The high-resolution gravimetric geoid of North Iberia: NIBGEO

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Abstract. The gravimetric geoid computed in the northern part of Iberia, is presented in this paper. This computation has been performed considering two study windows fitted to the areas with higher density of gravity data, to reduce the computation errors associated to the scarcity of gravity data, as much as possible. The bad influence of a bathymetry with poorer resolution than the topography is also reduced considering the smallest marine area possible. Moreover, the computation of this gravimetric model is based on the most recent geopotential model: EIGEN-GL04C (obtained in 2006). The method used in the computation of the new gravimetric geoid has been the Stokes integral in convolution form. The terrain correction has been applied to the gridded gravity anomalies, to obtain the corresponding reduced anomalies. Also the indirect effect has been taken into account. Thus, a new geoid model has been calculated and it is provided as a data grid in the GRS80 (Geodetic Reference System of 1980), distributed for the northern part of Iberia from 40 to 44 degrees of latitude and –10 to 4 degrees of longitude, on a 161x561 regular grid with a mesh size of 1.5’x1.5’. This new geoid and the previous geoid IGG2005 (Iberian Gravimetric Geoid 2005), are compared with the geoid undulations measured for 8 points of the European Vertical Reference Network (EUVN) on Iberia. The new geoid shows an improvement in precision and reliability, fitting the geoidal heights of these EUVN points with more accuracy than the previous geoid. Moreover, this new geoid has a smaller standard deviation (12.6 cm) than that obtained by any previous geoid developed for the Iberian area up to date. This geoid obtained for the northern part of Iberia will complement the previously obtained geoid for South Spain and the Gibraltar Strait area (SOSGIS), both geoids jointly will give a complete picture of the geoid for Spain and the Gibraltar Strait area. This new model will be useful for orthometric height determination by GPS over this study area, because it will allow orthometric height determination in the mountains and remote areas, in which levelling has many logistic problems. This new model contributes to our knowledge of the geoid, but the surrounding areas must be better known to constrain the lithospheric and mantle models.

Keywords: Gravity, Geoid, FFT, EUVN, North Spain.

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1. Introduction

In the northern part of Iberia, a previous study has taken as objective the geoid computation (Corchete et al., 2005). In this study, the IGG2005 geoid was calculated. This geoid was obtained as a data grid in the GRS80 reference system, distributed for the Iberian area from 35 to 44 degrees of latitude and –10 to 4 degrees of longitude and provided as a 361x561 regular grid with a mesh size of 1.5’x1.5’. Nevertheless, the bad influence of a bathymetry with much less resolution than the topography and the scarcity of gravity data in some areas of Iberia (like Portugal), gave as consequence a geoid with a poor accuracy (27.8 cm of standard deviation). On the other hand, the computation of IGG2005 is based on the EIGEN-CG01C geopotential model, but since the publication of IGG2005 a new geopotential model (EIGEN-GL04C) is available. Moreover, the new ETOPO1 (global relief model with topography and bathymetry) has updated the previous relief model ETOPO2 used in the computation of IGG2005. Logically, these new models (EIGEN-GL04C and ETOPO1) represent improvements that must be included in any new geoid to be computed now.

Thus, it would be very desirable to obtain a more accurate geoid solution for the northern part of Iberia based on these new models, considering study windows fitted to the areas with higher density of gravity data, to reduce the computation errors associated to the scarcity of gravity data, as much as possible. Also, the bad influence of a bathymetry, with poorer resolution than the topography, would be reduced considering the smallest marine area possible. This new geoid will be very desirable because it will complement the previously obtained geoid for SOuth Spain and the GIbraltar Strait area (SOSGIS). SOSGIS (Corchete et al., 2008) as this latter is also calculated in the GRS80 reference system and covers an area from 34 to 40 degrees of latitude and –8 to 0 degrees of longitude, i.e. it is located immediately south of the present study area. Thus, the new geoid and SOSGIS jointly will give a complete picture of the geoid for the whole area of Spain and the Gibraltar Strait, with more precision than the previous geoid IGG2005.

