# A. N. MARCHENKO<sup>1</sup>, O. V. KUCHER<sup>2</sup>, D. A. MARCHENKO<sup>3</sup>

- <sup>1</sup> National University "Lviv Polytechnic", Institute of Geodesy, Lviv, Karpinsky St., 6, Ukraine, e-mail: march@pancha.lviv.ua, phone: +380509801609)
- <sup>2</sup> Research Institute of Geodesy and Cartography, Kyiv, Chervonoarmiiska St, 69, Ukraine, e-mail: kucher@gki.com.ua
- <sup>3</sup> Carpathian Branch of Subbotine's Institute of Geophysics, Lviv, Naukowa St., 3B, Ukraine, e-mail: dmitriy.marchenko@gmail.com

## REGIONAL QUASIGEOID SOLUTIONS FOR THE UKRAINE AREA

### Abstract.

The goal. The UQG2012 regional quasigeoid solution of an accuracy better than 2 cm with respect to the GPS-levelling data of the 1st and 2nd order was constructed by means of the least squares collocation method. In the first iteration the gravimetry-only quasigeoid UQG2011 was developed from the gravity anomalies for the subsequent detection of gross errors in GPS-leveling data. All terrain reductions were based on the 3"x3" digital terrain model SRTM3.

Scientific significance. Thus, the final UQG2012 solution consists of gravity anomalies and quasigeoid heights at the points of a 2'×3' grid evaluated by means of the collocation method applied to the set of 4070 GPS-leveling quasigeoid heights plus the above mentioned gravimetry data. The comparison of the UQG2012 solution with all independent GPS-leveling data (given in the Baltic height system) shows a good agreement with rms < 4 cm. This noise level corresponds to an estimated accuracy of the quasigeoid UQG2012 for the Ukraine and Moldova area higher than 4-5 cm with respect to GPS-leveling points of different orders. The evaluation of the UQG2012 solution with independent GPS-leveling data of the 1st and 2nd orders gives a significantly better agreement with rms of about 1.5 cm. Finally, the comparison with the European quasigeoid EGG08 leads to differences of about 20-50 cm in certain areas and to the total mean shift of 25 cm caused by the different height systems used.

Key words. Regional quasigeoid - Gravimetry - Altimetry - GNSS/leveling

#### 1. Introduction

Since 1997 the gravimetric quasigeoid EGG97 [Denker and Torge, 1998] became the most representative solution for the Ukraine and Moldova MOLDGEO2004 MOLDGEO2005 and quasigeoids with an accuracy better than 10 cm were based on the fit of EGG97 to the Moldavian dense set of GPS/leveling (preferably of the 4th order) data given in the Baltic 1977 height system [Marchenko and Monin, 2004]. In this case the EGG97 solution has been preliminarily transformed to the Baltic height system and used as an information additionally verified by Ihde et. al [Ihde et. al, 2000]. A similar approach in the Ukraine area led to pessimistic results due to a low accuracy of the EGG97 quasigeoid in some of the Ukrainian regions (such as Crimea), which is probably caused by the digital terrain model (DTM) of ca. 5'×5' resolution applied in [Denker and Torge, 1998] for this area.

Thus, one can say that the first high-resolution gravimetric quasigeoid EGG97 (1'×1.5' grid), including the East European and Ukraine areas, was constructed in 1997 [Denker and Torge, 1998]. This solution was essentially improved as the EGG07 (quasi)geoid [Denker et al., 2007] and subsequently (2008) as the EGG08 (quasi)geoid within the European Gravity and Geoid Project (EGGP). An independent gravimetric (UQG2006 – Ukrainian QuasiGeoid 2006) and combined (UQG2007 – from ca. 160000 gravity anomalies, ca. 640000 altimetry measurements and ca. 3000 GPS/leveling points)

solution were derived (for a 2'×3' grid) in 2006 and 2007, respectively, see [Marchenko, Kucher, Renkevych, 2007] and [Marchenko and Kucher, 2008]. The quasigeoid models EGG07, EGG08, UQG2006 and UQG2007 have a variable accuracy of 5 – 10 cm in different areas estimated from the comparison with independent GPS/leveling control points.

