A new high-resolution gravimetric geoid for South Spain and the Gibraltar Strait area: SOSGIS

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Abstract. A new gravimetric geoid is computed for South Spain and the Gibraltar Strait area. This geoid is located just in the junction between two tectonic plates (Euro-Asiatic and African plates) and in the junction of two gravimetric geoids: IGG2005 (the Iberian Gravimetric Geoid obtained in 2005) and MORGEO (the MORoccan GEOid). IGG2005 is the Iberian geoid and MORGEO is the Moroccan geoid, both geoids have been previously obtained. The new geoid is the gravimetric geoid solution that connects the two above-mentioned geoids, getting a more accurate and reliable picture of this area than the other previous geoids. The method used is the Stokes integral in convolution form, which shows to be an efficient method to reach the proposed objective. The terrain correction and the indirect effect have been taken into account. The new geoid is obtained as a regular grid (with a mesh size of 1.5'x1.5') in the GRS80 reference system, covering the study area from 34 to 40 degrees of latitude and from –8 to 0 degrees of longitude. This gravimetric geoid and the previous geoids: IGG2005 and MORGEO; are compared to the geoid undulations derived at the validation points located on the study area (4 GPS/levelling points measured on Morocco and 5 points of the European Vertical Reference Network (EUVN) measured on Iberia). As it is expected, the new geoid is a more precise and reliable model, fitting the geoidal heights of these validation points with more accuracy than the other previous geoids. This new model will be useful for orthometric height determination by GPS in the mountains and remote areas, where levelling has many logistic problems. Also, it can be interesting for other geophysical purposes different to the height measurements, because it can provide a constraint for the density distribution, the thermal state of Lithosphere and the viscosity in the mantle. Such details can be inferred from a geoid model and the seismic velocity structure.

Keywords: Gravity, Geoid, FFT, GPS/Levelling, EUVN, Gibraltar strait, South Spain.

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1. Introduction

In South Spain and the Gibraltar Strait area, two previous studies have taken as objective the geoid computation (Corchete et al., 2005; Corchete et al., 2007). IGG2005, calculated by Corchete et al. (2005), is provided as a data grid in the GRS80 reference system covering Iberia from 35 to 44 degrees of latitude and –10 to 4 degrees of longitude, in a 361x561 regular grid with a mesh size of 1.5’x1.5’. MORGEO, calculated by Corchete et al. (2007), is provided as a data grid in the GRS80 reference system covering Morocco from 27 to 37 degrees of latitude and –14 to 0 degrees of longitude, in a 401x561 regular grid with a mesh size of 1.5’x1.5’. Unfortunately, these previous geoids only cover partially the studied area (south Spain and Gibraltar Strait) that corresponds to the junction between two plates. Furthermore, since the publication of IGG2005 and MORGEO, a new geopotential model (EIGEN-GL04C) is available. Logically, this new global model represents an improvement that must be included in a new geoid for this interesting area.

This new geoid covers completely South Spain and the Gibraltar Strait area and it is extended from 34 to 40 degrees of latitude and –8 to 0 degrees of longitude. This model is provided as a data grid in the GRS80 reference system, in a 241x321 regular grid with a mesh size of 1.5’x1.5’. The Stokes integral in convolution form (Haagmans et al., 1993), is used to obtain this new geoid. The necessary terrain correction is applied to obtain the reduced gravity anomalies and the corresponding indirect effect is taken into account (Schwarz et. al., 1990; Sideris, 1990). Finally, the computed geoid for SOuth Spain and the GIbraltar Strait area (SOSGIS) is compared to IGG2005 and MORGEO, to demonstrate the improvement in precision and reliability attained by this new geoid.

2. Data set

For the gravimetric geoid computation the data sets needed are: (1) free-air gravity anomalies; (2) a geopotential model; (3) a high-precision DTM; and (4) observed geoid undulations. The data sets used for the computation of the SOSGIS geoid are detailed below.

