Regional quasigeoid solution for the Moldova area from GPS/levelling data

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Introduction

According to the Molodensky theory of the determination of the Earth's figure and its gravitational field the geodetic height H can be represented in the following way (see, for instance, Heiskanen and Moritz, 1967):

$$H = H^{\gamma} + \zeta \,, \qquad (1)$$

where H^{γ} is the normal height and ζ is the anomaly height or the so-called quasigeoid height. Normal heights H^{γ} of some auxiliary surface, the telluroid, can be found with respect adopted ellipsoid via the geopotential number $C_P = W_0 - W_P$ using the Molodensky condition

$$W_0 - W_P = U_0 - U_{\tilde{P}} \,, \qquad (2)$$

where W_0 and W_P are the gravity potential at the reference origin point and some current point P on the Earth's surface, respectively; U_0 is the normal potential of adopted ellipsoid ($U_0 = W_0$); $U_{\tilde{P}}$ is the normal potential at the point $\tilde{P}$ of the telluroid; the points P and $\tilde{P}$ are situated on the same ellipsoidal normal; the distance $P\tilde{P}$ represents the anomaly height ζ in the expression (1).

In a short span of time the development of modern GNSS technologies has led to a straightforward application of the equation (1) in the following form

$$\zeta = H - H^{\gamma} \,, \qquad (3)$$

which allows to "measure" the quasigeoid height ζ if H and H^{γ} are known in the same points from GNSS, precise levelling, and gravity observations.

The equation (3) gives today important information for the establishment of height reference systems on the whole and for the development of the European Vertical Reference System (EVRS), in particular. Traditionally gravimetric solutions for ζ have a high relative resolution and require usually additional transformation to some absolute level through the so-called bias and tilt parameters, which are equivalent to the well-known datum shift parameters. A valuable decision to develop a new European quasigeoid (in the frame of the EGGP project) instead of the well-known EGG97 model (Denker and Torge, 1997) was adopted at the last General Assembly of the IUGG (Sapporo, 2003) assuming in this case a wide application of existing sets of GNSS/levelling data for a combined quasigeoid solution.
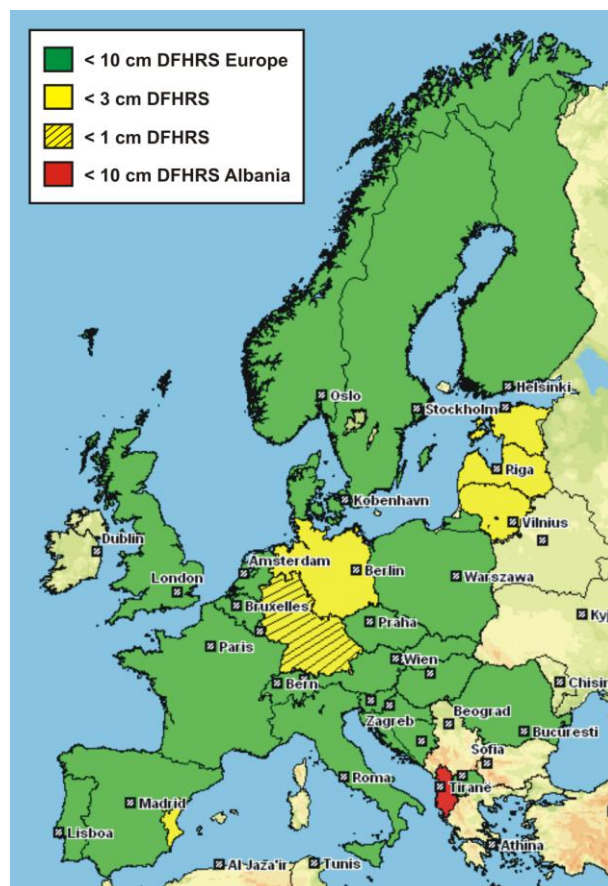
Fig.1 Achieved accuracy of the conversion of the geodetic heights H into the normal heights H^{γ} in the frame of the DFHRS concept (Jäger, 2004)