This paper focuses on the new quasigeoid solutions with accuracy of about 1.5 cm with respect to GPS/leveling points of the 1st and 2nd order. The UQG2012 model for the Ukraine and Moldova area was constructed on the basis of the least squares collocation method with regularization [Neyman, 1979; Moritz, 1980]. Since kernel functions corresponding to radial multipoles were applied [Marchenko, 1998; Marchenko and Lelgemann 1998], only singular point harmonic functions were used for regional quasigeoid solutions on the basis of gravimetry, satellite altimetry, and GPS/leveling data. Terrain reductions rest on the 3"×3" digital terrain model SRTM3 (Jarvis et al., 2006) having (in comparison with leveling data) over one order better accuracy than DTM GEBCO and ETOPO1. The solution UQG2012 was constructed by the least squares collocation method applied within two iterations. In the first iteration the gravimetry-only quasigeoid UQG2011 was obtained from the digitized gravity anomalies (continental area) and gravity anomalies in the Black Sea and Azov Sea areas derived from altimetry data of ERS-1, ERS-2, TOPEX/POSEIDON, JASON-1, ENVISAT, GFO missions (1992 – 2005), additionally to the national

and BGI marine gravimetry data. The UQG2011 quasigeoid was used especially for an effective detection of gross errors in GPS-leveling data.

As a result, the final UQG2012 model consists of gravity anomalies and quasigeoid heights at the points of a 2'×3' grid which were evaluated by means of the least squares collocation method applied to the heterogeneous data set: 4070 GPS-leveling quasigeoid heights plus previous gravimetry data. Comparisons of the UQG2012 solution with GPS-leveling data show a good agreement with rms of about 1.5 cm at the GPS-points of the 1st and 2nd orders. This noise level corresponds to an estimated geoid accuracy better than 5 cm for the Ukraine area and to the adopted requirements for the combined adjustment of the Ukrainian geodetic network based on the terrestrial and satellite GNSS data.

### 2. Initial data. Preprocessing

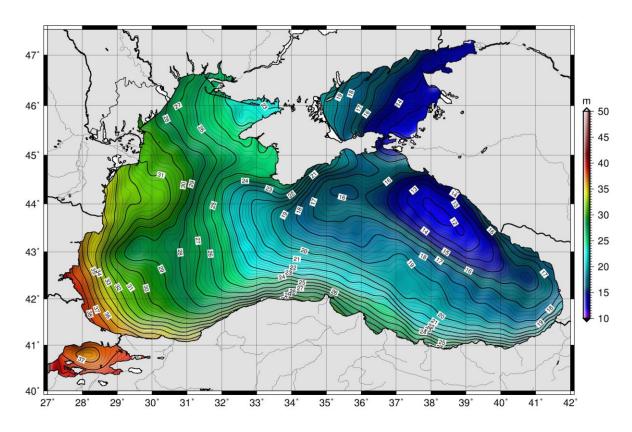
All the initial free air gravity anomalies  $\Delta g$  were used as two individual sets of land and marine (Black

Sea region)  $\Delta g$ . Let's start with a brief description of the marine  $\Delta g$  basically derived from the following altimetry and gravimetry data in the Black Sea area:

Subset 1 represents 643128 TOPEX/POSEIDON, ERS-1, ERS-2, JASON-1, ENVISAT and GFO Sea Surface Heights (SSH) taken for the period 1992-2005 and corrected by CSL AVISO for different geophysical phenomena and instrumental effects.

