combined quasigeoid solutions

The traditional remove-restore technique was applied to get rid of the long wavelength constituent of the gravity field adopted in this study according to the EIGEN-CG01C gravity field model up to 360 degree/order (Reigber et al., 2006). In the case of gravimetry-only data the residuals $\delta\Delta g$ were initially computed

$$\delta\Delta g = \Delta g_F - \Delta g_{\text{EIGEN-CG01C}} \,, \qquad (1)$$

where Δg_F is the Faye anomaly; $\Delta g_{\text{EIGEN-CG01C}}$ is the EIGEN-CG01C gravity anomaly. Then the prediction of the residual anomaly heights $\delta\zeta_P$ was made at some point P inside the studying area by applying the collocation with regularization

$$\delta\zeta_P = \mathbf{C}_{\delta\zeta,\delta\Delta g}(\mathbf{C} + \alpha\mathbf{C}_{\text{nn}})^{-1}\mathbf{l} \,, \qquad (2)$$

where in the case of gravimetry data, $\mathbf{l}$ is the q-vector consisting of the components $\delta\Delta g_i$ (i=1, 2,…q); q is a number of observations; $\mathbf{C}$ is the ($q \times q$) - covariance matrix of the residual gravity anomalies $\delta\Delta g$; $\mathbf{C}_{\Delta\delta g,\delta\zeta}$ is the ($1 \times q$) - cross-covariance matrix between $\delta\Delta g$ and $\delta\zeta$; $\mathbf{C}_{\text{nn}}$ is the ($q \times q$) – covariance matrix of the measurement noise $\mathbf{n}$; α is the regularization parameter or weight factor constraining the variability of the solution (Neyman, 1979; Moritz, 1980). Obviously the collocation method will correspond to α =1 in Eq. (2).

After application of the regularization/collocation via Eq. (2) the anomaly heights ζ are restored at the chosen grid in the following way

$$\zeta = \zeta_{\text{EIGEN-CG01C}} + \delta\zeta \quad , \qquad (3)$$

where $\zeta_{\text{EIGEN-CG01C}}$ is the contribution of the EIGEN-CG01C gravity field model in the quasigeoid heights.

For further use of Eq. (2), the following problems have to be solved:

- The construction of the analytical covariance function K (P, Q) of the anomalous potential T.
- The choice of an appropriate method for the computation of the regularization parameter α.

In this study we will apply such reproducing kernels $K(P,Q)$, which are described only by singular point harmonic functions (Marchenko, 1998; Marchenko and Lelgemann, 1998):

$$K_{\text{n}}(P,Q) = \left[\frac{GM}{R}\right]^2 \beta_{\text{n}}\sigma^{\text{n+1}}v_{\text{n}} \quad , \quad \sigma = \frac{R_{\text{B}}^2}{r_{\text{P}}r_{\text{Q}}} \quad , \qquad (4)$$

where R is the Earth's mean radius; R_{B} is the Bjerhammar's sphere radius; r_{P} and r_{Q} are the geocentric distances to the external points P and Q; GM is the product of the gravitational constant G and the planet's mass M; v_{n} is the dimensionless potential of radial multipole of the degree n; β_n represents some dimensionless coefficient (Marchenko, 1998; Marchenko et al., 2001). The traditional determination of the parameter α in Eq. (2) requires a special iterative process and the inversion of a matrix with a dimension equal to the number q of observations (Neyman, 1979). So, when a number of observations are large we come to a time consuming procedure. To avoid this difficulty we will use another estimation of the parameter α:

$$\alpha = 1 + \sqrt{1 + \text{Trace}(\mathbf{C}\mathbf{C}_{\text{nn}}) / \text{Trace}(\mathbf{C}_{\text{nn}}\mathbf{C}_{\text{nn}})} \,, \qquad (5)$$

verified by Marchenko and Tartachynska (2003) for the inversion of altimetry data SSH into gravity anomalies.

