URUGEOIDE 2000 PROJECT
A GRAVIMETRIC GEOID FOR URUGUAY

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ABSTRACT

The data, methodology and results of the UruGeoide 2000 project, aiming a determination of an accurate gravimetric geoidal model for Uruguay (latitude -30º to -35º and longitude 301,5º to 307º), are presented on this paper.

Data types used were a high degree geopotential model, namely the EGM96 spherical harmonic coefficient set; a set of 15860 gravity stations and a 2 km x 2 km digital terrain model for the uruguayan terrestrial data, completed with bathymetric and GTOPO30 model data for the rest of the area.

The remove-restore technique (Sideris, 1997) was successfully applied, combining the three mentioned datasets. The external masses to the geoid was consider it with the second Helmert condensation method, taking in account the indirect effect. The gravimetric contribution was calculated using residual free-air anomalies in the spherical Stokes formula, evaluated with 1D-FFT as suggested by (Haagmans *et al.*, 1993).

The geoid model was referred both to WGS84 (G873) geodetic system and Montevideo 1948.0 vertical system and has been tested using a set of GPS/leveling geoid undulations, obtaining a 0.25 m and 2 ppm absolute and relative precision respectively. After a bias and tilt were estimated, a standard error of 0.13 m was found. Results are presented in form of graphics and contour maps.

1. INTRODUCTION

The GPS (Global Positioning System) is today one of the most surveying tools used on geodesy and topography. Its offers a simple method to get good precision for point position with low costs of execution. The tridimensional cartesian coordinates obtained for a station are easily transformed in geodetic (φ, λ, h) or planes ones (E, N, h), however the elipsoidal altitude h, are physical meaningless, being refereed just to an arbitrary ellipsoid surface. The reduction of the ellipsoidal altitude to a orthometric one, referred to a specific geoid or zero local level, require of the accurate knowledge of the separation of the two surfaces, ellipsoid and geoid, namely geoidal undulation, N. On a global basis, the modern geopotential models of high degree, offer geoidal undulations with some ± 1m error, but when a better precision is needed, its must be used gravimetric and topographic data of high resolution.

In Uruguay the SGM (Military Geographical Survey), as the official cartographic institution of the country, started the systematic use of the GPS observation techniques for mapping purposes in 1993. Since that time it was recognized the needed of an accurate transformation of the ellipsoidal altitudes to the local defined Montevideo Vertical Datum (1948.0). In 1994 a joint project SGM/USP (São Paulo University, Brazil), allows to calculate a geoidal model for Uruguay. The model was based on the geopotential model GEMT2 up to degree 36, combined with gravimetric anomalies. Terrain effects were not took in account it due to lack of a detailed DTM. Some tests of the model showed systematic errors of up to 2 m on the geoidal undulation calculated. New geopotential models (Lemoine *et al.*, 1998), as wee as gravimetric and topographic data (Subiza, 1998a) resulted in a new project, aiming a high-resolution geoidal model for Uruguay and named UruGeoide 2000. The main characteristics of the project are:

Its *objective* is calculate a high resolution geoidal model for Uruguay, located at latitude -30º to -35º and longitude 301.5º to 307º, with the main application being the transformation between ellipsoidal and orthometric altitudes.

The *calculation method* of the geoidal model is the gravimetric one, through the spherical Stokes formula, with rigorous kernel and evaluated via the one dimensional Fast Fourier Transform (1D-FFT).

The *main technique* used is the remove-restore technique (Sideris, 1997), combining a high degree of a geopotential model (EGM 96 up to 360 degree), gravimetric anomalies and a digital terrain model. In order to consider the external masses to the geoid, the second condensation method is applied; taking in account the indirect effect associated (Heiskanen e Moritz, 1985).

2. THEORICAL CONCEPTS

The problem of obtain geoidal undulations using gravimetric anomalies was solved by Georg Stokes in 1849, through the well known formula, named after him:

$$N = \frac{R}{4\pi\,\gamma}\int \Delta g\; S(\psi)\; dS\,, \tag{1}$$

being N the geoidal undulation, R a mean terrestrial radius, γ the theoretical gravity value, Δg the gravity anomaly and $S\,(\psi)$, the so-called Stokes' function, depending only on the spherical distance between the calculated point and the anomaly being integrated. The integral is carried out onto the whole geoidal surface.