However, a current application of the gravimetric EGG97 solution is possible with better accuracy in the regional/local scale after the mentioned additional transformation of EGG97 to GNSS/levelling network, using, for example, "Height Solution of the Vertical Reference Network (EUVN)" (Ihde et al., 2000) where the EUVN/UELN adjusted normal heights H^{γ} and

GPS-derived anomaly heights ζ were estimated with 5 cm accuracy on continental level. In fact such approach has applied by (Jäger and Kälber, 2003) for the realization of the so-called DFHRS concept (Digital-FEM-Height-Reference-Surface) based on the finite elements method (FEM) for the patch-continuous approximation of the geodetic heights H using the equation (1) and some height reference surface (HRS) represented in reality by transformed quasigeoid heights ζ. Fig. 1 demonstrates achieved accuracy of the conversion of the geodetic heights (as one decimeter model of the EVRS) into the EUVN normal heights H^γ by means of the transformed EGG97 anomaly heights. According to Fig. 1 the development of the EVRS for such countries as Ireland, Yugoslavia, Bulgaria, Greece, Turkey, Russia, Byelorussia, the Ukraine, and Moldova up to now is not realized with accuracy < 10 cm.

This paper focuses on the construction of the quasigeoid model in the Moldova area for further conversion of the geodetic heights H into normal heights H^γ with accuracy <10 cm. Because appropriate gravimetry data are not available in the Moldova Republic, such model, called here by the MOLDGEO2004A solution, is based only on GPS/levelling anomaly heights (3) within Moldova and the modified EGG97 solution, surrounded this area. Note also that Moldavian GPS network represents today ~ 1200 GPS stations of zero, 1st, 2nd, and 3rd order with geodetic coordinates referred to the ETRS89 system and known normal heights in the same points obtained from old geometric levelling of 2nd, 3rd, and 4th order (97%). All normal heights are given in the Baltic 1977 height system, which is adopted in Moldova Republic.

An essential difference from the mentioned DFHRS concept (where information about the quasigeoid heights are postulated) is the direct application of the least squares collocation method (Moritz, 1980) with regularization (Marchenko et al., 2001; Marchenko and Tartachynska, 2003) for the prediction of quasigeoid heights and their inversion to gravity anomaly from some irregular distribution (Fig.2) to the regular grid $(1'\times1.5')$. The EGG97 solution has transformed preliminary via datum shift parameters to the Baltic 1977 system. By this the modified EGG97 anomaly heights surrounded the Moldova area are used as additional – supported information to get a better results along the Moldavian border.

Data

Because anomaly heights (derived from the expression (3)) are adopted as basic set of initial data, a special attention was made to the preparation of such type of data taken from different sources. The identification process of the same geodetic points with known geodetic (GPS-derived) coordinates and normal heights has led to the creation of special database realized in the Oracle Database System.