The new geoid will be computed as a 161x561 regular data grid in the GRS80 reference system, with a mesh size of 1.5’x1.5’, completing the picture of the geoid for Spain from 40 to 44 degrees of latitude and –10 to 4 degrees of longitude. This new geoid will be computed using the Stokes integral in convolution form. The necessary terrain correction will be applied to obtain the gridded reduced gravity anomalies. The corresponding indirect effect will be taken into account. After the computation of this North IBerian GEOid (NIBGEO), it will be compared with the IGG2005 geoid, to demonstrate the improvement in precision and reliability attained by the new geoid.

2. Data set

For the gravimetric geoid computation the necessary data sets are: (1) free-air gravity anomalies; (2) a geopotential model; (3) a high precision Digital Terrain Model (DTM); and (4) observed geoid undulations. The data sets used for computation of the North IBerian GEOid (NIBGEO) are detailed below.

Land and marine gravity data bank. The land and marine gravity data used in this study has been provided by the National Geophysical Data Center (NGDC) and the Bureau Gravimetrique International (BGI). NGDC contributed with a data set consisting of 15266 points distributed over Iberia. The BGI data set has 39892 points in the study area (26539 on land and 13353 at sea). The whole data set consisted of 55158 points of free-air gravity anomalies (41805 on land and 13353 at sea), distributed in the study area from 40 to 44 degrees on latitude and -10 to 4 degrees on longitude. The accuracy of all these data ranges from 0.1 to 0.2 mgal. The compiled gravity data were checked to remove repeated points, leaving 49485 points distributed over the study area, as shown in Figure 1. All the data were converted to the GRS80 reference system and the atmospheric correction was taken into account (Wichiencharoen, 1982; Kuroishi, 1995). It should be noted that two overlapping data windows were considered (Figure 1 and 3), to avoid some zones of Iberia (like Portugal) with scarcity of gravity data, trying to include the zones of Iberia with higher data density, as much as possible. Thus, the loss of accuracy arisen in the computation of the previous geoid IGG2005, due to the scarcity of gravity data in some zones of Iberia, has been avoided.

Geopotential model. The EIGEN-GL04C model (Förste et al., 2006) is an upgrade of the EIGEN-CG03C model (Förste et al., 2005). This model is a combination of the GRACE (Gravity Recovery and Climate Experiment) and LAGEOS (LAser GEOdynamics Satellite) mission solution adding a 0.5 x 0.5 degrees gravimetry and altimetry surface data. The surface data are identical to EIGEN-CG03C set except for the geoid undulations over the oceans. The EIGEN-GL04C geopotential model represents a major advance in modelling the Earth’s gravity and geoid. Therefore, this global model is the geopotential model that must be used for the computation of the long-wavelength contribution to the geoid and the gravity anomaly, to obtain a high-precision geoid in the study area.

Digital terrain model (DTM). Any gravimetric geoid computation based on Stokes’ integral must use anomalies that have been reduced to the geoid, usually by means of Helmert’s second method of condensation (Heiskanen and Moritz, 1967). This involves the computation of the terrain correction and the indirect effect on the geoid, which are computed from a DTM. A DTM is also necessary to compute the Residual Terrain Model reduction (RTM reduction or also called RTM correction) for the point anomalies, in order to obtain smooth gravity anomalies, which are more easily gridded. For the present study, a new elevation model for the whole study area, with a 3’’x 3’’ spacing, has been obtained from the Shuttle Radar Topography Mission (SRTM) elevation data and the ETOPO1 bathymetry data, following the process described by Corchete and Pacino (2007). To minimize the loss of accuracy associated to the low resolution of the ETOPO1 bathymetry, the data windows were selected so that they include the marine data as small as possible. Figure 2 shows the elevation model resulting of this computation process.

EUVN points used as a control data set. The height data of the 8 EUVN points existing for the study area (Corchete et al., 2005), have been used as a control data set to check the computed geoid. Figure 3 shows the geographical distribution of these points and Table 1 their coordinates and heights.