Integrals as (1), are called convolution integrals, being easily evaluated in the frequency domain via Fast Fourier Transform methods, allowing to process great amount of data, a must for regional geoids. In order to apply the Stokes' formula, same modifications has to be made. Thus the integral is replace by a discrete summation of discrete data, where the spatial density information is usually not better than 5'x 5'. Besides, the integral is apply over a restricted area and not over the whole geoidal surface. The remove-restore technique solve these problems, being described as follows:

In the remove step, are obtained the residual gravimetric anomalies using:

$$\Delta g^{res} = \Delta g_{AL} - \Delta g_{MG} - \Delta g_{H}\,. \tag{2}$$

being Δg_{AL} the Free-air gravimetric anomalies (corrected by the atmospheric effect) Δg_{MG} is the geopotential model contribution and Δg_{H} is the topographic contribution (direct effect), accounting for the Helmert's second condensation method. The formulae for the mentioned contribution are:

$$\Delta g_{MG} = \bar{g}\sum_{n=2}^{n_{máx}}(n-1)\sum_{m=0}^{n}\left(\bar{C}_{\mathrm{nm}}\cos m\lambda_P + \bar{S}_{nm}\,\mathrm{sen}\, m\lambda_P\right)\bar{P}_{nm}\,\mathrm{sen}\,\varphi_P \tag{3}$$

$$\Delta g_H = -c = \frac{1}{2} G\,\rho \int_E \frac{H^2 - H_P^2}{l^3} dxdy \quad - H_P G\,\rho \int_E \frac{H - H_P}{l^3} dxdy \tag{4}$$

where $\bar{g}$ is a mean terrestrial gravity value; $\overline{C}, \overline{S}$ and $\overline{P}$ are the normalized coefficient of the geopotential model used and the normalized Legendre associated functions of first kind respectively; φ_P and λ_P, are the point geodetic coordinates; n and m are the geopotential model degree and order; c is the so-called *classical terrain correction*; G stands for the universal gravitational constant; ρ is the density of the terrestrial masses; H is the station orthometric altitude; l, is the distance between the calculation point and the integrated one and finally E is the restricted area of the integral.

In the second step the residual geoidal undulation are calculated, applying the Stokes formula in the following convolution form:

Considering that in order to calculate the undulation on a parallel φ_l, using data of another parallel φ_j, ψ changes only λ_k-λ_i, and Δg^{res} changes with λ_j, the Stokes' formula can be expressed as:

$$N_2(\varphi_l, \lambda_k) = \frac{R}{4\pi\gamma} \sum_{j=0}^{N\text{-}1} \left[\sum_{i=0}^{M-1} \Delta g^{res}(\varphi_j, \lambda_i) S(\varphi_l - \varphi_j, \lambda_k - \lambda_{\mathrm{i}}) \cos\varphi_j \, \Delta\lambda \right] \Delta\varphi \tag{5}$$

with $\varphi_l = \varphi_1, \varphi_2, ..., \varphi_n$. Using the addition theorem of the Fourier Transform, eq. (5) is evaluated by FFT as:

$$N_2(\varphi_l, \lambda_k) = \frac{R}{4\pi\gamma} \; \mathrm{F}_1^{-1} \left\{ \sum_{j=0}^{N-1} \mathrm{F}_1 \left[\Delta g^{res} \left(\varphi_j, \lambda_i \right) \cos\varphi_j \right] \mathrm{F}_1 \left[S \left(\varphi_l, \varphi_j, \Delta\lambda \right) \right] \right\} \tag{6}$$

In (6), F_1 and F_1^{-1}, are the direct and inverse Fourier transform, given the geoidal undulations for all points located at parallel φ_l. These results are identical to the classical numerical integration.

In the restore step, the geopotential model contribution and the terrain indirect effect are added to the residual geoidal undulations, obtaining the final geoidal undulations according to the formula:

$$N = N_1 + N_2 + N_3 \,, \tag{7}$$

N_1 is given by the geopotential model, in this case EGM96 up to degree and order of 360, calculated with the general formula:

$$N_1 = R \sum_{n=2}^{n_{máx}} \sum_{m=0}^{n} \left(\overline{C}_{\mathrm{nm}} \cos m\lambda_P + \overline{S}_{nm} \operatorname{sen} m\lambda_P \right) \overline{P}_{nm} \operatorname{sen} \varphi_P \tag{8}$$