Land and marine gravity data set. The land and marine gravity data used in this study has been provided in by the National Geophysical Data Center (NGDC), the Bureau Gravimetrique International (BGI) and the United States Geological Survey (USGS). NGDC contributes with a data set consisting of 15266 points (all over Iberia), the BGI data set in the study area has 49992 points (8639 terrestrial and 41353 sea-gravity points) and USGS provides 21822 points in Morocco and the Gibraltar strait area. The whole data set consists of 87080 points of free-air gravity anomalies (45727 terrestrial and 41353 sea-gravity points) distributed, in the study area, from 34 to 40 degrees on latitude and -8 to 0 degrees on longitude. The accuracy of these data ranges from 0.1 to 0.2 mgal. The compiled gravity data have been checked to remove repeated points, letting 85245 points distributed over the study area as it is shown in Figure 1. The data window is taken so that it excludes as much as possible the zones of Iberia and Africa where the gravity measurements are scarce. Thus, the loss of accuracy occurred in the computation of the previous geoids (IGG2005 and

MORGEO), due to the scarcity of gravity data for countries as Portugal and Algeria, has been avoided. All the data have been converted into the GRS80 reference system and the atmospheric correction has been taken into account (Wichiencharoen, 1982; Kuroishi, 1995).

Geopotential model. The EIGEN-GL04C model (Förste et al., 2006) is an upgrade of the EIGEN-CG03C model (Förste et al., 2005). This model is a combination of the GRACE (Gravity Recovery and Climate Experiment) and LAGEOS (LAser GEOdynamics Satellite) mission solution adding a 0.5 x 0.5 degrees gravimetry and altimetry surface data. The surface data are identical to EIGEN-CG03C set except for the geoid undulations over the oceans. The EIGEN-GL04C geopotential model represents a major advance in the modelling of the Earth's gravity and geoid. Thereby, this model is the geopotential model that must be used for the computation of the long-wave contribution to the geoid and the gravity anomaly, to obtain a high-precision geoid in the study area.

Digital terrain model (DTM). Any gravimetric geoid computation based on the Stokes's integral must use anomalies that have been reduced to the geoid, usually by means of the Helmert's second method of condensation (Heiskanen and Moritz, 1967). This involves the computation of the terrain correction and the indirect effect on the geoid, which are computed from a DTM. A DTM is also necessary to compute the Residual Terrain Model reduction (RTM reduction) for the point anomalies in order to obtain smooth gravity anomalies, which are more easily gridded. For the present study, a new elevation model for the whole study area, with a 3''x 3'' spacing, is obtained from the Shuttle Radar Topography Mission (SRTM) elevation data and the ETOPO2 bathymetry data, following the process described by Corchete et al. (2005). To minimize the loss of accuracy associated to the low resolution of the ETOPO2 bathymetry, the data window is selected so that it includes as few marine data as possible.

GPS/levelling and EUVN points used as a control data set. To test the accuracy of our model we compare its undulations with the GPS/levelling and the EUVN data shown in Table 1. Points 1 to 5 have been supplied by a previous study developed in Iberia by Corchete et al. (2005). Points 6 to 9 have been observed using GPS/levelling in the Gibraltar strait area by Benaim (1991). Figure 2 shows the GPS/levelling and EUVN points available for testing of the geoid model.

3. Methodology and processing

In the present study, the computation method of Corchete et al. (2005) is applied for the calculation of the new gravimetric geoid. In this paper, only a brief review of the principal concepts of this methodology is presented.

Gravity data gridding. Since the gravity data set consists of point data anomalies distributed randomly, an interpolation process should be applied to obtain a regular data grid. Before the interpolation, it is very suitable to remove the short-wavelength and the long-wavelength effects applying the well-known relationship (the RTM correction)

$$\Delta g_{red}^{pts} = \Delta g_{free}^{pts} - 2\pi G\rho(h - h_{ref})^{pts} + c^{pts} - \Delta g_{GM}^{pts} \quad (1)$$

where the superscript *pts* denotes each point randomly distributed over the study area, Δg_{free} is the free-air gravity anomaly, *G* is the Newton's gravitational constant, ρ is the density of the topography (2.67 g/cm^3) for the RTM correction on land or the density of the topography minus seawater density (2.67 – 1.03 = 1.64 g/cm^3) for marine RTM, *h* is the elevation, h_{ref} denotes the elevation of the reference surface (this reference surface is obtained by applying of a 2D low-pass filter with a resolution of 60', to the elevations field), *c* is the terrain correction computed for land and marine points, and Δg_{GM} is the gravity anomaly computed from the geopotential model EIGEN-GL04C.