3. Methodology and processing

The computation method for the calculation of a gravimetric geoid detailed by Corchete et al. (2005) was followed. In this paper only a brief review of this computation process is presented. The geoid will be computed in the each window shown in Figure 3, to avoid the loss of accuracy arisen in the computation of the previous geoid IGG2005, due to the scarcity of gravity data and the low resolution of the ETOPO1 bathymetry. After that, both geoid solutions will be merged computing the media of the geoid heights in the overlapping area.

Gravity data gridding. Since the gravity data set consists of point anomalies distributed randomly, an interpolation process must be applied to obtain a regular data grid. Before this interpolation, it is very suitable to remove the short-wavelength and the long-wavelength effects applying the well-known relationship (the RTM correction)

$$\Delta g_{red}^{pts} = \Delta g_{free}^{pts} - 2\pi k\rho(h - h_{ref})^{pts} + c^{pts} - \Delta g_{GM}^{pts} \qquad (1)$$

where the superscript *pts* denotes each point randomly distributed over the study area, Δg_{free} is the free-air gravity anomaly, *k* is Newton's gravitational constant, ρ is the density of the topography (2.67 g/cm^3) for the RTM correction on land or the density of the topography minus seawater density (2.67 – 1.03 = 1.64 g/cm^3) for marine RTM, *h* is the elevation, h_{ref} denotes the elevation of the reference surface (this reference surface is obtained by applying a 2D low-pass filter with a resolution of 60', to the elevation field), *c* is the terrain correction computed for land and marine points, and Δg_{GM} is the gravity anomaly computed from the geopotential model EIGEN-GL04C. The same constant value for the density of the topography (2.67 g/cm^3) is always used in the RTM and terrain corrections, because these corrections have not sensitivity to small changes in this value. The error in the computation of the RTM and terrain corrections, associate to the consideration of a media value for the density of the topography instead of the exact value given by the density distribution of the topography masses in the study area, is negligible if a good media value for this density is considered (Torge, 1989).

When the smooth anomalies have been obtained by (1), it can be observed that some points have gravity anomalies with high values. These erroneous values are associated to bad gravity data points. To avoid the inclusion of these bad data in the computation process, the gravity anomalies given by (1) greater than 50 mgal have been removed. Thus, 957 points have been removed from the total data set (49485 points), leaving 48528 points for the interpolation on a regular grid. This regular grid has been obtained by using Kriging-based routines which are a part of OriginLab software package (© 1991-2003 OriginLab Corporation). The gridded data are distributed over the study area in both overlapping windows (shown in Figure 3) from 40 to 44 degrees of latitude and –10 to 4 degrees of longitude, on a 161x561 regular grid with a mesh size of 1.5’x1.5’.

Finally, RTM must be restored in the gridded anomalies to obtain the true free-air anomalies relative to EIGEN-GL04C. This RTM effect can be restored by

$$\Delta g_{\mathrm{free}}^{\mathrm{grid}} = \Delta g_{\mathrm{red}}^{\mathrm{grid}} + 2\pi k\rho(h - h_{\mathrm{ref}})^{\mathrm{grid}} - c^{\mathrm{grid}} \qquad (2)$$

where the superscript *grid* denotes each point of the regular grid considered (161 x 561 = 90321 points), Δg_{free} is the free-air gravity anomaly, Δg_{red} is the gravity anomaly reduced by (1) and gridded.

Geoid computation. This new geoid has been computed by the classical remove-restore technique. Following this method, the geoid models for the areas shown in Figure 3 are obtained by the sum of three terms

$$N = N_1 + N_2 + N_3 \qquad (3)$$

The first term N_1 is the geopotential model contribution to the geoid undulation. This term can be computed from a spherical harmonic expansion by (Heiskanen and Moritz, 1967; Corchete et al., 2005). The second term N_2 is the indirect effect of Helmert’s second method of condensation reduction on the geoid. N_2 consists of two terms in planar approximation (Sideris, 1990). This planar approximation can be easily written in convolution form (Schwarz et al., 1990) and computed by a FFT procedure. The third term N_3 is the contribution of the residual gravity. This term can be calculated by means of the Stokes integral (Heiskanen and Moritz, 1967) written in convolution form by using of 1D FFT (Haagmans et al., 1993).