Because of data gaps in the corrected SSH (Black Sea region) the following additional set of point gravimetry data is used to support the SSH-only solution for  $\Delta g$ :

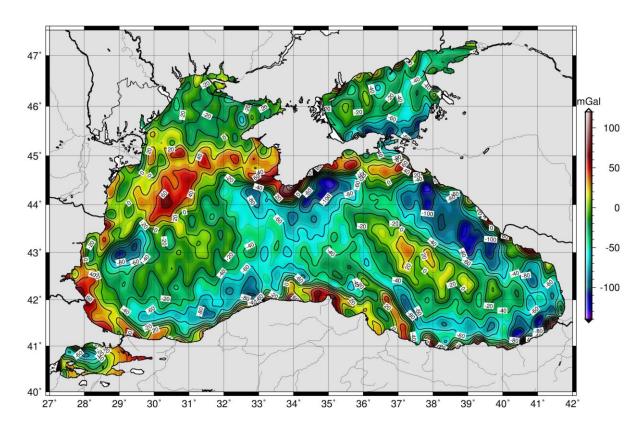
Subset 2 represents 20207 values of BGI and national marine point gravimetry measurements in addition to land gravimetry surrounding the Black Sea area.



**Fig. 1.** AVISO corrected SSH [m] from TOPEX/POSEIDON, ERS1, ERS2, JASON-1, ENVISAT and GFO altimetry, filtered for residual temporal effects

Fig. 1 illustrates AVISO corrected SSH from six satellite missions in the Black Sea area, which were filtered for residual temporal effects (and here also referred to the GRS80 ellipsoid). It has to be pointed out, that the surface in Fig. 1 differs from the geoid due to small deviations of Sea Surface Topography (SST) and various residual effects averaged in time.

Fig. 2 demonstrates the result of processing the above two subsets by regularization method into a  $2'\times2'$  grid of the gravity anomalies. They were obtained by Marchenko and Yarema [Marchenko and Yarema, 2006] and in the following we call them as the Set 1 (marine  $\Delta g$ ).



**Fig. 2.** Gravity anomalies [mGal] from six satellites missions (1992 – 2005) and BGI gravimetry plus national marine gravimetry measurements in the Black Sea area

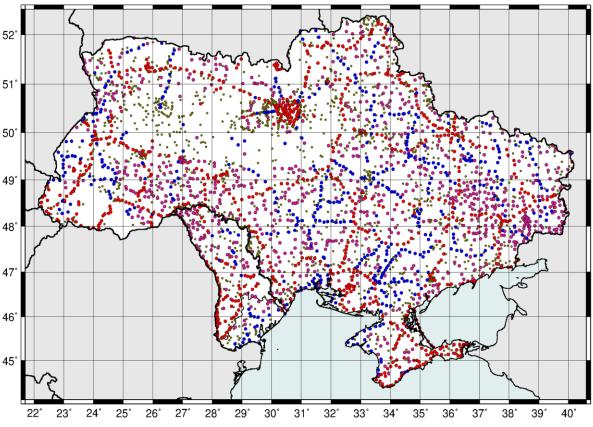


Fig. 3. Distribution of GPS/leveling data of the I (•), II (•), III (•), and IV (•) orders

The second set (Set 2) of free air gravity anomalies at the points of the  $2'\times3'$  grid was derived from the digitized Bouguer anomalies covering the Ukraine and Moldova area. Then classical terrain reduction was applied for the conversion of the free air anomalies  $\Delta g$  to the Faye anomalies  $\Delta g_F$ . This reduction was based on the DTM SRTM3 given for a 3"×3" grid [Jarvis et al., 2006]. Subsequently, the Faye anomalies  $\Delta g_F$  were adopted as a basic initial information for quasigeoid computations. Note that the classical terrain reduction was derived by using the numerical integration up to a radius of 167 km and has maximal values of ca. 50 mGal in the Ukraine area considered.

Finally, Fig. 3 illustrates the Set 3 containing 4070 points of GPS/leveling data of different orders covering the Ukraine and Moldova area. Set 3 was formed after the detection of gross errors in GPS/leveling data given in the Baltic 1977 height system.