In (7), N_2 was obtained with (6) and N_3 is the indirect effect of the Helmert method of condensation, calculated as:

$$N_3 = -\frac{\pi\, G\, \rho}{\gamma} H_P^2 - \frac{G\rho}{6\gamma} \int_E \frac{H^3 - H_P^3}{l^3} dxdy \,. \tag{9}$$

In order to evaluate the geoidal model, GPS observations over benchmarks were used. The geoidal undulation at that points (N_{GPS}), related both ellipsoidal (h) and orthometric altitudes (H), as:

$$N_{GPS} = h - H \tag{10}$$

If a geoidal undulation calculated from a geoidal model (N_{GEOI}), has the same geodetic reference system as the one obtained through (10), it is possible to compare both in an absolute form, as:

$$DN = N_{GPS} - N_{GEOI} = h - H - N_{GEOI} \,. \tag{11}$$

A better evaluation is made in a relative form, comparing the height differences at two stations:

$$\Delta DN = \Delta N_{GPS} - \Delta N_{GEOI} = (h_2 - h_1) - (H_2 - H_1) - (N_{GEOI}^2 - N_{GEOI}^1) \,, \tag{12}$$

and relating the calculated value to the distance between station in order to get the relative error in one km.

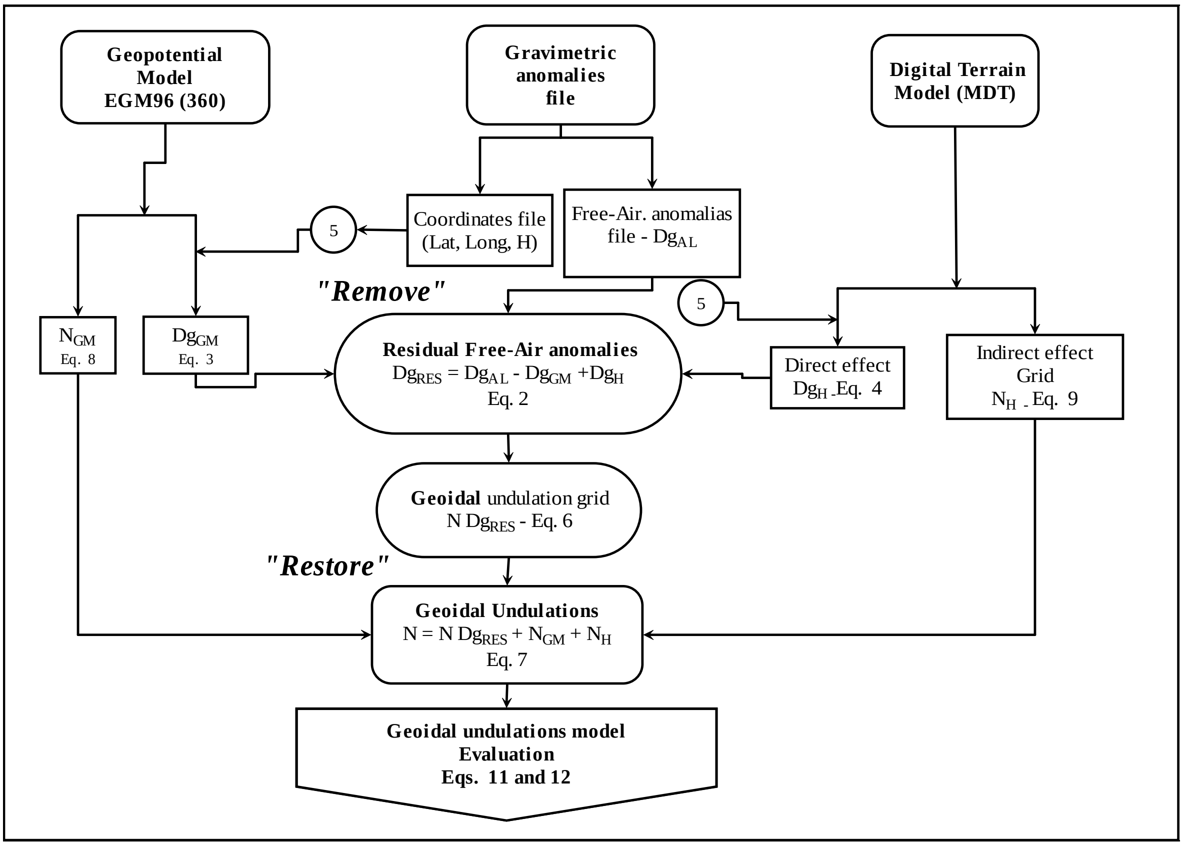

Figure 1. Procedure to calculate a gravimetric geoidal model via FFT and using the remove-restore technique.