Thus, the geoid solutions for the areas shown in Figure 3 are obtained summing of all previously computed terms according to equation (3). Figure 4 shows the difference between the geoid heights obtained for both geoids, in the overlapping area. It should be noted that the two geoids show a very good coincidence in the overlapping area as the difference in geoid heights is small (approximately 5 cm). Finally, both geoid solutions are merged, computing the media of the geoid heights in the overlapping area, to obtain the North IBerian GEOid (NIBGEO). This geoid with a mesh size of 1.5’x1.5’ (4x14 degrees over the study area), is shown in Figure 5. As it can be seen in Figure 5, an important indirect effect appears clearly in some contours, making these contour lines more rugh in areas with high mountains (for latitudes from 42 to 43 ºN and longitudes from -1 to 3 ºE). Corchete et al. (2006) have demonstrated that a geopotential geoid model can fail in these regions, being the gravimetric geoid a more reliable model in areas with high mountains. This new model and a simple FORTRAN program for PC can be obtained from the internet address http://airy.ual.es/www/ NIBGEO_english.htm. This computer program allows the computation of the geoid height (using this geoid model) in any point over the study area shown in Figure 5.

Geoid validation. The new geoid NIBGEO has been checked by comparison with the geoid undulations measured for the 8 validation points located in the study area (the 8 EUVN points shown in Figure 3). Table 1 shows the geoid undulations predicted by NIBGEO and IGG2005 for these validation points and their differences with these heights. The statistics of these differences are shown in Table 2. In this Table, it should be noted that the new geoid NIBGEO shows an improvement in precision and reliability, fitting the geoidal heights measured for the validation points better than IGG2005.

4. Conclusions

The computation methods based on the FFT have allowed the calculation of a new gravimetric geoid for the northern part of Iberia, which is a major advance in the modelling of the geoid for Iberia. The gravimetric geoid determination has been carried out by means of the Stokes integral in convolution form. This method, which has previously been shown to be an efficient method to compute a high-resolution geoid, yielded a regular gridded geoid of 161x561 points (90321 points) in the GRS80 reference system, with a mesh size of 1.5’x1.5’, distributed from 40 to 44 degrees of latitude and –10 to 4 degrees of longitude. The new geoid shows less discrepancy with the geoid undulations measured for the validation points (8 EUVN points available on the northern part of Iberia), than the other previous geoid IGG2005. The new geoid has a standard deviation of 12.6 cm (Table 2). This is the smallest error obtained by any geoid solution for Iberia and surrounding area, up to date. Nevertheless, an important problem arises from the gravity data. New gravity data are needed because some of the existing gravity data are very old (a lot of these points were measured long time ago, from 1950 until now). For this reason, the computation of a gravimetric geoid with a centimetre precision is not possible with the present gravity data. This centimetre precision in the geoid model can be obtained, when new gravity data will be available to replace the oldest measurements of the gravity data compilations for Iberia. Updating of these international compilations is needed to supply new gravity data measured with the most modern technology, to replace the oldest measurements that, obviously, do not have the accuracy of the modern gravimeters. In spite of this, a new geoid model has been obtained and it will be useful for the orthometric height determination by GPS over the northern part of Iberia, because it will allow the orthometric height determination in mountains and remote areas, where levelling has many logistic problems. This geoid obtained for the northern part of Iberia complements the previously obtained geoid for South Spain and the Gibraltar Strait area (SOSGIS), because both geoids jointly give the complete picture of the geoid for Spain and the Gibraltar Strait area, with more precision than the previous geoid IGG2005. Thus, NIBGEO and SOSGIS contribute to our knowledge of the geoid, but the surrounding areas must be better known to constrain the lithospheric and mantle models.