After the smoothing procedure given by (1), some erroneous values could be detected. These spurious outliers are removed when the gravity anomalies given by (1) have an absolute value larger than 50 mgal. Thus, 1523 points from the total data set (consisting of 85245 points) have been removed, letting 83722 points for the interpolation in a regular grid. This regular grid is calculated by using Kriging-based routines, which are part of OriginLab software package (© 1991-2003 OriginLab Corporation). The gridded data are distributed over the study area from 34 to 40 degrees of latitude and –8 to 0 degrees of longitude, in a 241x321 regular grid with a mesh size of 1.5'x1.5' and 77361 points. Finally, RTM must be restored in the gridded anomalies to obtain the true free-air anomalies relative to EIGEN-GL04C. This RTM effect can be restored by

$$\Delta g_{free}^{grid} = \Delta g_{red}^{grid} + 2\pi G\rho(h - h_{ref})^{grid} - c^{grid} \quad (2)$$

where the superscript *grid* denotes each point of the regular grid considered (241 x 321 = 77361 points), Δg_{free} is the free-air gravity anomaly, Δg_{red} is the gravity anomaly reduced by (1) and gridded. It should be noted that $2\pi G\rho(h - h_{ref})$ and *c* are computed in the same way as before, but now over a regular grid of points.

Geoid computation. The new geoid is computed by the classical remove-restore technique. Following this method, the geoid model is obtained by the sum of three terms

$$N = N_1 + N_2 + N_3 \quad (3)$$

The first term N_1 is the global field contribution to the geoid undulation. This term can be computed from a spherical harmonic expansion (Heiskanen and Moritz, 1967; Corchete et al., 2005). Figure 3a shows the results obtained by application of such spherical harmonic expansion (the maximum degree considered is 360), to the grid points of the study area. The second term N_2 is the indirect effect of Helmert's second method of condensation reduction on the geoid. N_2 consists of two terms in planar approximation (Sideris, 1990). This planar approximation can be easily written in convolution form (Schwarz et al., 1990) and computed by a FFT procedure, obtaining the result shown in Figure 3b. The third term N_3 is the contribution of the residual gravity. This term can be obtained by means of the Stokes integral (Heiskanen and Moritz, 1967) written in convolution form by using 1D FFT (Haagmans et al., 1993). The result of this integration procedure is shown in Figure 3c.

The gravimetric geoid solution (SOSGIS geoid) is reached by the sum of all previously described terms according to (3). This geoid with a mesh size of 1.5’x1.5’ and extended 6x8 degrees over the study area, is shown in Figure 4. In this Figure, it should be noted that the contour lines are more rugged in areas with high mountains, because of the indirect effect (term N_2 of the equation (3)). Corchete et al. (2006) have shown that a global geoid model fails in mountainous area where gravimetric geoid models are more reliable and accurate. This new model and a simple FORTRAN program for PC can be obtained from the internet address http://airy.ual.es/www/SOSGIS_english.htm. This computer program allows the computation of the geoid height (using this geoid model) in any point over the study area shown in Figure 4.

Geoid validation. The geoid undulations predicted by other previous geoids available for this area (IGG2005 and MORGEO) are compared with the undulations predicted by the SOSGIS geoid. Table 2 shows the geoid undulations predicted by the geoids: SOSGIS, MORGEO and IGG2005, for the validation points: 4 GPS/levelling and 5 EUVN points. Table 3 shows the differences obtained subtracting the geoid heights of the validation points to the heights predicted by the above-mentioned geoids. The statistics of these differences is shown in Table 4. The new geoid SOSGIS shows an improvement in precision and reliability with respect to the other previous geoids, fitting the geoidal heights of the validation points better than any other geoid, as it can be seen in Table 4.

4. Conclusions

The computation methods based on the FFT analysis have allowed us to compute a new gravimetric geoid for South Spain and the Gibraltar Strait area, which is a major advance in the modelling of the geoid for this region. This gravimetric geoid is calculated by means of the Stokes integral in convolution form, obtaining a regularly gridded geoid of 241x321 points (77361 points) in the GRS80 reference system, with a mesh size of 1.5’x1.5’, distributed over the study area from 34 to 40 degrees of latitude and –8 to 0 degrees of longitude. The new geoid shows a rms discrepancy of 17 cm with the geoid undulations of the 9 validation points, which is 30% and 50% better than the MORGEO and IGG2005 models (see Table 4). Nevertheless, this geoid is only a first step in the way to obtaining a geoid with centimetric precision. The centimeter accuracy can be approached when new gravity data (at some zones with scarce gravity data coverage) and a high-resolution bathymetry will be available for South Spain and the Gibraltar Strait area. In spite of this, the SOSGIS geoid has better accuracy than any previous geoid solutions obtained in Iberia or Morocco. This new model will be useful for orthometric height determination by GPS, because it allows the orthometric height determination in the mountains and remote areas, where levelling has many logistic problems. Furthermore, this new model can be interesting for several geophysical purposes, because it can provide a constraint for the density distribution and the thermal state of Lithosphere, through some elementary relations between the mass distributions inside Earth and the geoid. The geoid also can be used to infer details of the viscosity structure in the top 1000 km of the mantle, joint to a tomographic structure beneath the study area (Ricard et al., 1984; Ricard et al., 2006). A combination of temperature profiles and viscosity in the mantle can be inferred from the geoid and the seismic velocity structure. On the other hand, previous studies developed in different areas of Earth have shown the relation between plate motions and undulations in the geoid. In these studies, it has been demonstrated that the geoid exhibit lineated anomalies, trending in the direction of the absolute plate motion. So, the new geoid obtained is a major contribution for the knowledge of Geophysics in the junction between the Euro-Asiatic and African plates.

