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The first high-precision gravimetric geoid of Hungary: HGG2013

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Abstract

The first geoid computed with a very high precision (centimetre precision) for Hungary, is presented in this paper. The computation of this gravimetric geoid is based on the most recent geopotential model: EGM2008 (Earth Gravitational Model released in 2008). The method used in the computation of this new gravimetric geoid has been the Stokes integral in convolution form. The terrain correction has been applied to the gridded gravity anomalies, to obtain the corresponding reduced anomalies. Also the indirect effect has been taken into account. Thus, a new geoid model has been calculated and it is provided as a data grid in the GRS80 (Geodetic Reference System of 1980), distributed for the study area from 45 to 49 degrees of latitude and 16 to 23 degrees of longitude, on a 161x281 regular grid with a mesh size of 1.5'x1.5'. This new high-precision geoid and the global geoid EGM2008; are compared with the geoid undulations measured for 18 GPS/levelling points on Hungary. The new geoid shows an improvement in precision and reliability, fitting the geoidal heights of these GPS/levelling points with more accuracy than the global geoid. Moreover, this new geoid has a smaller standard deviation (3.6 cm) than that obtained by any previous geoid developed for Hungary (and the surrounding area) up to date. This geoid obtained for Hungary will complement the high-precision geoids obtained for some European countries, because the new geoid and these others jointly will give the complete picture of the high-precision geoid for Europe. This new model will be useful for orthometric height determination by GPS over this study area, because it will allow orthometric height determination in the mountains and remote areas, in which levelling has many logistic problems.

Keywords: Gravity, Geoid, FFT, GPS/levelling, Hungary

1. Introduction

A previous study developed by [1] has taken as objective the geoid computation for Hungary. In this study, the EGG9401 geoid was calculated. This geoid was obtained as a data grid in the GRS80 reference system, distributed for the Hungarian area from 44 to 50 degrees of latitude and 14 to 25 degrees of longitude and provided as a 241x265 regular grid with a mesh size of 1.5'x2.5'. Nevertheless, since the publication of EGG9401 a new geopotential model (EGM2008) is available and a new high-resolution DTM (Digital Terrain Model), for the whole Earth topography, is supplied by the Shuttle Radar Topography Mission (SRTM). Logically, these new models (EGM2008 and SRTM) represent improvements that must be included in any new geoid to be computed now. Thus, it would be very desirable to obtain a new high-precision geoid for Hungary based on these new models, because this Hungarian geoid would be useful for orthometric height determination by GPS over this study area, allowing the orthometric height determination in the mountains and remote areas, in which levelling has many logistic problems. Also, this new geoid will be very desirable because it will complement some European geoids calculated with high-precision. This new geoid and these others will give jointly a complete picture of the high-precision geoid for Europe, with more precision than the global geoid EGM2008.

The new geoid will be computed as a 161x281 regular data grid in the GRS80 reference system, with a mesh size of 1.5'x1.5', completing the picture of the European geoid for Hungary from 45 to 49 degrees of latitude and 16 to 23 degrees of longitude. This new geoid will be computed using the Stokes integral in convolution form. The necessary terrain correction will be applied to obtain the gridded reduced gravity anomalies. The corresponding indirect effect will be taken into account. After the computation of this Hungarian Gravimetric Geoid (HGG2013), it will be compared with the global geoid EGM2008, to demonstrate the improvement in precision and reliability attained by the new geoid.

2. Data set

For the gravimetric geoid computation the necessary data sets are: (1) free-air gravity anomalies; (2) a geopotential model; (3) a high precision Digital Terrain Model (DTM); and (4) observed geoid undulations. The data sets used for computation of the Hungarian Gravimetric

Geoid released in 2013 (HGG2013) are detailed below.

Land gravity data bank. The land gravity data used in this study has been provided by the Bureau Gravimetrique International (BGI). The BGI data set has 2145 points in the study area. This data set of free-air gravity anomalies is distributed in the study area from 45 to 49 degrees on latitude and 16 to 23 degrees on longitude, as it is shown in Figure 1. The accuracy of all these data ranges from 0.1 to 0.2 mgal. All the data were converted to the GRS80 reference system and the atmospheric correction was taken into account [5, 7].