3. DATASETS USED

The geodetic reference system adopted was the WGS84 (G873) (NIMA, 1999), thus the main purpose of the geoidal model i.e. the transformation between ellipsoidal altitudes and orthometric one, is well achieved. The new geodetic reference system adopted in Uruguay, based on SIRGAS 1995.4 is compatible to a cartographic level with WGS84. The gravimetric and topographic datasets were previously transformed from the old local reference system to the new one. The vertical reference system in Uruguay, was defined in 1948.0, having a systematic bias not well established yet of about +0,5 m, when compared to others regional/global vertical datum (Subiza, 1998b). The Fig. 2 shows the geographical data and type distribution used.

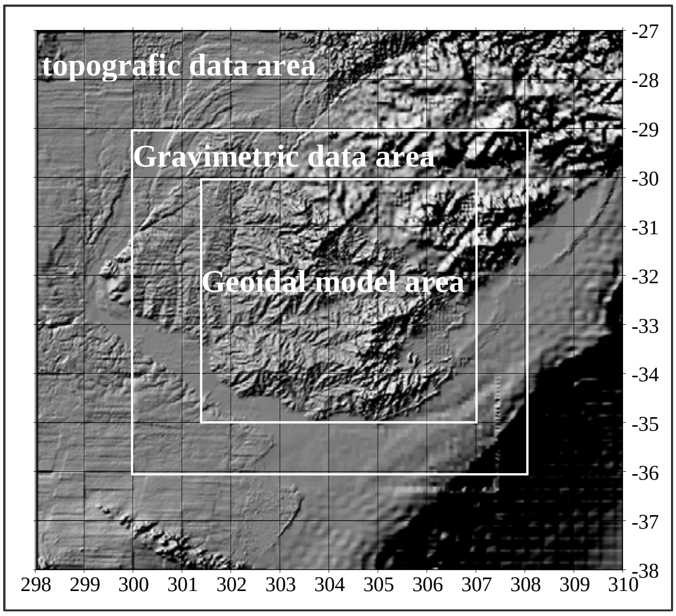

Figure 2. Geographical data and type distribution

The terrestrial gravity data density in Uruguay is better than 5' x 5', but at the remained area can reach 10' x 10' or worse. The gravimetric data were collected from the SGM databank, BGI (Bureau Gravimetrique Internationale), SCGGSA (South American Sub-Commission for the Geoid and Gravimetry) and IfE-PFA3 (Hanover University). The marine area was completed with the global marine Free-Air anomalies model GMGA9706 (Hwang *et al.*, 1997). A total of 15860 gravity stations were used, being 5873 marine and 9987 marine ones.

The Fig. 3 shows the geographical distribution of the gravity data and Table 1 shows the correspondent statistics.

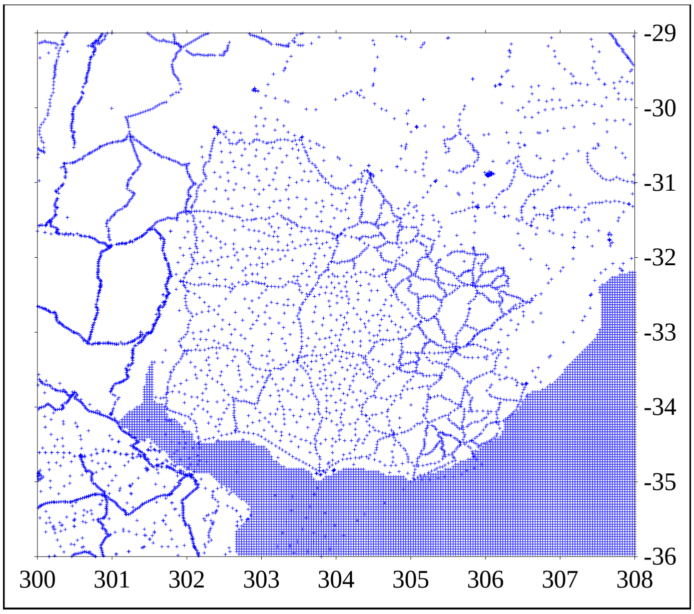

Figure 3. Free-Air gravity anomalies

Table 1. Statistic values of the Free-Air anomalies file.