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Heading for tables

Table 1. GPS/levelling and EUVN points used as control data set.

Table 2. Geoid heights predicted by the available geoids over the study area, for the 9 validation points listed in Table 1.

Table 3. Differences between the geoid heights predicted by the available geoids and the geoid heights listed in Table 1.

Table 4. Statistics of the differences listed in Table 3.

Figure captions

Figure 1. Geographical distribution of the observed free-air gravity data over the study area (85245 points).

Figure 2. Geographical distribution of the GPS/levelling and the EUVN points used as control data set (triangles).

Figure 3. (a) The EIGEN-GL04C geoid model computed for the study area. The contour interval is 1.0 m. (b) The indirect effect on the geoid (plotted positive). The grey scale is non-linear. (c) The residual geoid undulation.

Figure 4. The geoid for SOuth Spain and the GIbraltar Strait area (SOSGIS) as a sum of the terms given by the equation (3). The contour interval is 1.0 m.

Table 1.

Point (n.)	Latitude (ºN)	Longitude (ºE)	h (m)	H (m)	N = h-H (m)
1	38.87886596	-7.05170516	229.938	174.929	55.009
2	36.46434412	-6.20564687	84.180	38.940	45.240
3	38.68675554	-4.11048923	763.215	709.899	53.316
4	36.85247066	-2.45921661	125.046	74.248	50.798
5	38.33891914	-0.48123106	60.345	9.998	50.347
6	35.78836458	-5.90952502	360.060	318.890	41.170
7	35.82213664	-5.67800456	287.030	245.880	41.150
8	35.84421129	-5.56325737	116.160	74.740	41.420
9	35.90087486	-5.46126042	396.770	355.090	41.680

(h = ellipsoidal height, H = orthometric height and N = geoid height)

Table 2.

Point (n.)	Latitude (ºN)	Longitude (ºE)	SOSGIS (m)	MORGEO (m)	IGG2005 (m)
1	38.87886596	-7.05170516	55.091	--	55.163
2	36.46434412	-6.20564687	45.235	44.860	45.210
3	38.68675554	-4.11048923	53.520	--	53.773
4	36.85247066	-2.45921661	50.551	50.516	50.465
5	38.33891914	-0.48123106	50.298	--	50.469
6	35.78836458	-5.90952502	41.324	41.320	41.569
7	35.82213664	-5.67800456	41.405	41.422	41.783
8	35.84421129	-5.56325737	41.649	41.573	41.997
9	35.90087486	-5.46126042	41.931	41.838	42.185