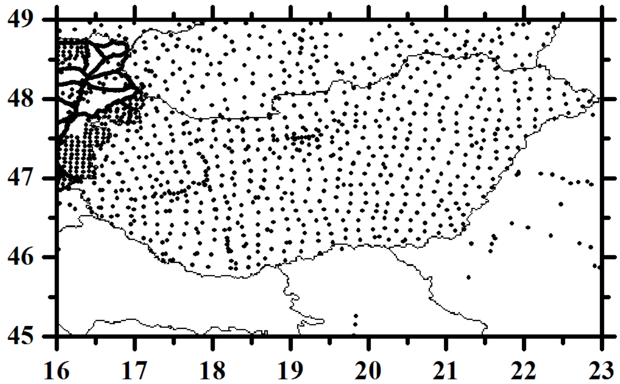


Figure 1. Geographical distribution of the gravity data over the study area (2145 free-air gravity anomalies).

Geopotential model. The EGM2008 geopotential model represents a major advance in modelling the Earth's gravity and geoid [6]. Therefore, this global model is the geopotential model that must be used for the computation of the long-wavelength contribution to the geoid and the gravity anomaly, to obtain a high-precision geoid in the study area.

Digital terrain model (DTM). Any gravimetric geoid computation based on Stokes' integral must use anomalies that have been reduced to the geoid, usually by means of Helmert's second method of condensation [4]. This involves the computation of the terrain correction and the indirect effect on the geoid, which are computed from a DTM.

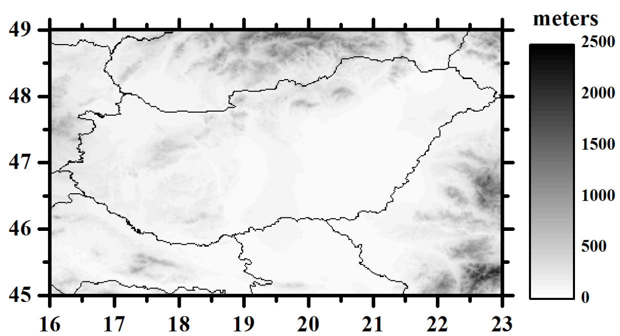


Figure 2. Topographic digital model used in this study (90 m x 90 m mesh size).

A DTM is also necessary to compute the Residual Terrain Model reduction (RTM reduction or also called RTM correction) for the point anomalies, in order to obtain smooth gravity anomalies, which are more easily gridded. For the present study, the elevation model provided by the Shuttle Radar Topography Mission (SRTM) with a 3''x 3'' spacing was used. Figure 2 shows this elevation model plotted for the study area.

GPS/levelling points used as a control data set. The height data of the 18 GPS/levelling points existing for the study area have been used as a control data set to check the computed geoid [1]. Figure 3 shows the geographical distribution of these points and Table 1 their coordinates and heights. The orthometric heights have been computed from the normal heights through a well-known formula [2, 4].

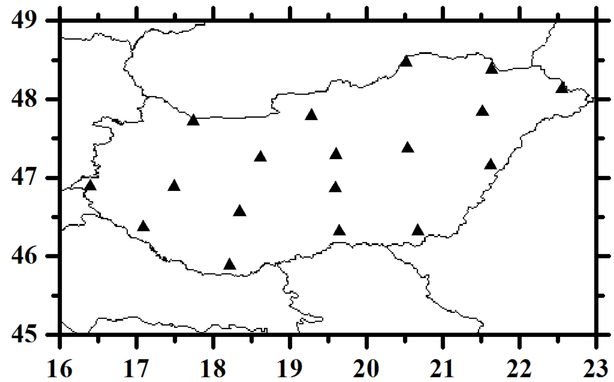


Figure 3. Geographical distribution of the GPS/levelling points used as control data set (triangles).

3. Methodology and processing

After The computation method for the calculation of a gravimetric geoid detailed by [2] was followed. In this paper only a brief review of this computation process is presented.

Gravity data gridding. Since the gravity data set consists of point anomalies distributed randomly, an interpolation process must be applied to obtain a regular data grid. Before this interpolation, it is very suitable to remove the short-wavelength and the long-wavelength effects applying the well-known relationship (the RTM correction)

$$\Delta g_{red}^{pts} = \Delta g_{free}^{pts} - 2\pi k\rho(h - h_{ref})^{pts} + c^{pts} - \Delta g_{GM}^{pts} \quad (1)$$

where the superscript *pts* denotes each point randomly distributed over the study area, Δg_{free} is the free-air gravity anomaly, *k* is Newton's gravitational constant, ρ is the density of the topography (2.67 g/cm^3), *h* is the elevation (shown in Figure 2), h_{ref} denotes the elevation of the reference surface (this reference surface is obtained by applying a 2D low-pass filter with a resolution of 60', to the elevation field) and *c* is the terrain correction computed at each point.