The additional comparisons of the observed anomaly heights ζ with the global gravitational field EGM96 (360,360) and EGG97 quasigeoid were made for the detection of gross-errors (due to antenna height etc.). As a result, part of sites with significant deviations was deleted and we got a final set of 1200 GPS/levelling stations with known geodetic coordinates in the ETRS89 system and the normal heights H^γ given in the Baltic 1977 height system. Accuracy of chosen H^γ is estimated from 4 cm to 8 cm. Fig. 2 illustrates the distribution of initial GPS/levelling data and the additional 30 GPS/levelling control points adopted below for the independent comparison (due to their exclusion from data processing).
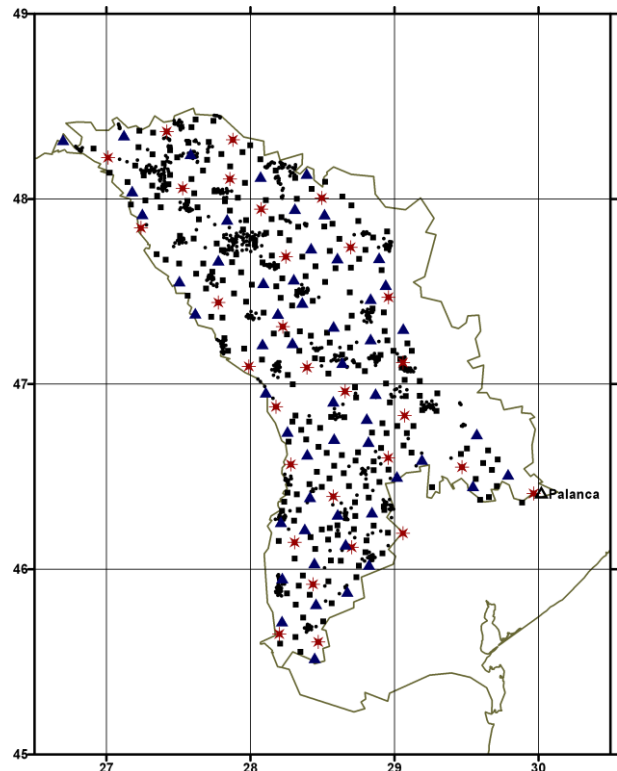
Fig. 2. Distribution of the GPS-derived anomaly heights: ☞ – zero order site; % – 1st order sites; ! – 2nd order sites; • – 3rd order sites; ☞ – control sites

The comparison of 1200 values of ζ with the gravimetric solution EGG97 leads to the following statistics: mean deviation E=-23 cm, RMS=10.2 cm. A significant mean deviation E=-23 cm can be treated by different height systems adopted in Moldova Republic (Baltic 1977) and for the construction of the EGG97 quasigeoid (UELN network referred to Amsterdam system). For proper comparison the simplest datum shift transformation (Heiskanen and Moritz, 1967) was applied

$$\zeta_{\text{GPS}} = \zeta_{\text{EGG97}} + \delta x \cos\phi \cos\lambda +$$
$$\delta y \cos\phi \sin\lambda + \delta z \sin\phi, \qquad (4)$$

where ϕ and λ are the geodetic latitude and longitude, respectively; $\zeta_{\rm GPS}$ and $\zeta_{\rm EGG97}$ are the anomaly heights derived from the relationship (3) and quasigeoid heights EGG97 accordingly; ($\delta x, \delta y, \delta z$) are the well-known datum shift parameters, which are found from the least squares adjustment and based on 1200 observational equations of the kind (4) when ($\delta x, \delta y, \delta z$) are considered as unknowns.

The transformed or modified anomaly heights $\zeta_{\rm EGG97}^{\rm mod}$ of EGG97 can be found immediately after the estimation of ($\hat{\delta x}, \hat{\delta y}, \hat{\delta z}$):

$$\zeta_{\rm EGG97}^{\rm mod} = \zeta_{\rm EGG97} + \hat{\delta x}\cos\phi\cos\lambda +$$
$$\hat{\delta y}\cos\phi\sin\lambda + \hat{\delta z}\sin\phi. \quad (5)$$

The comparison of 1200 $\zeta_{\rm GPS}$ with modified by (5) $\zeta_{\rm EGG97}^{\rm mod}$ leads to new statistics: mean deviation E=0, standard deviation=9.8 cm, maximal and minimal deviations are equal to 37 cm and −38 cm, respectively. Analyzing the results of these two comparisons we come to the conclusion about the accordance between $\zeta_{\rm GPS}$ and $\zeta_{\rm EGG97}$ ($\zeta_{\rm EGG97}^{\rm mod}$) within 10 cm level in the studying area. For this reason the anomaly heights $\zeta_{\rm EGG97}^{\rm mod}$ surrounded the Moldova are included as measurements simultaneously with 1200 quantities $\zeta_{\rm GPS}$ providing in this manner 10 cm level of quality of quasigeoid heights along the Moldavian border. Thus, the following data sets used for the MOLDGEO2004A solution are:

- Set1 – 1200 GPS/levelling anomaly heights $\zeta_{\rm GPS}$ with adopted mean accuracy 6 cm;

- Set2 – modified anomaly heights $\zeta_{\rm EGG97}^{\rm mod}$ surrounded the Moldova area with adopted mean accuracy 10 cm.