### 3. Gravimetric and combined quasigeoid solutions

The traditional remove-restore technique was applied to eliminate the long wavelength constituent of the gravity field. It was represented by the EGM2008 gravity field model [Pavlis et al., 2008] used up to degree and order 720. In the case of gravimetry-only data the residuals  $\delta\Delta g$  were initially computed as

$$\delta \Delta g = \Delta g_F - \Delta g_{EGM2008}, \qquad (1)$$

where  $\Delta g_F$  is the Faye anomaly;  $\Delta g_{\rm EGM2008}$  is the is the  $(1 \times q)$  - cross-covariance matrix between  $\delta \Delta g$  and  $\delta \zeta$ ;  $\mathbf{C}_{\rm nn}$  is the  $(q \times q)$  - covariance matrix of the EGM2008 gravity anomaly up to degree and order 720. Then the prediction of the residual anomaly height  $\delta \zeta_P$  at a point P inside the studying area was made by applying the collocation with regularization

$$\delta \zeta_P = \mathbf{C}_{\mathcal{K}, \delta \Delta g} (\mathbf{C} + \alpha \mathbf{C}_{nn})^{-1} \mathbf{I}, \qquad (2)$$

where in case of gravimetry data,  $\mathbf{l}$  is the q-vector consisting of the components  $\delta\Delta g_i$  (i=1, 2,...q); q is a number of observations;  $\mathbf{C}$  is the ( $q \times q$ ) - covariance matrix of the residual gravity anomalies  $\delta\Delta g$ ;  $\mathbf{C}_{\Delta\delta g,\,\delta\zeta}$  is the cross-covariance matrix;  $\mathbf{C}_{nn}$  is the ( $q \times q$ ) - covariance matrix of measurement noise  $\mathbf{n}$  and  $\alpha$  is the regularization parameter or weight factor constraining the variability of the solution [Neyman, 1979; Moritz, 1980]. Obviously, the collocation method corresponds to  $\alpha$ =1 in Eq. (2).

After applying the regularization (collocation) via Eq. (2), the anomaly heights  $\zeta$  at the chosen grid are restored in the following way

$$\zeta = \zeta_{\text{EGM2008}} + \delta \zeta , \qquad (3)$$

where  $\zeta_{\rm EGM2008}$  is the contribution of the EGM2008 gravity field model to quasigeoid heights to the same order. For further use of Eq. (2), the following two problems have to be solved: (a) building the ACF (analytical covariance function) K(P,Q) of the anomalous potential T; (b) the choice of an appropriate method for the computation of the regularization parameter  $\alpha$ .

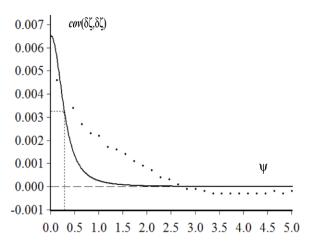
In this study we apply reproducing kernels K(P,Q), which are represented by singular point harmonic functions only [Marchenko and Lelgemann, 1998]:

$$K_{\rm n}(P,Q) = \left[\frac{GM}{R}\right]^2 \beta_{\rm n} \sigma^{\rm n+1} v_{\rm n}, \sigma = \frac{R_{\rm B}^2}{r_{\rm p} r_{\rm O}}, (4)$$

where R is the Earth's mean radius;  $R_{\rm B}$  is the Bjerhammar's sphere radius;  $r_{\rm P}$  and  $r_{\rm Q}$  are the geocentric distances to the external points P and Q; GM is the product of the gravitational constant G and the mass M of the Earth;  $v_{\rm n}$  is the dimensionless potential of a radial multipole of degree n and  $\beta_n$  represents a dimensionless coefficient [Marchenko, 1998; Marchenko et al., 2001]. The traditional determination of the parameter  $\alpha$  in Eq. (2) requires a specific iterative process and the inversion of a matrix with a dimension equal to the number q of observations [Neyman, 1979]. So, when a number of observations is large we come to a time consuming procedure. To avoid this difficulty we may use an estimate