Table 1. The 18 GPS/levelling points used as a control data set (validation points), the geoid heights predicted by the available geoids over the study area and the differences between the geoid heights predicted by the available geoids and the geoid heights.

Point (n.)	Latitude (ºN)	Longitude (ºE)	h (m)	H* (m)	H (m)	N = h-H (m)	EGM2008 (m)	EGM2008 (m)	EGM2008 - N	HGG2013 - N
1	47.789600511111	19.281523475000	291.842	248.260	248.260	43.582	43.453	43.560	-0.129	-0.022
2	47.370721563889	20.536939522222	136.086	94.145	94.145	41.941	41.860	41.953	-0.081	0.012
3	47.720804241667	17.741209069444	163.035	119.170	119.171	43.864	43.747	43.890	-0.117	0.026
4	47.255687955556	18.619247602778	234.717	190.533	190.532	44.185	44.037	44.144	-0.148	-0.041
5	47.291598602778	19.601825844444	270.638	227.727	227.731	42.907	42.747	42.852	-0.160	-0.055
6	46.563324816667	18.346772433333	209.630	165.196	165.196	44.434	44.354	44.483	-0.080	0.049
7	45.883615330556	18.217166188889	314.459	269.704	269.708	44.751	44.640	44.792	-0.111	0.041
8	46.865781405556	19.594987430556	167.002	123.827	123.827	43.175	43.056	43.171	-0.119	-0.004
9	46.313565755556	19.646300063889	171.045	127.207	127.206	43.839	43.774	43.891	-0.065	0.052
10	46.319565477778	20.670774511111	142.569	99.910	99.910	42.659	42.631	42.705	-0.028	0.046
11	47.157644219444	21.620348030556	137.573	96.286	96.285	41.288	41.310	41.330	0.022	0.042
12	47.838072933333	21.513441547222	161.680	121.331	121.333	40.347	40.272	40.379	-0.075	0.032
13	48.467895702778	20.520160044444	456.304	414.121	414.125	42.179	42.083	42.219	-0.096	0.040
14	48.129550394444	22.549150850000	193.706	154.721	154.724	38.982	38.852	38.937	-0.130	-0.045
15	48.377019286111	21.632354141667	155.897	115.916	115.916	39.981	39.849	39.965	-0.132	-0.016
16	46.883900811111	17.491353661111	191.738	146.298	146.296	45.442	45.309	45.434	-0.133	-0.008
17	46.368945858333	17.090630550000	285.434	240.274	240.280	45.154	45.033	45.125	-0.121	-0.029
18	46.889588625000	16.395943983333	325.647	279.730	279.737	45.910	45.804	45.919	-0.106	0.009

(h = ellipsoidal height, H* = normal height, H = orthometric height, N = geoid height, EGM2008 = Pavlis et al. (2008), HGG2013 = This paper)

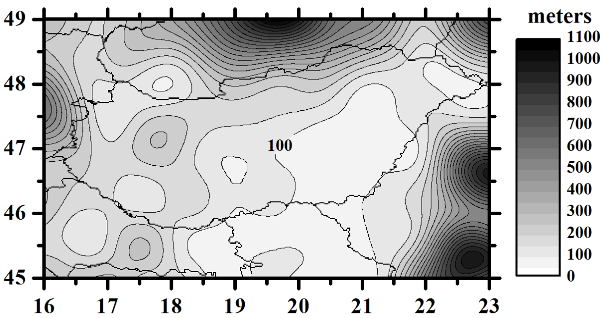


Figure 4. Reference surface (3” x 3” mesh size) obtained by low-pass filtering of the elevations showed in Figure 2. Contour interval: 50 m.

Δg_{GM} is the gravity anomaly computed from the geopotential model EGM2008, considering 360 as maximum degree in the spherical harmonic expansion (i.e. subtracting only the long-wavelength effect in the point data). Figure 4 shows the reference surface (h_{ref}) obtained after filtering of this elevations model. Figure 5 shows the gravity anomaly (Δg_{GM}) computed from the geopotential model EGM2008.

When the smooth anomalies have been obtained by (1), it can be observed that some points have gravity anomalies with high values. These erroneous values are associated to bad gravity data points. To avoid the inclusion of these bad data in the computation process, the gravity anomalies given by (1) greater than 30 mgal have been removed. Thus, 24 points have been removed from the total data set (2145 points), leaving 2121 points for the interpolation on a regular grid.