MOLDGEO2004A regional gravity field

For further use of bilinear interpolation with expected accuracy < 10 cm all following computations were done at the grid nodes (1′×1.5′). As well-known such grid size provides a necessary methodological accuracy of (quasi)geoid interpolation using simplest formulas. According to this method of interpolation after the coordinate transformation

$$X = (x - x_1)/(x_2 - x_1),$$
$$Y = (y - y_1)/(y_2 - y_1), \quad (6)$$

the given function $Z(x,y) = Z(X(x),Y(y))$ can be considered within the unite square

$$[0 \le X \le 1], \quad [0 \le Y \le 1]. \quad (7)$$

Then, the function $Z(X,Y)$ can be represented in the following way

$$Z(X,Y) = a_0 + a_1 X + a_2 Y + a_3 XY, \quad (8)$$

with the next expressions for the coefficients

$$a_0 = Z_1, \; a_1 = Z_2 - Z_1, \; a_2 = Z_4 - Z_1,$$
$$a_3 = Z_1 + Z_3 - Z_2 - Z_4, \quad (9)$$

where Z_1, Z_2, Z_3, Z_4 are the given values of $Z(X,Y)$ at the 4 nodes of a regular grid. In view of the inequalities (7) the following numbering of initial data points for interpolation can be adopted

$$Z_1 = Z(0,0), \quad Z_2 = Z(1,0),$$
$$Z_3 = Z(1,1), \quad Z_4 = Z(0,1). \quad (10)$$
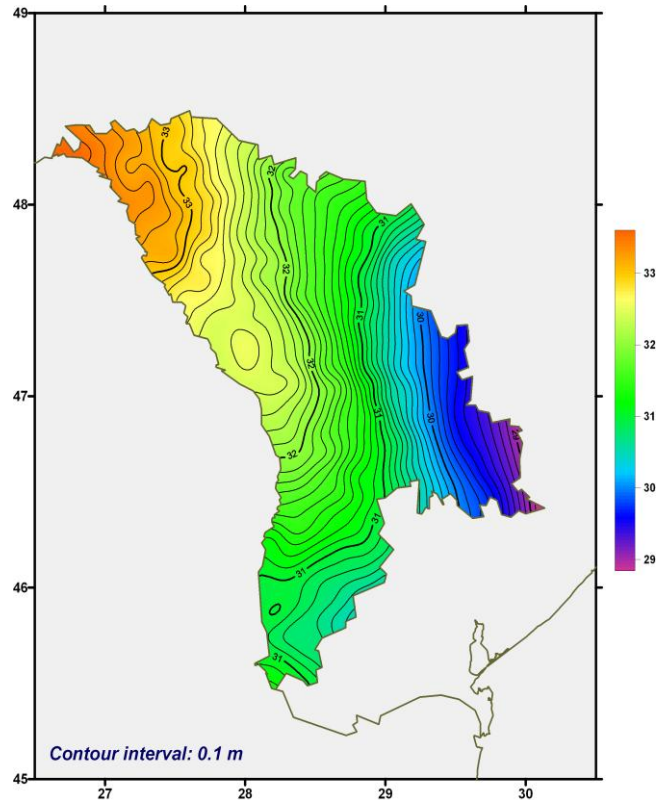
Fig. 3. MOLDGEO2004A anomaly heights $\zeta_{\rm 2004A}$ (Contour interval: 0.1 m)

Thus, if the anomaly heights ζ are known at the nodes of (1′×1.5′) grid, the expressions (6) to (10) become our working tool for the computation of the anomaly heights $\zeta = Z(X,Y)$. On the fist step the rectangular area from $45^{\rm o}$N to $49^{\rm o}$N in latitude and from $26.5^{\rm o}$E to $30.5^{\rm o}$E in longitude (see, Fig. 3) was filled by the $\zeta_{\rm EGG97}^{\rm mod}$ values at the nodes of (1′×1.5′) grid. On the final step the values $\zeta_{\rm EGG97}^{\rm mod}$ within the Moldova area are replaced at the same grid points by some other quantities ζ derived

by the regularization method from the measured ζ_{GPS} anomaly heights. To be consistent all comparisons after the mentioned replacement were based on the interpolation formula (8) and the *combined* grid with $\zeta_{\mathrm{EGG97}}^{\mathrm{mod}}$ adopted around Moldova and $\zeta = \zeta_{2004\mathrm{A}}$ derived from ζ_{GPS} within Moldova area.