15.860 stations	**Maximum**	**Minimum**	**Mean**	**St. Deviation**	**Range**
Latitude	-29°	-36°	34.97°	-	7°
Longitude	308°	300°	304,71°	-	8°
Altitude (m)	692,9	-37,5	35,46	75,46	730,4
Free-Air (mGal)	100,639	-35,788	13,317	16,246	136,427

The topographic data were compiled from the SGM (46014 terrestrial and marine altimetry datapoints, resolution of 2 km x 2 km), USP (41881 terrestrial and marine altimetry datapoints, resolution of 2.5' x 2.5'), from the global topographic model GTOPO30 (UGSC, 1997) (190351 datapoints, resolution of 1' x 1') and finally were added the altitudes from the gravity stations used. Some missing bathymetric values were detected at SE corner of the total area. Two grids with 1.5' and 3' regular resolution were calculated with the final topographic of 263376 datapoints. Table 2 and 3 shows the statistics from these files.

Table 2. Statistics values from final combination of topographic files

Source	Nr.	Maximum (m)	Maximum (m)	Mean (m)	St. Deviation	Resolution
GTOPO30	190351	732	1	68,15	58,34	1'
USP	21228	1340	-2459	219,96	359,85	2,5'
SGM	46014	425	-2300	104,13	85,03	2 km
Gravity st.	5783	629,9	-37,5	97,01	98,01	5'~10'
DTM	263376	1340	-2459	87,52	126,37	≈ 1,3'

Table 3. Statistics of the derived files

File	Maximum (m)	Minimum (m)	Mean (m)	St. Deviation	Resolution	Resolution (km)	Raw x Column
DTM15	1340,0	-2390,9	- 0,4	421	1,5'	≈ 2,8	441 x 480
DTM30	1340,6	-2258,8	+0,4	420	3'	≈ 5,5	221 x 240

4. RESULTS

In order to apply the remove-restore technique, the geopotential model EGM96 (up to 360 degree and order) and the topographic direct effect contributions for each gravity station observed were calculated.

Using eq. (2) the residual Free-Air gravimetry anomalies were obtained, being show on Fig. 4 and its statistics on Table 4.

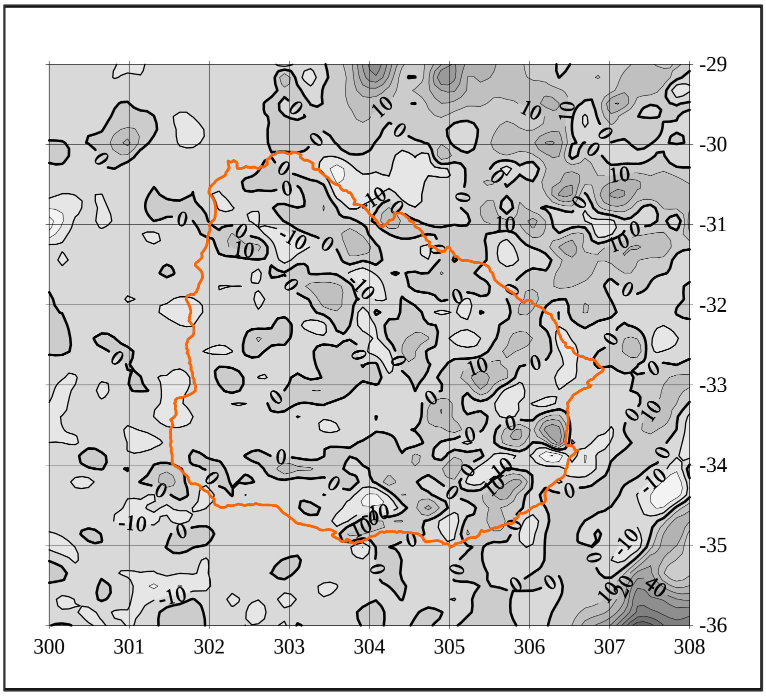

Figure 4. Residual Free-Air anomalies (in mGal)

Table 4. Residual Free-Air gravity anomalies.

15860 Stations	Latitude (°)	Longitude (°)	Altitude (m)	Free-Air anomaly	Terrain effect	EGM96 Free-air	Residual anomaly
Max.	-29	308	692,9	100,639	103,080	51,066	96,737
Min.	-36	300	-37,5	-35,788	0	-27,664	-59,139
Mean	-34,07	304,7	35,5	13,318	3,764	13,063	4,019
St. Deviation	-	-	75,7	16,247	9,483	13,901	12,577
Range	7	8	730,4	136,427	103,080	78,730	155,876

Using the Free-Air anomalies with Stokes' formula in the one-dimensional convolution form of eq. (6), the residuals geoidal undulations are obtained.