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Heading for tables

Table 1. The 8 EUVN points used as a control data set (validation points), the geoid heights predicted by the available geoids over the study area and the differences between the geoid heights predicted by the available geoids and the geoid heights.

Table 2. Statistics of the differences listed in Table 1.

Figure captions

Figure 1. Geographical distribution of the gravity data over the study area (49485 free-air gravity anomalies).

Figure 2. Topographic and bathymetric digital model used in this study (90 m x 90 m mesh size). Heights above 2000 meters and below -3000 meters are shown in white and black colours, respectively.

Figure 3. Geographical distribution of the EUVN points used as control data set (triangles). The two overlapped data windows considered in this study are shown with solid and dashed lines.

Figure 4. Difference between the geoid heights obtained for each geoid computed in the areas shown in Figure 3. This difference is calculated in the overlapping area. The contour interval is 0.05 m.

Figure 5. The North IBerian GEOid (NIBGEO) obtained by merging of the geoid solutions computed for the windows shown in Figure 3. The media of the geoid heights for both geoid solutions has been computed in the overlapping area. NIBGEO is plotted joint to SOSGIS (Corchete et al., 2008) to show the total area covered by both geoids. The contour interval is 1.0 m.

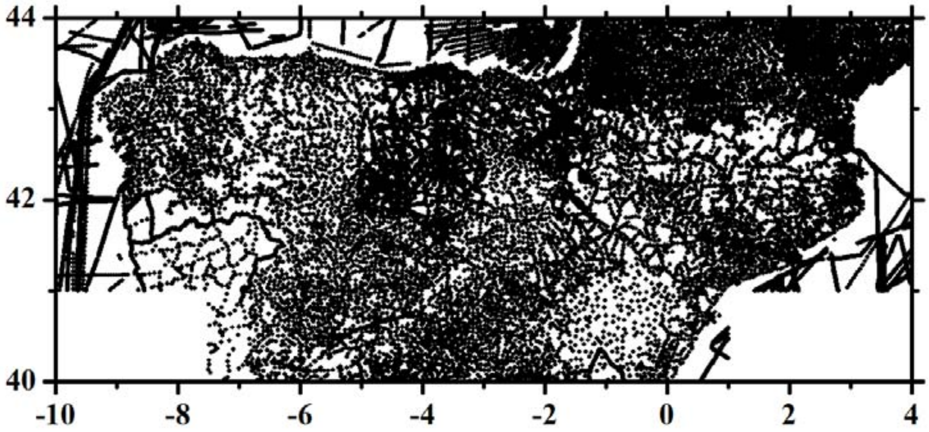
Table 1.

Point (n.)	Latitude (ºN)	Longitude (ºE)	h (m)	H (m)	N = h-H (m)	NIBGEO (m)	IGG2005 (m)	NIBGEO - N	IGG2005 - N
1	41.02211188	-6.94117046	221.758	165.953	55.805	55.725	55.689	-0.080	-0.116
2	41.19636707	-8.70701314	70.077	14.865	55.212	55.069	55.118	-0.143	-0.094
3	43.36438216	-8.39893488	66.965	12.122	54.843	54.824	54.767	-0.019	-0.076
4	40.42723144	-4.24918772	815.091	762.177	52.914	52.938	52.559	0.024	-0.355
5	43.46140399	-3.78943353	59.288	8.968	50.320	50.381	49.872	0.061	-0.448
6	41.72154644	-1.02463175	269.637	219.293	50.344	50.629	50.644	0.285	0.300
7	40.82088686	0.49236036	107.811	57.708	50.103	50.097	49.959	-0.006	-0.144
8	41.35091848	2.15739914	67.662	18.170	49.492	49.521	49.494	0.029	0.002

(h = ellipsoidal height, H = orthometric height and N = geoid height, NIBGEO = This paper; IGG2005 = Corchete et al. (2005))

Table 2.