$$\alpha = 1 + \sqrt{1 + \frac{\operatorname{Trace}(\mathbf{CC}_{nn})}{\operatorname{Trace}(\mathbf{C}_{nn}\mathbf{C}_{nn})}}, \quad (5)$$

verified in [Marchenko et al., 2001; Marchenko, 2003] for the inversion of SSH altimetry data into gravity anomalies.



**Fig. 4.** Empirical covariance function (dotted curve) and analytical covariance function (solid curve) of the residuals  $\delta \zeta$  [m<sup>2</sup>],  $\psi$  is the spherical distance [in degree]

Recalling now the processing of BGI gravimetry and satellite altimetry data, in the following we used the Set 1 (i.e. the  $2'\times2'$  grid of gravity anomalies)

together with the Set 2 and the Set 3 of GPS/leveling data for constructing the gravimetric quasigeoid UQG2012 by means of the regularization/collocation method. The empirical covariance function (ECF) was computed on the basis of the residual gravity anomalies  $\delta \Delta g$ . It was then approximated by some specific reproducing kernels or analytical covariance functions ACF, derived from radial multipole potentials by the Kelvin transformation [Marchenko, 1998], that provide the covariance propagation in  ${\bf R}^3$  to geoid heights and other functionals of the anomalous potential. The modified Poisson kernel without zero degree harmonics showed the best fit to ECF [Marchenko and Lelgemann, 1998]. Therefore, it was selected (Fig. 4).

The comparison of the UQG2012 solution with independent GPS/leveling control points demonstrates a good agreement with St. Dev. < 4 cm. It is worth mentioning, that only differences between GPS/leveling data of 1<sup>st</sup>, 2<sup>nd</sup>, and 3<sup>rd</sup> orders and UQG2012 correspond to the same level of agreement (5 cm in terms of r.m.s deviation) as in the case of independent GPS/leveling control points (Table 1).

Table 1

#### Differences between GPS/leveling points and UQG2012

	GPS/leveling points				
Statistics (cm)	I order	II order	III order	All points	
Min	-8.1	-6.7	-17.9	-22.9	
Max	6.2	6.8	17.6	23.6	
Mean	-0.3	-0.1	0.9	0.1	
Std. dev.	1.4	1.6	5.5	5.4	

Table 2

### Differences between GPS/leveling points and EGM2008

	GPS/leveling points				
Statistics (cm)	I order	II order	III order	All points	
Min	-38.0	-30.3	-28.0	-38.0	
Max	33.4	25.7	25.6	33.4	
Mean	1.7	1.6	2.1	1.1	
Std. dev.	8.6	7.5	8.5	8.7	

An additional evaluation of the UQG2012 solution was deduced from its comparison with the recent high-resolution gravity field model EGM2008 up to degree 2190. For proper comparison a simplest datum shift transformation was applied, statistics are given in Table 2. Major differences are observed in

the Black Sea basin, Carpathians and Crimea area with values greater than 50 cm, together with smaller differences in a few scattered zones. Nevertheless, the deviation between GPS/leveling points of 1st and 2nd orders and the UQG2012 quasigeoid in the Carpathian and Crimean regions are smaller than  $\pm 10$  cm.