The anomaly heights $\zeta_{2004\mathrm{A}}$ and gravity anomalies $\Delta g_{2004\mathrm{A}}$ (called here as the MOLDGEO2004A solution) are constructed in the form of $(1'\times1.5')$ grid using the well-known remove-restore procedure with the additional improvement of covariance function by iterations.

On the fist step from initial anomaly heights $\zeta = \zeta_{\mathrm{GPS}}$ and $\zeta = \zeta_{\mathrm{EGG97}}^{\mathrm{mod}}$ the contribution of the global gravitational model EGM96 (360, 360) up to degree/order 360 was removed

$$\delta\zeta = \zeta - \zeta_{\mathrm{EGM96}}. \qquad (11)$$
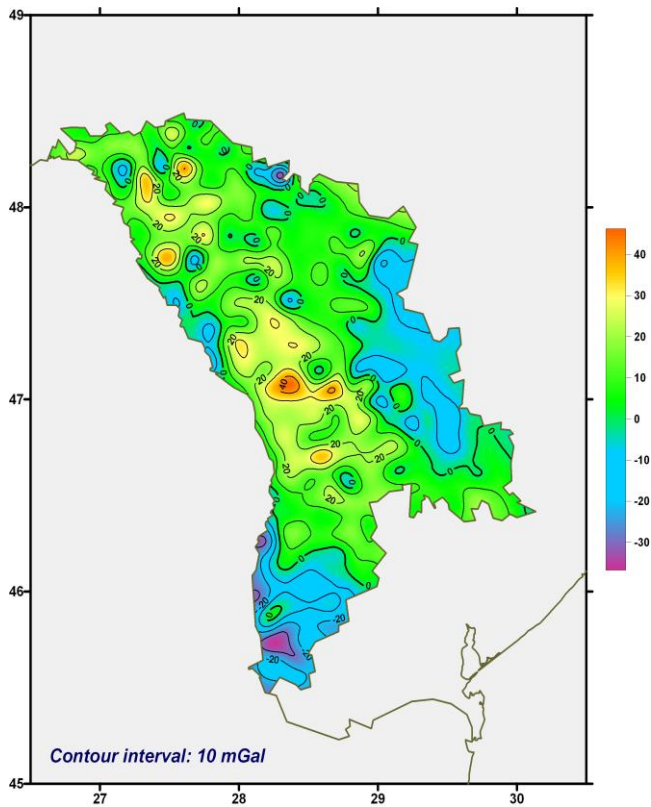
Fig. **4**. MOLDGEO2004A gravity anomalies $\Delta g_{2004\mathrm{A}}$ (Contour interval: 5 mGal)

First empirical (ECF) and analytical (ACF) covariance functions are constructed for the *residual anomaly heights* (11). Note that the modified Poisson kernel without zero degree harmonic was applied successfully for the ECF representation. Having such ACF for $\delta\zeta$ (and through the covariance propagation also for $\delta\Delta g$)

the regularization method is used straightforward to predict $\delta\zeta$ and $\delta\Delta g$ at the grid points $(1'\times1.5')$ within the Moldova area. Then the obtained $\delta\Delta g$ were applied for the computation of new ECF and ACF, which are based on the *residual $\delta\Delta g$ gravity anomalies*. The second – final prediction of $\delta\zeta$ and $\delta\Delta g$ was made, despite a change of essential parameters of these new ECF and ACF, with the same initial data and new ACF (also adopted in the form of the modified Poisson kernel). As a result, the solution MOLDGEO2004A was restored after the second prediction