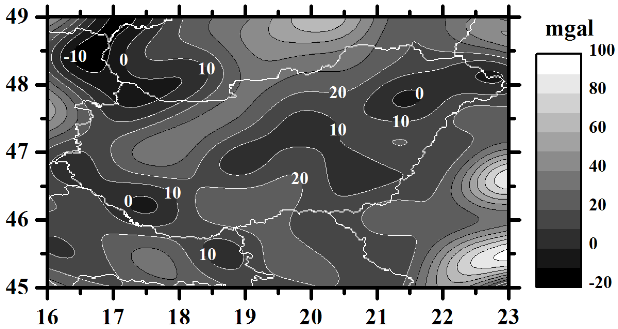


Figure 5. Gravity anomaly based on the EGM2008 model (considering the maximum degree of the harmonic expansion equals to 360). Contour interval: 10 mgal.

This regular grid has been obtained by using Kriging-based routines which are a part of OriginLab software package (© 1991-2003 OriginLab Corporation, Northampton, MA 01060 USA). The gridded data are distributed over the study area from 45 to 49 degrees of latitude and 16 to 23 degrees of longitude, on a 161x281 regular grid with a mesh size of 1.5'x1.5', as it is shown in Figure 6.

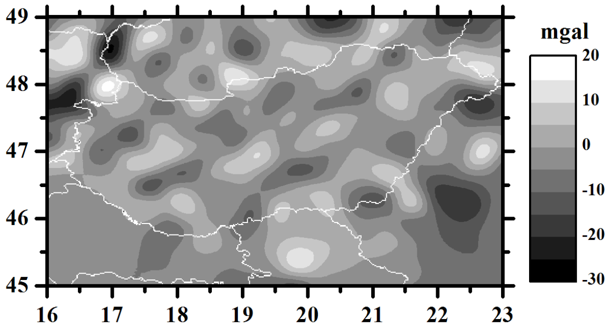


Figure 6. Free air gravity anomalies reduced by (1) and gridded over the study area with 1.5'x1.5' mesh size. Contour interval: 5 mgal.

Finally, RTM must be restored in the gridded anomalies to obtain the true free-air anomalies relative to EGM2008. This RTM effect can be restored by

$$\Delta g_{free}^{grid} = \Delta g_{red}^{grid} + 2\pi k\rho(h - h_{ref})^{grid} - c^{grid} \qquad (2)$$

where the superscript *grid* denotes each point of the regular grid considered (161 x 281 = 45241 points), Δg_{free} is the free-air gravity anomaly, Δg_{red} is the gravity anomaly reduced by (1) and gridded (Figure 6).

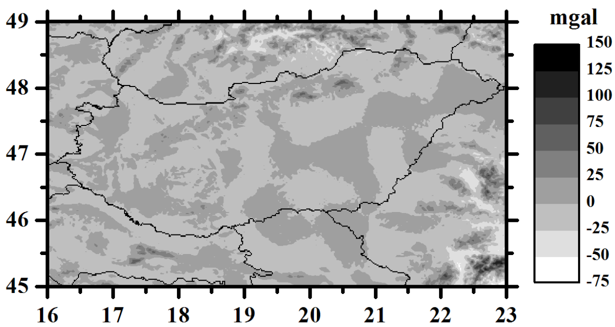


Figure 7. RTM principal term $2\pi k\rho(h - h_{ref})$ computed from the topographic model shown in Figure 2, considering the reference surface shown in Figure 4. Contour interval: 25 mgal.

Figure 7 shows the values of the term $2\pi k\rho(h - h_{ref})$ and Figure 8 shows the terrain correction considered in (2). Figure 9 shows the free-air gravity anomaly (Δg_{free}) computed by (2), i. e. the true free-air anomalies relative to EGM2008.

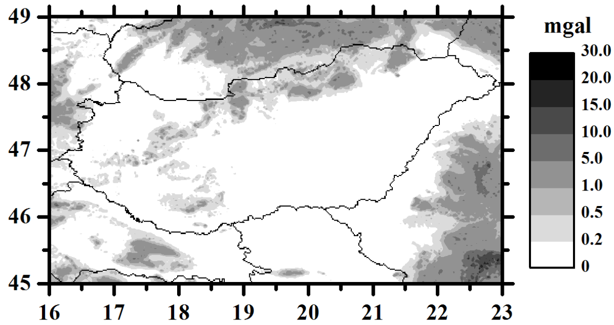


Figure 8. The terrain correction computed from the elevations shown in Figure 2.