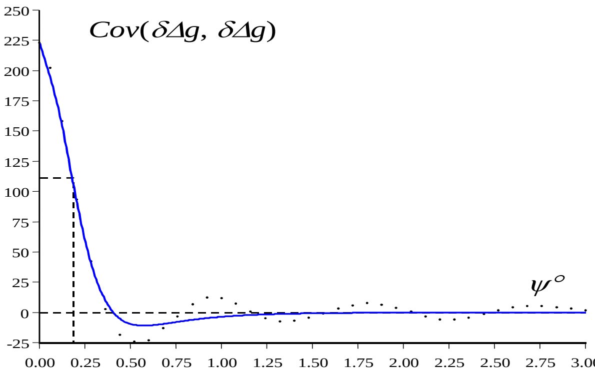

Fig. 5. Empirical (dotted) and analytical (solid) covariance functions of the residuals $\delta\Delta g$ (mGal^2)

Taking into account the processing of BGI gravimetry and satellite altimetry data, gravity anomalies at the grid points 2′×2′ were used as the Set 1 together with the Set 2 for the construction of the gravimetric quasigeoid UQG2006 (Marchenko et al., 2007) by means of the regularization/collocation. The empirical covariance function (ECF) was computed on the basis of the residual gravity anomalies $\delta\Delta g$. Then, this ECF was approximated by some reproducing kernels or analytical covariance functions ACF, derived from radial multipole potentials by the Kelvin transformation (Marchenko, 1998), that provides the covariance propagation in $\mathbf{R}^3$ to geoid heights and other functionals of the anomalous potential. Because of better fit to ECF of the modified Poisson kernel without harmonic of zero degree (Marchenko and Lelgemann, 1998), this one was selected (Fig. 5).

Although the UQG2006 solution is based on the Faye anomalies computed using DTM GEBCO, this quasigeoid was applied for further detection of gross errors in GPS/leveling data due to antenna height, etc. As a result, part of sites with significant deviations was deleted and we got the final Set 3 of 3000 GPS/leveling stations with known geodetic coordinates in the ETRS89 system and the normal heights given in the Baltic 1977 height system. Fig. 6 demonstrates results of such a comparison where histogram of these differences reflects a fit to normal distribution. Comparisons of the gravimetric UQG2006 quasigeoid with independent GPS/leveling data show the agreement with Std.Dev. < 8 cm. Fig. 7 illustrates an essential improvement of the quasigeoid UQG2006 in the area of Crimea Mountain in comparison with EGG97.

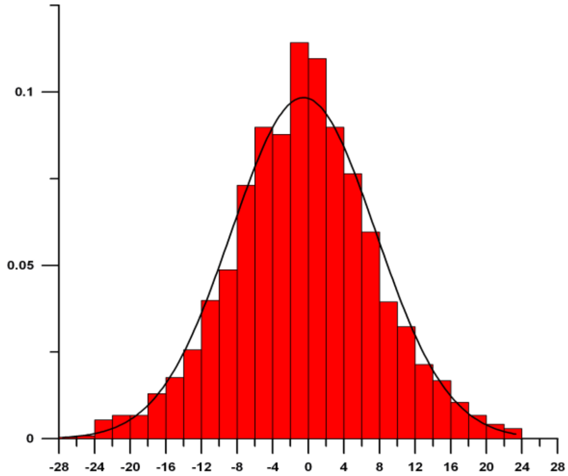

Fig. 6. Histogram of differences $\Delta\zeta$ between GPS-derived and computed according to UQG2006 quasigeoid heights. Axes: *Y* is the relative frequency; *X* is the intervals (cm).

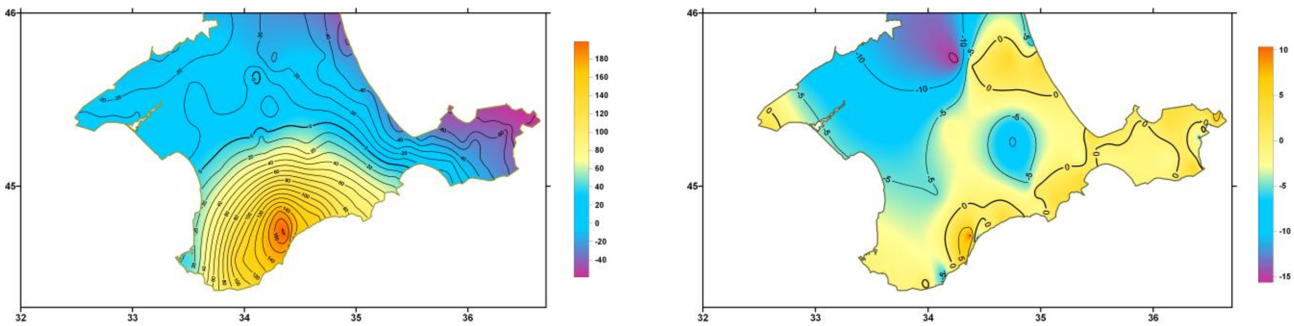

Fig. 7. Differences (cm) between the measured and GPS-derived normal heights based on the EGG97 quasigeoid (left) and the UQG2006 solution (right) in the Crimea area