Differences	Mean (m)	Std. dev. (m)
NIBGEO - N	0.019	0.126
IGG2005 - N	-0.116	0.226



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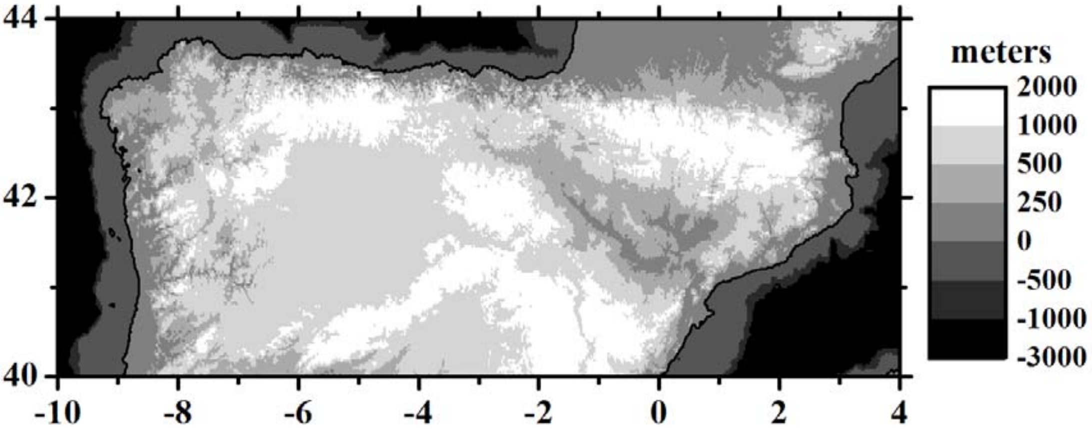

Fig. 2.

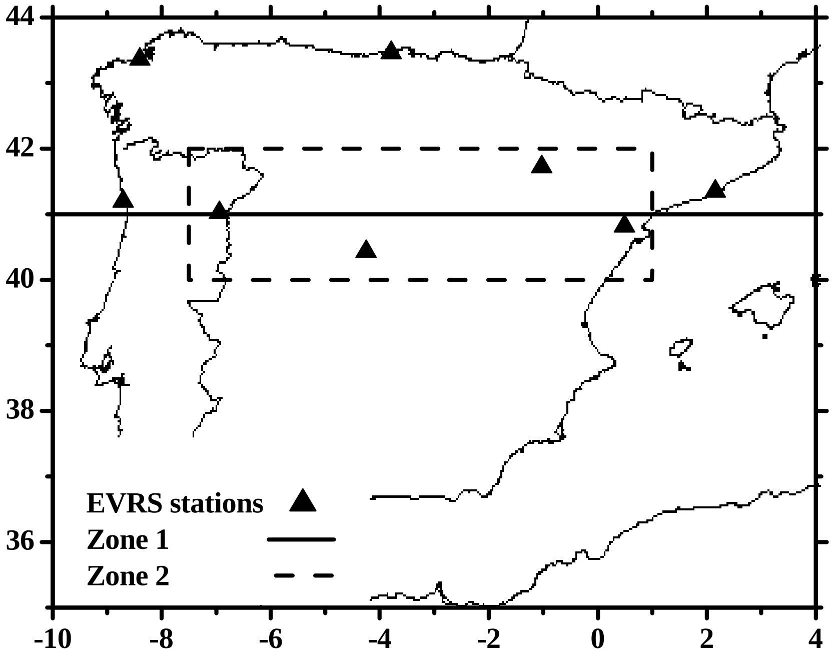

Fig. 3.