Table 3

	GPS/leveling points				
Statistics (cm)	I order	II order	III order	All points	
Min	-4.7	-0.6	-3.7	-7.9	
Max	54.3	51.8	55.7	55.7	
Mean	25.6	26.0	26.8	25.5	
Std. dev.	7.9	7.6	9.3	8.7	

### Differences between GPS/leveling points and EGG08

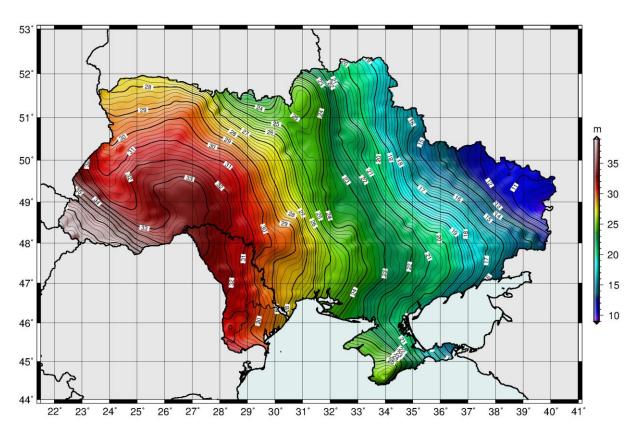


Fig. 5. Heights [m] of the UQG2012 quasigeoid solution

Another evaluation rests on the comparison with the EGG08 geoid model [Denker et al., 2007]. In this connection we have to note that the EGG08 quasigeoid is referred to the Amsterdam tide gauge. Therefore, one can observe a shift of 25 cm concerning GPS/leveling control points which are referred to the Baltic 1977 system (Table 3). The resulting quasigeoid solution UQG2012 for the Ukraine and Moldova area is shown in the Fig.5.

### Conclusions

Considering the results discussed above we can formulate the following conclusions.

• The application of the regularization method to updated satellite altimetry, gravimetry, terrain and GPS/leveling data leads to a significantly improved quasigeoid solution UQG2012 for the Ukraine and Moldova area.

- A comparison of the UQG2012 model with independent GPS/leveling data given in the Baltic height system shows a good agreement with standard deviation < 4 cm. This noise level corresponds to an expected quasigeoid accuracy of about 5 cm for the Ukraine and Moldova area.
- In contrast to previous geoid models the combined UQG2012 solution provides considerably better agreement with the independent GPS/leveling control stations since Std. Dev. < 4 cm represents an improvement by 50%. This progress can be partly explained by the application of the 3"×3" DTM SRTM3 (instead of the 1'×1' DTM GEBCO) for terrain reductions.
- A comparison of the UQG2012 quasigeoid with the global gravity field model EGM2008 up to degree 2190 leads to an agreement within  $\pm 10$  cm (in terms of standard deviation) and differences having values greater than 50 cm in the Black Sea area, and Carpathian and Crimean Mountains.

#### References

- Denker, H., Torge W. The European Gravimetric Quasigeoid EGG97, An IAG supported continental enterprise. In: IAG Symposium "Geodesy on the Move – Gravity Geoid, Geodynamics, and Antarctica". Proceedings, 1998, Vol. 119, Springer Verlag. pp. 249-254.
- Denker H., J.-P. Barriot, R. Barzaghi, D. Fairhead, R. Forsberg, J. Ihde, A. Kenyeres, U. Marti, M. Sarrailh and I. N. Tziavos (2007) *The Development of the European Gravimetric Geoid Model EGG07* M.G.Sideris (ed.), Observing our Changing Earth, Int. Assoc. of Geodesy, Symp. 132. pp. 177-186, Springer Verlag, Berlin-Heidelberg 2009.
- Ihde J., Adam J., Gurtner W., Harsson B. G., Sacher M., Schlüter W., Wöppelmann G. *The Height Solution of the European Vertical Reference Network (EUVN)*. Veröffentlichungen der Bayerischen Kommission für die Internationale Erdmessung der Bayerischen Akademie der Wissenschaften, Astronomisch-Geodatische Arbeiten, München, Heft Nr. 61 (IAG/EUREF Publication No. 9, Ed. by J.A. Torres and H. Hornik). 2000, pp. 132-145.
- Jarvis A., H. I. Reuter, A. Nelson, E. Guevara. Hole-filled seamless SRTM data V3, International Centre for Tropical Agriculture (CIAT), 2006, available from http://srtm.csi.cgiar.org.
- Marchenko, A. N. Parameterization of the Earth's Gravity Field: Point and Line Singularities. Lviv Astronomical and Geodetic Society, Lviv, 1998.
- Marchenko A. Regional gravity field model from satellite altimetry, gravimetry and GPS-leveling data in the Black sea. Bolletino di Geodesia e Scienze Affini. ANNO LXII, No 4, 2003, pp. 246-259.
- Marchenko A. N., Barthelmes F., Meyer U., Schwintzer P. Regional geoid determination: An