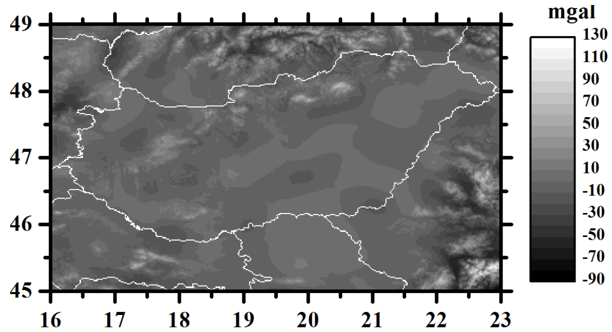
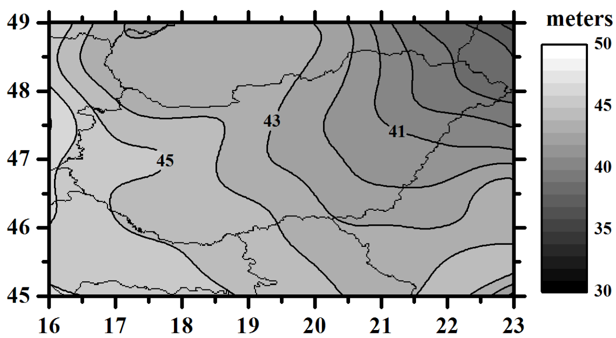


Figure 9. Free-air anomalies relative to EGM2008 computed by (2). Contour interval: 10 mgal.

Geoid computation. This new geoid has been computed by the classical remove-restore technique. Following this method, the geoid model for the study area is obtained by the sum of three terms

$$N = N_1 + N_2 + N_3 \qquad (3)$$

The first term N_1 is the geopotential model contribution to the geoid undulation. This term has been computed from the geopotential model EGM2008, considering 360 as maximum degree in the spherical harmonic expansion (i.e. only considering the long-wavelength contribution to the geoid height). The second term N_2 is the indirect effect of Helmert's second method of condensation reduction on the geoid. The third term N_3 is the contribution of the residual gravity. Figure 10 shows the values of the three terms of formula (3).



(a)

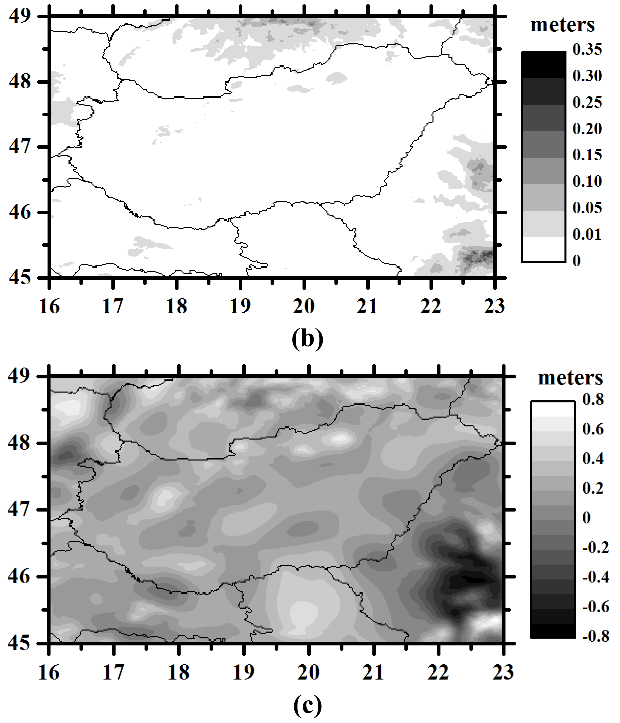


Figure 10. (a) The EGM2008 geoid model computed for the study area, considering the maximum degree of the harmonic expansion equals to 360. Contour interval: 1 m. (b) The indirect effect on the geoid (plotted positive). (c) The residual geoid undulation. Contour interval: 0.1 m.

The geoid solution for the study area is shown in Figure 11 and it is obtained summing all previously computed terms (Figure 10) according to equation (3). This Hungarian Gravimetric Geoid (HGG2013) with a mesh size of 1.5'x1.5' is extended 4x7 degrees over the study area.