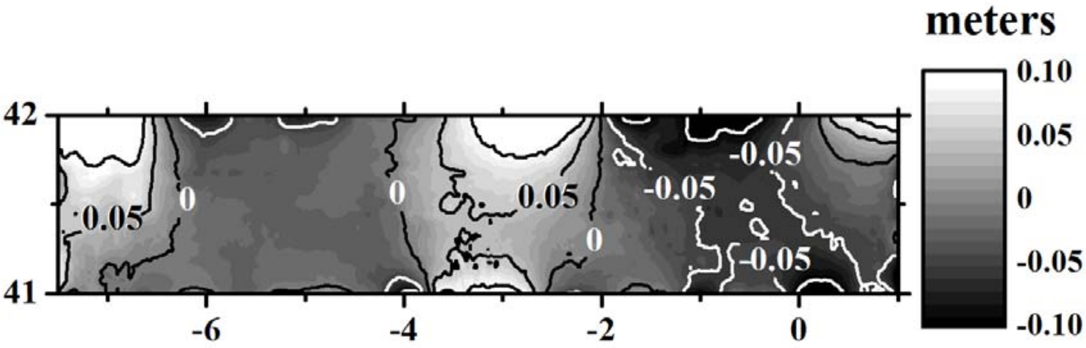
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Fig. 4.

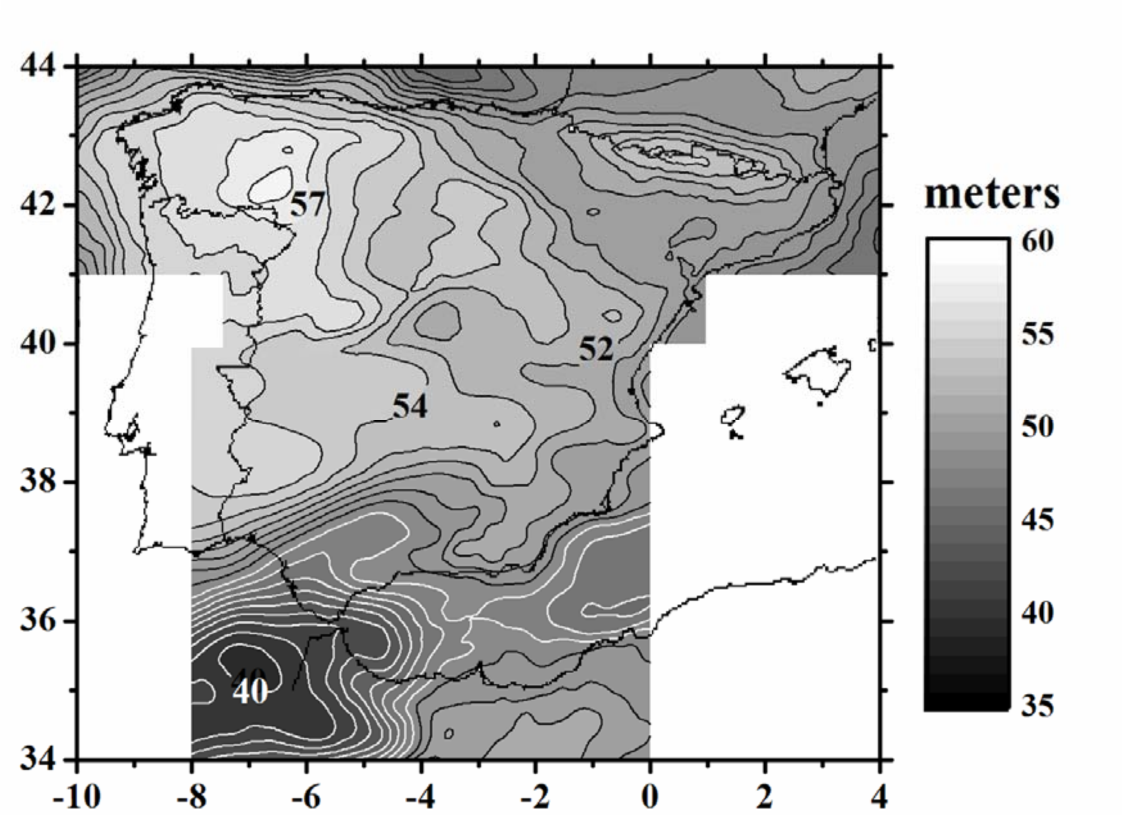
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Fig. 5.