The minimum and maximum values of this component were 0.8 and 3 m respectively, being shown on Fig. 5

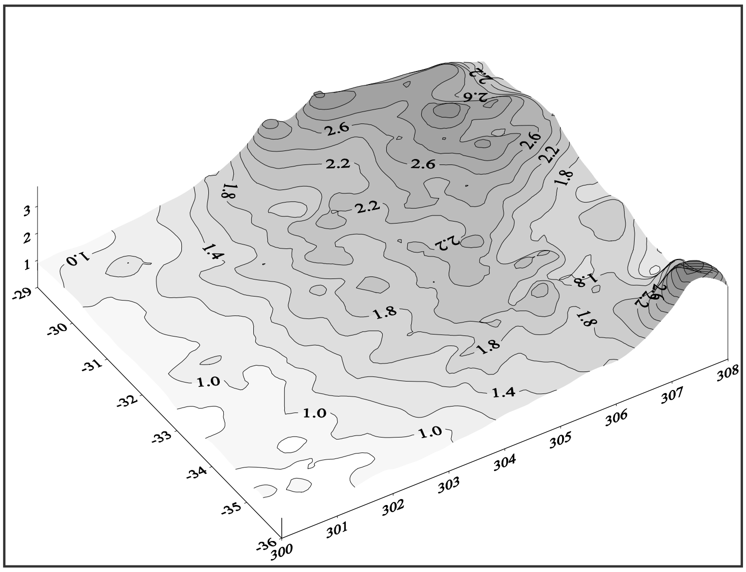

Figure 5. Residual geoidal undulations (in m)

Finally the reintroduction of the wavelengths corresponding to the geopotential model and the indirect effect of the Helmert second condensation method, allows getting the final geoidal undulations. The indirect effect values were sub centimetric ones and thus neglected.

As the process used was FFT, the results are obtained directly in gridded form, ready for practical use. Fig 6 and 7 show the geopotential model EGM96 (up to degree and order of 360) contribution and the final geoidal model.

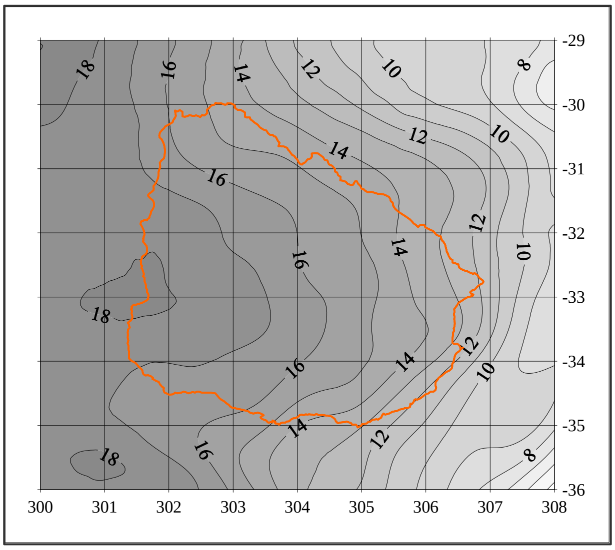

Figure 6. EGM96 (360) geoidal undulations

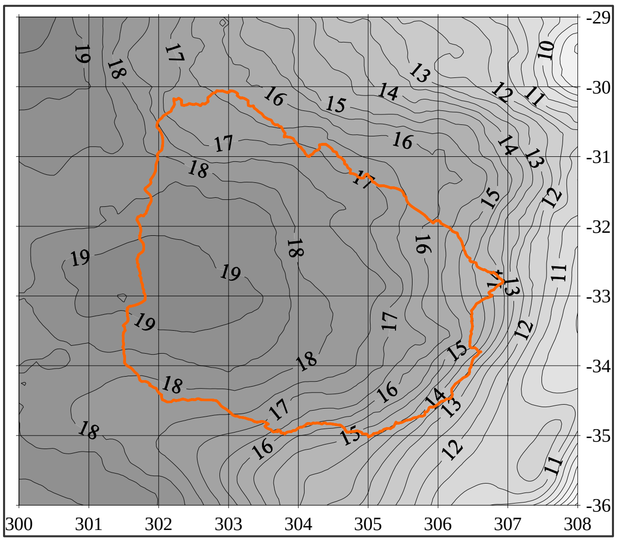

Figure 7. Final geoidal undulations

5. EVALUATION

A systematic bias between the geoid model and the vertical datum for Uruguay and Argentina, ranging from -1.05 to -1.95 m was found. This bias is caused mainly by different local vertical definitions, systematic geopotential coefficient errors of long wavelengths and error at the national leveling networks.