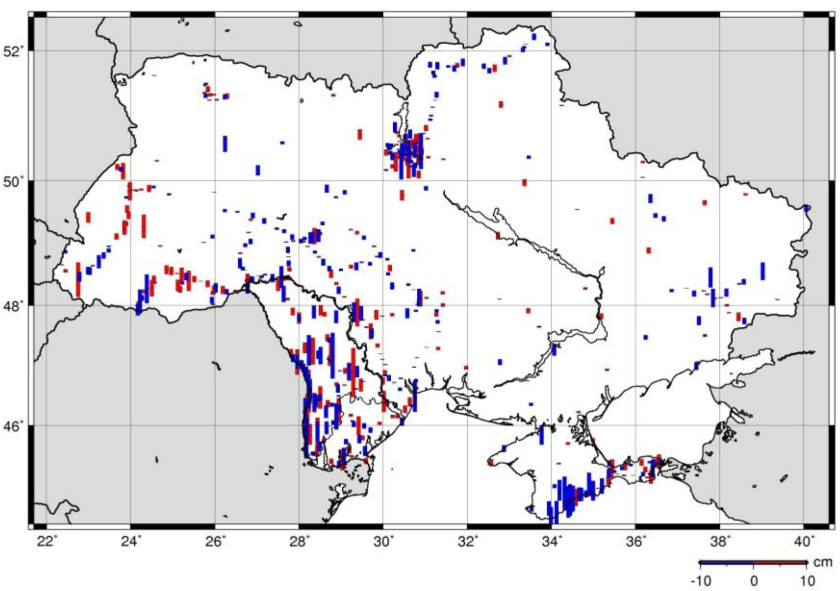

Fig. 8. Differences between GPS/leveling data of 1st and 2nd orders and UQG2007 quasigeoid in the Ukraine and Moldova area. (Statistics: Std.Dev. = 2.8 cm; Mean = –0.1 cm; Min = –12.3 cm; Max = 9.3 cm)

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Table 1. Statistic of differences (cm) between GPS-derived quasigeoid heights and the UQG2007 quasigeoid

Statistics	GPS/leveling points of different orders in Ukraine and Moldova area				
	1st (348 pts.)	2nd (242 pts.)	3rd (493 pts.)	4th (1917 pts.)	Total: 3000 pts.
Min.	-8.6	-12.3	-15.1	-19.7	-19.7
Max.	7.5	9.3	16.2	17.7	17.7
Mean	-0.3	0.2	-0.4	-0.6	-0.7
Standard deviation	2.5	3.0	4.8	5.9	5.5

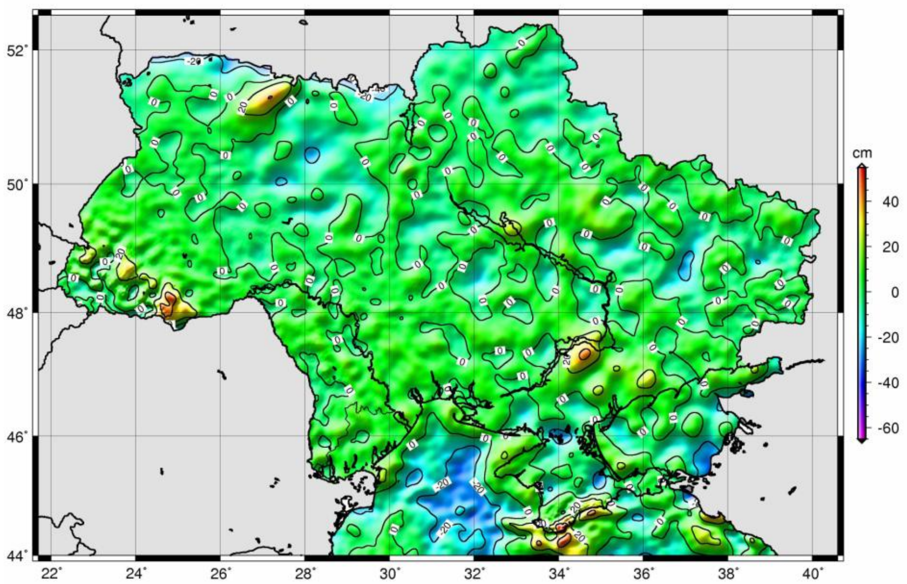

Fig. 9. Differences (cm) between the UQG2007 and EGM2008 solutions after datum shift transformation.