$$\left.\begin{array}{l} \zeta_{2004\mathrm{A}} = \delta\zeta + \zeta_{\mathrm{EGM96}}, \\ \Delta g_{2004\mathrm{A}} = \delta\Delta g + \delta\Delta g_{\mathrm{EGM96}}, \end{array}\right\} \qquad (12)$$

at the above – mentioned grid points $(1'\times1.5')$. Fig. 3 and Fig. 4 illustrate the obtained quasigeoid heights and gravity anomaly, respectively.
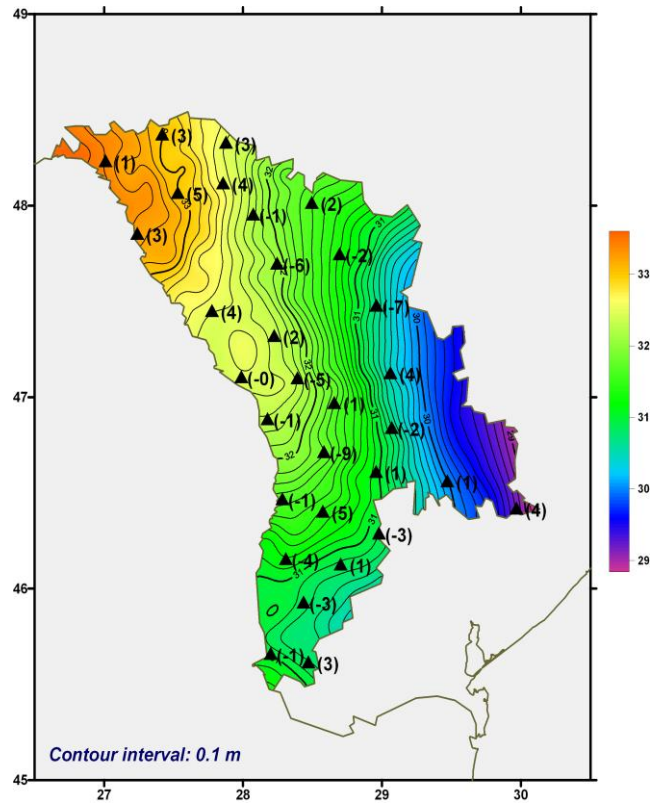
Fig. **5**. Comparison of MOLDGEO2004A solution with anomaly heights at the 30 independent control points. Differences between observed and interpolated anomaly heights are shown in brackets in cm

Conclusions

For further application of bilinear interpolation (8) to the function $\zeta = Z(X,Y)$ the rectangular grid $(1'\times1.5')$ was combined from MOLDGEO2004A solution (Fig.3) and the $\zeta_{\mathrm{EGG97}}^{\mathrm{mod}}$ values surrounded the Moldova area. So that, all comparisons were done between observed

ζ_{GPS} and interpolated anomaly heights. First comparison was made for 30 scattered points adopted as independent GPS/levelling control sites. Fig.5 demonstrates results of this verification where differences between observed and interpolated anomaly heights are shown in brackets [cm]. Note here a small values of obtained statistics: mean deviation E=0.8 cm, standard deviation σ=2.6 cm. Second test was done for the whole initial Set1 of 1200 GPS/levelling points with known ζ_{GPS}. The results are: mean deviation E=-0.2 cm, standard deviation σ=6.9 cm. This standard deviation can be decreased to the level from 6 cm to 5 cm if some part of doubtful points (from 30 to 50, respectively) will be deleted from the initial data set1.

On the one hand, the MOLDGEO2004A solution for quasigeoid heights and gravity anomalies at $(1'\times1.5')$ grid points allows now the computation of anomaly heights and by this the conversion of the geodetic heights H into normal heights H^{γ} in the Baltic 1977 height system (adopted in Moldova) with accuracy better than 10 cm. On the other hand, there is a real possibility to achieve a 5 cm–level of accuracy of representation of quasigeoid heights after an additional examination of geodetic coordinates and normal heights of GPS/levelling data adopted as initial information.

Acknowledgments

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