- application to airborne gravity data in the Skagerrak, GeoForschungsZentrum Potsdam, Scientific Technical Report, STR01/07, 2001
- Marchenko A. N., O. Kucher, O., Renkevych Determination of the UQG2006 quasigeoid in the Ukraine area. Geodesy and Cartography, Kyiv, No 2, 2007. pp. 3-13 (in Ukrainian).
- Marchenko A. N., Lelgemann D. A classification of reproducing kernels according to their functional and physical significance, IGeS Bulletin. Milan, 1998, No 8, pp. 49-52.
- Marchenko A. N., Monin I. I. Regional Quasigeoid Solution for the Moldova Area from GPS/levelling data, Report on the Symposium of the IAG Subcomission for Europe (EUREF), Bratislava, 2-5 June 2004, Mitteilungen des Bundesamtes für Kartographie und Geodäsie, Band35, Frankfurtn am Main, 2005 pp. 297-301.
- Marchenko A., Kucher O. *On the quasigeoid solutions for the Ukraine and Moldova area*. Paper presented at the EUREF Symposium 2008, Brussels, Belgium, June, 2008. (in press), available in: http://www.euref.eu/symposia/2008Brussels/Symposium2008-Brussels.html.
- Marchenko A. N., Yarema N. P. Determination of the mean gravity potential  $W_0$  and its time-variations, Geodesy and Cartography, Kyiv, No 6, 2006, pp. 2-9 (in Ukrainian).
- Moritz, H. Advanced Physical Geodesy. Wichmann, Karlsruhe, 1980.
- Neyman, Yr. Variational method of Physical Geodesy. Nedra, Moscow, (in Russian), 1979.
- Pavlis N. K, Holmes S. A, Kenyon S. C, Factor J. K. An Earth Gravitational Model to Degree 2160: EGM2008, Paper presented at the EGU General Assembly 2008, Vienna, Austria, April, 2008.

### О. М. МАРЧЕНКО, О. В. КУЧЕР, Д. О. МАРЧЕНКО

- <sup>1</sup> Національний університет "Львівська політехніка", Інститут геодезії, Львів, вул. Карпінського,6, Україна, ел. пошта: march@pancha.lviv.ua, тел. +380509801609
- <sup>2</sup> Науково-дослідний інститут геодезії і картографії, Київ, вул. Велика Васильківська, 69, Україна, ел. пошта: kucher@gki.com.ua

### РЕГІОНАЛЬНІ РОЗВ'ЯЗКИ КВАЗІГЕОЇДА ДЛЯ РЕГІОНУ УКРАЇНИ

#### Анотація.

**Мета**. Розв'язок UQG2012 регіонального квазігеоїда точністю краще, ніж 2 см по відношенню до даних GPS-нівелювання 1-го і 2-го порядку був побудований за допомогою методу середньої квадратичної колокації. У першій ітерації гравіметричний квазігеоїд UQG2011 був обчислений за даними аномалій Фая для подальшого виявлення грубих помилок в даних GPS-нівелювання. Редукція за рельеф обчислена на основі 3"х3" цифрової моделі місцевості SRTM3.