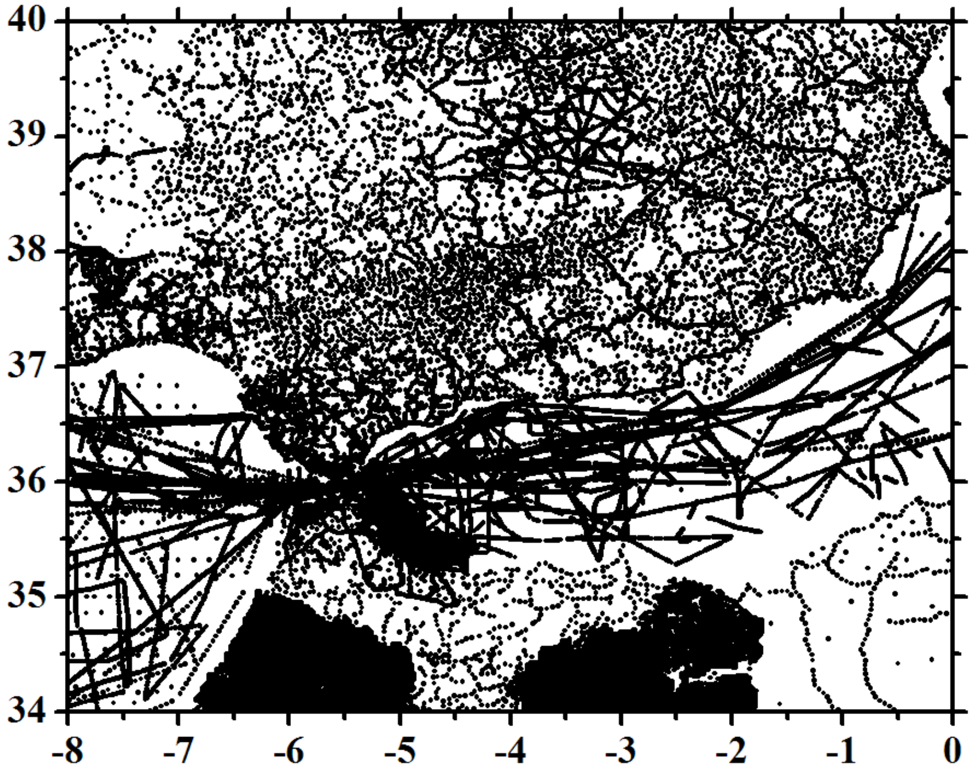
(MORGEO = Corchete et al. (2007); IGG2005 = Corchete et al. (2005))

Table 3.

Point (n.)	SOSGIS - N (m)	MORGEO - N (m)	IGG2005 - N (m)
1	0.082	--	0.154
2	-0.005	-0.380	-0.030
3	0.204	--	0.457
4	-0.247	-0.282	-0.333
5	-0.049	--	0.122
6	0.154	0.150	0.399
7	0.255	0.272	0.633
8	0.229	0.153	0.577
9	0.251	0.158	0.505

Table 4.

Differences	Mean (m)	Std. dev. (m)
SOSGIS - N	0.097	0.170
MORGEO - N	0.012	0.271
IGG2005 - N	0.276	0.320



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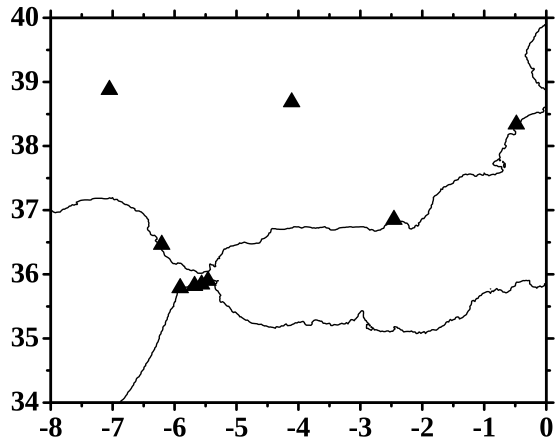