The following Table presents the results of the evaluation made after the systematic biases was removed, using two datasets of relative and absolute of GPS observations over bench marks in Uruguay and one dataset of absolute observations for the Argentinean area calculated. At Uruguay were used 24 relative and 22 absolute GPS stations, meanwhile in Argentina were used a total of 29 absolute ones. The values presented at the Table are for the relative data: mean difference between GPS geoidal undulation and geoidal model one in m, mean value in ppm and its standard deviation. For absolute data the values are maximum and minimum difference between GPS geoidal undulation and geoidal model one in m, range of the difference and standard deviation of the zero mean value.

			UruGeoide2000
Relative evaluation Uruguay		**Mean Diff.**	0.26 m
		ppm	1.99
		St. Deviation	2.24
Absolute evaluation	**Uruguay**	**máximum**	0.31 m
		minimum	-0.46 m
		Range	0.77 m
		St. Deviation	0.25
	Argentina	**máximum**	0.21 m
		minimum	-0.14 m
		Variación	0.35 m
		St. Deviation	0.10

Table 5.

Table 5 shows that the UruGeoide 2000 has in Uruguay, a relative precision of 2 ppm, with an absolute error of 0.25 m at one sigma level of confidence. For the Argentinean dataset the results are better, probably because the GPS observations were made exclusively at the 1st. Order Leveling Network. The model at that region has an error of 0.1 m at one sigma level of confidence.

A further evaluation included not only a bias but also a tilt in order to account for the differences between the gravimetric geoid and GPS/levelling undulations. It was used 8 GPS/Levelling stations, covering the Uruguay area and the following transformation function (Denker *et al.*, 1997 and Lyscowicz & Forsberg, 1997):

$$N = N_{GEOID} + \cos\varphi \cos\lambda \, X + \cos\varphi \,\mathrm{sen}\,\lambda \, Y + \mathrm{sen}\,\varphi \, Z \tag{13}$$

where X, Y, and Z are the translation parameters between the implied GPS and the geoidal system, in a cartesian mode.

The two following tables 6 and 7 show the calculated parameters, the correspondent bias and tilt for the UruGeoide 2000 model and the results of a similar than Table 5 absolute evaluation. A clearly 50 % improvement could be seen in the range and in standard deviation of the difference (Table 7).

	Parameters	Bias&tilt
Uruguay	X= +0.2288 m	-1.952 m
	Y= +5.6881 m	N-S= +0.108"
	Z= -3.5652 m	E-W=+0.178"

Table 6

UruGeoide 2000		
Uruguay	**maximum**	0.09 m
	minimum	-0.28 m
	Range	0.37 m
	St. Deviation	0.131

Table 7

An interpolation program, using the *inverse distance* method within a cell (Franke, 1982) and these cartesian parameters mentioned, was implemented for practical purposes for the SGM's field GPS surveys.

6. CONCLUSIONS

The main purpose of this paper was the presentation of the UruGeoide 2000 project. The remove-restore technique was successfully applied, with main application through the FFT, an approximation not used before for Uruguay. In the calculation process was used a combination of the EGM96 geopotential model, gravimetric data and a 2 x 2 km resolution DTM.

As a subproduct of the research, the Uruguayan Gravimetric Database was transformed from a text form to a relational database with better possibilities for control and search information.

The UruGeoide 2000 model has a relative precision of 2 ppm and an absolute one of 0,25. Using the modeling parameters of Table 6, a 0,13 m standard deviation error could be expected. The results represent an improvement of one order, over the former model and geopotential models alone calculations.

In order to achieve future improvements for geoidal models in Uruguay, more attention must be paid to the DTM and areas lack of high-resolution gravimetric data. The CHAMP and GRACE satellite mission could be in great help in these matters. New methods/procedures as Fast collocation technique (FC) in geoidal calculations, will be also tested.

The geoidal model UruGeoide 2000 is available on demand at IGeS and SGM, Uruguay.

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