The UQG2007 combined solution was constructed on the basis of combined sets of the marine Δg (Set 1), the recomputed Faye gravity anomalies Δg_F in the land area (new Set 2), and the Set 3 of GPS/leveling data. After preliminary verification of the DTM SRTM3 (Jarvis A. et al., 2006) by comparison with leveling data this model was fitted to the Baltic 1977 height system. Then recompilation of Δg_F using the modified 3″×3″ digital terrain model SRTM3 produces the new Set 2 of the Faye anomalies. As a result, the application of this DTM instead of the GEBCO model gave two times larger maximal values of the classical terrain reduction (Fig. 3) in mountain areas of Carpathians and Crimea.

It has to be pointed out, that from the comparison of the UQG2006 quasigeoid with 3000 GPS/leveling data follows a level of agreement of the quasigeoid and leveling of different orders. In general leveling of the 1st order characterizes in the Ukraine and Moldova area by accordance about 3 cm with UQG2006; 2nd order – 4 cm; 3rd order – 6 cm; and 4th order – 8 cm. These values were chosen as weight factors in the subsequent processing. Therefore, the initial Set 1 and new Set 2 of marine and land gravity anomalies (mean accuracy ~3 mGal) supplemented by the Set 3 were applied for the construction of the UQG2007 combined solution using the approach discussed before, which was extended to the processing of *heterogeneous* data according to (Moritz, 1980; Marchenko et al., 2001). A similar to Fig. 5 analytical covariance function was based on the recomputed Faye anomalies Δg_F (new Set 2) and the marine Δg (Set 1).

The comparison of the UQG2007 solution with independent GPS/leveling control points demonstrates a good accordance with Std.Dev. < 4 cm. Table 1 illustrates differences between GPS-derived anomaly heights applied for the UQG2007 solution. It should be mentioned, that only differences between GPS/leveling data of 1st, 2nd, and 3rd orders and UQG2007 (Table 1) correspond to the same level of agreement within 5 cm (in terms of r.m.s deviation) as in the case of *independent* GPS/leveling control points.

Fig. 8 shows differences between a most accurate common set of GPS/leveling data of 1st and 2nd orders and the UQG2007 quasigeoid. The additional evaluation of the UQG2007 solution was provided by the comparison with the recent high-resolution gravity field model EGM2008 (Pavlis et al., 2008) up to 2190 degree. For proper comparison the simplest datum shift transformation was applied and we got the following statistics: mean deviation = 0 cm, standard deviation = ±10 cm, Min = –66 cm; Max = 56 cm. According to Fig. 9 major differences are observed in the Black Sea basin, Carpathians and Crimea areas with values more than 50 cm together with smaller differences in few scattered zones. Nevertheless, the deviation between GPS/leveling points of 1st and 2nd orders and the UQG2007 quasigeoid in the Carpathian and Crimean regions are smaller than ±10 cm according to Fig. 8.

Conclusions

The use of the regularization method applied to updated satellite altimetry, gravimetry, terrain, and GPS/leveling data leads to the significantly improved quasigeoid solutions UQG2006 and UQG2007 for the Ukraine and Moldova area. As a result, in summary we can conclude.

- Comparisons of the UQG2006 and UQG2007 solutions with independent GPS/leveling data given in the Baltic height system show a good agreement with Std.Dev. < 8 cm and Std.Dev. < 4 cm, respectively. This noise level corresponds to quasigeoid accuracy about 5 cm (UQG2007) and 10 cm (UQG2006) for the Ukraine and Moldova area.
- The gravimetry-only solution UQG2006 was used successfully for the detection of gross errors in GPS/leveling data, the estimation of weight factors for further use in the combined solution, and the adjustment of the Ukrainian geodetic network based on the combination of GNSS observations and classic terrestrial data.
- The combined UQG2007 solution provides significantly better agreement (than UQG2006) with independent GPS/leveling control stations because Std.Dev. < 4 cm corresponds to the improvement up to 50%. This progress can be explained partly by the application for terrain reductions the 3″×3″ DTM SRTM3 instead of the 1′×1′ DTM GEBCO.
- Comparison of the UQG2007 quasigeoid with the global gravity field model EGM2008 up to degree 2190 leads to accordance within ±10 cm (in terms of standard deviation) with great differences having values more than 50 cm in the Black Sea area, and Carpathian and Crimean Mountains.

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