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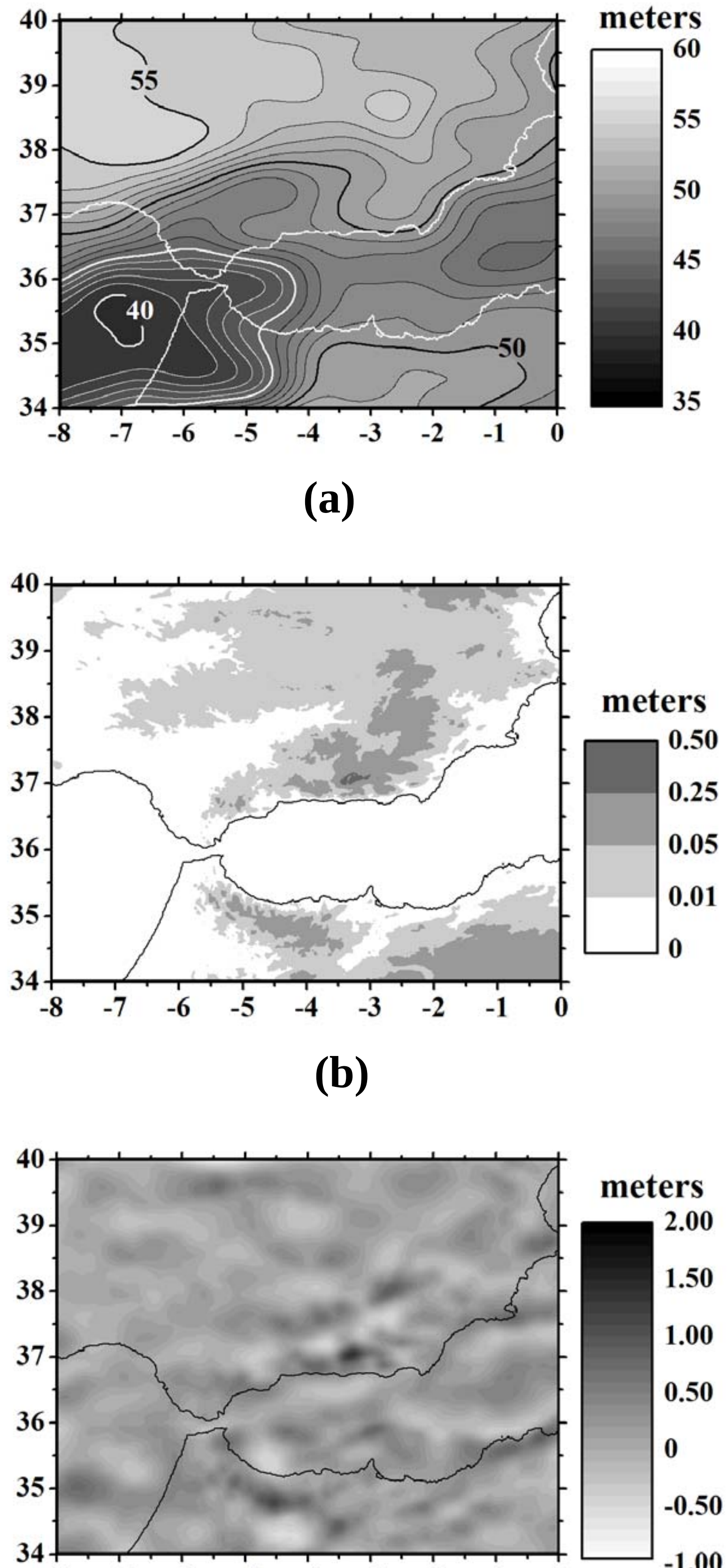
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Fig. 3.

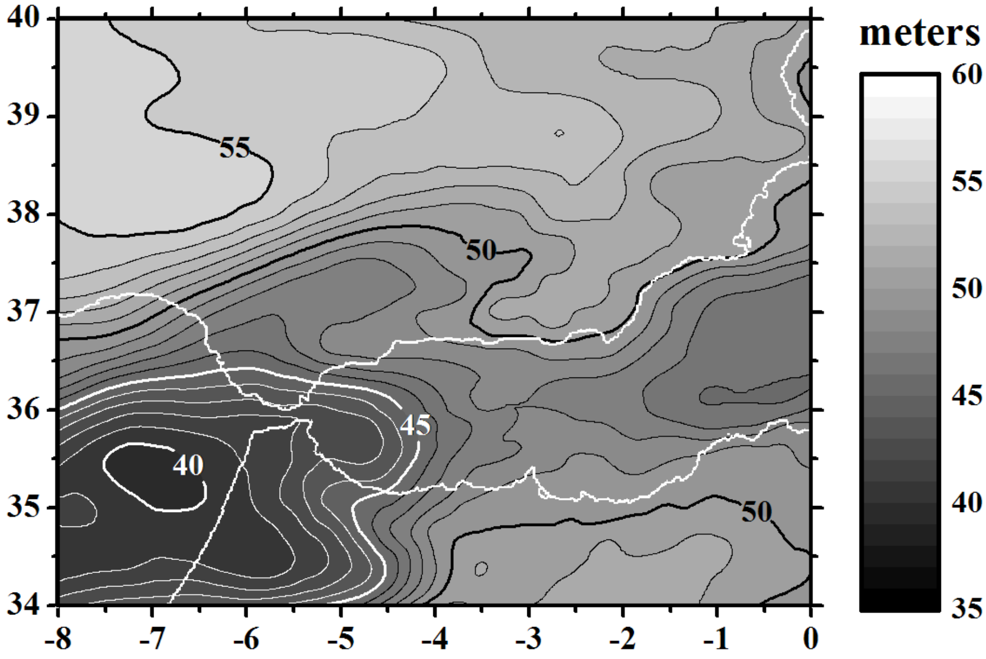
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Fig. 4.