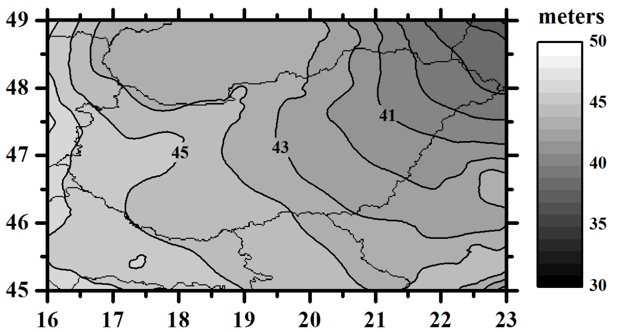


Figure 11. The Hungarian Gravimetric Geoid released in 2013 (HGG2013) obtained as a sum of the terms given by the equation (3). Contour interval: 1 m.

As it can be seen in Figure 11, an important indirect effect appears clearly in some contours, making these rougher contour lines in areas with high mountains. [3] have demonstrated that a geopotential geoid model (like EGM2008) can fail in these regions, being the gravimetric geoid a more reliable model in areas with high mountains. This new model and a simple FORTRAN program for PC can be obtained from the internet address: http://airy.ual.es/www/HGG2013.htm. This computer program allows the computation of the geoid height (using the HGG2013 model) in any point over the study area shown in Figure 11.

Geoid validation. The new geoid HGG2013 has been checked by comparison with the geoid undulations measured for the 18 validation points located in the study area (the 18 GPS/levelling points shown in Figure 3). Table 1 shows the results of this comparison. The geoid height predicted by the EGM2008 model, for any point on the earth, can be easily obtained by means of useful software available from the internet address: http://earth-info.nga.mil/GandG/wgs84/gravitymod/egm2008/index.html.

Table 2. Statistics of the differences listed in Table 1.

Differences	Mean (m)	Std. dev. (m)
EGM2008 - N	-0.101	0.045
HGG2013 - N	0.007	0.036

This software allows the computation of the spherical harmonic expansion up to the maximum degree defined for this global model (degree 2190 and order 2159). The geoid heights obtained from the EGM2008 model for the validation points have been computed using this software (Table 1). In Table 1, the differences between the geoid heights predicted by the available models (EGM2008 and HGG2013) and the geoid heights measured for the validation points also are shown. The statistics of these differences are shown in Table 2. In this Table, it should be noted that the new geoid HGG2013 shows an improvement in precision and reliability, fitting the geoidal heights measured for the validation points better than EGM2008.

4. Conclusions

The computation methods based on the FFT have allowed the calculation of a very precise geoid for Hungary, which is a major advance in the modelling of the geoid for Europe. The gravimetric geoid determination has been carried out by means of the Stokes integral in convolution form. This method, which has previously been shown to be an efficient method to compute a high-resolution geoid, yielded a regular gridded geoid of 161x281 points (45241 points) in the GRS80 reference system, with a mesh size of 1.5'x1.5', distributed from 45 to 49 degrees of latitude and 16 to 23 degrees of longitude. The new geoid shows less discrepancy with the geoid undulations measured for the validation points (18 GPS/levelling points available on the study area), than the most recent global geoid (EGM2008). The new geoid has a standard deviation of 3.6 cm (Table 2). This is the smallest error obtained by any geoid solution for Hungary and surrounding area, up to date. Nevertheless, an important problem arises from the scarcity of the gravity data for the south and east of the study area (Figure 1). For this reason, the computation of a gravimetric geoid with a centimetre precision, for the whole study area, is not possible with the present gravity data. This centimetre precision in the geoid model can be obtained for the south and east of the study area, when new gravity data will be available to cover these areas. Updating of these international compilations is needed to supply new gravity data measured for many areas in the world, in which the scarcity of gravity data is a severe problem for the development of new geophysical models. In spite of this, a new high-precision geoid has been obtained for Hungary and it will be useful for the orthometric height determination by GPS over Hungary, because it will allow the orthometric height determination in mountains and remote areas, where levelling has many logistic problems. On the other hand, this geoid obtained for Hungary will complement others high-precision geoids computed for some European countries, giving jointly the complete picture of the centimetre-precision geoid for Europe.

5. Acknowledgements

The author is grateful to the Bureau Gravimetrique International (BGI) for providing the gravity data used in this study. The United States Geological Survey (USGS) has supplied the elevation data required to compute the necessary terrain corrections, through the database: SRTM 90M (available by FTP internet protocol). The author is also grateful to the National Geospatial-Intelligence Agency (NGA), who provided the software and the data file with the coefficients of the harmonic expansion (available by HTTP internet protocol), used for the computation of the geoid height predicted by the EGM2008 model (for the validation points) and the computation the long-wavelength effects (in the geoid and the gravity anomaly).

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