**Наукова новизна та практична цінність.** Таким чином, остаточне рішення складається з UQG2012 аномалій сили тяжіння і квазігеоїда висот у вузлах 2'×3' рівномірної сітки, оцінених за допомогою методу колокації, застосованого до набору 4070 GPS-визначених висот квазігеоїда, плюс зазначених

<sup>&</sup>lt;sup>3</sup> Карпатське відділення Інституту геофізики імю Субботіна НАН України, вул. Наукова, 3-б, м. Львів, Україна, ел. пошта: dmitriy.marchenko@gmail.com

вище гравіметричних даних. Порівняння квазігеоїда UQG2012 з усіма незалежними даними GPSнівелювання (заданих в системі Балтійської 1977 висот) показує гарну згоду з RMS <4 см. Цей рівень шуму відповідає розрахунковій точності квазігеоїда UQG2012 для України та Молдови, ніж 4-5 см по відношенню до GPS-нівелювання точок різних класів. Оцінка рішення UQG2012 з незалежними даними GPS-нівелювання 1 і 2 класів дає значно кращу згоду з середньоквадратичним відхиленням близько 1,5 см. Нарешті, порівняння з Європейським квазігеоїдом EGG08 призводить до відмінностей близько 20-50 см в деяких районах і в загальному до середнього зсуву 25 см, викликаних різними системами висот, що використовуються.

Ключові слова: Регіональний квазігеоїд; гравіметрія; альтиметрія; ГНСС/нівелювання.

### А. Н. МАРЧЕНКО, А. В. КУЧЕР, Д. А. МАРЧЕНКО

- <sup>1</sup> Национальный университет "Львовская политехника", Институт геодезии, Львов, ул. Карпинского, 6, Украина, эл. почта: march@pancha.lviv.ua, тел. +380509801609
- <sup>2</sup> Научно-исследовательский институт геодезии и картографии, Киев, ул. Большая Васильковская, 69, Украина, эл. почта: kucher@gki.com.ua
- <sup>3</sup> Карпатское отделение Института геофизики им. Субботина НАН Украины, ул. Научная, 3-б, Львов, Украина, эл. почта: dmitriy.marchenko@gmail.com

### РЕГИОНАЛЬНЫЕ РЕШЕНИЯ КВАЗИГЕОИДА ДЛЯ РЕГИОНА УКРАИНЫ

#### Аннотация.

**Цель.** Решение UQG2012 регионального квазигеоида с точностью лучше, чем 2 см по отношению к данным GPS-нивелирования 1-го и 2-го порядка было построено с помощью метода средней квадратичной коллокации. В первой итерации гравиметрический квазигеоид UQG2011 был вычислен по данным аномалий Фая для дальнейшего выявления грубых ошибок в данных GPS-нивелирования. Редукция за рельеф основана на 3"х3"цифровой модели местности SRTM3.

Научная новизна и практическая ценность. Таким образом, окончательное решение состоит из UQG2012 аномалий силы тяжести и квазигеоида высот в узлах 2'×3' равномерной сетки, оцененных с помощью метода коллокации, примененного к набору 4070 пунктов GPS-нивелирования высоты квазигеоида, плюс указанных выше гравиметрических данных. Сравнение квазигеоида UQG2012 со всеми независимыми данными GPS-нивелирования (заданных в системе Балтийской +1977 высот) показывает хорошее согласие с RMS <4 см. Этот уровень шума соответствует расчетной точности квазигеоида UQG2012 для Украины и Молдовы, чем 4-5 см по отношению к GPS-нивелированию точек разных классов. Оценка решения UQG2012 с независимыми данными GPS-нивелирования 1 и 2 классов дает значительно лучшее согласие с среднеквадратичным отклонением около 1,5 см. Наконец, сравнение с Европейским квазигеоидом EGG08 приводит к различиям около 20-50 см в некоторых районах и в общем к среднему сдвигу 25 см, вызванных различными системами, используемых высот.

Ключевые слова: Региональный квазигеоид; гравиметрия; альтиметрия; ГНСС / нивелирование.