README file for the Colorado 1 cm geoid experiment

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1. Files in directory

(1) Gravimetric quasi-geoid model grid at a resolution of 1′×1′.
File: co_1min_zeta.txt
Format: latitude, longitude, height anomaly (m) (one grid-node per row)
Geographic Limits: 35.5°N – 39.5°N, 250.5°E – 257.5°E

(2) Gravimetric geoid model grid at a resolution of 1′×1′.
File: co_1min_N.txt
Format: latitude, longitude, geoid height (m) (one grid-node per row).
Geographic Limits: 35.5°N – 39.5°N, 250.5°E – 257.5°E

(3) Values at the GSVS17 test points.
File: gsvs17_gpsl_N_zeta_W.txt
Format: Mark ID, latitude, longitude, ellipsoidal height (m), geoid height (m), height anomaly (m), potential value W(P) (m^2/s^2). (one point per row)

2. Computation method

(1) Spectral combination approach

Considering a study area where both terrestrial and airborne gravity data are available, the gravimetric quasigeoid height (height anomaly) at the computation point can be decomposed into three components contributed from satellite, terrestrial and airborne gravity data

$$\zeta_{Gra} = \zeta_{Sat} + \zeta_{Ter} + \zeta_{Air} + \Delta + N_0 \quad (1)$$

where ζ_{Gra} is the gravimetric quasigeoid height (height anomaly), ζ_{Sat}, ζ_{Ter} and ζ_{Air} are the height anomaly contribution of the satellite gravity model, terrestrial and airborne gravity data, respectively, Δ is the geoid–quasigeoid correction term, and N_0 is the zero-degree term of geoid height.

Applying the remove-compute-restore procedure with a high degree reference gravity model and representing the high frequency gravity effects by the residual terrain model (RTM, Forsberg 1984), ζ_{Ter} and ζ_{Air} can be furtherly decomposed and Eq. (1) is written as

$$\zeta_{Gra} = \zeta_{Sat} + (\zeta_{Ter}^{Res} + \zeta_{Ter}^{Ref} + \zeta_{Ter}^{RTM}) + (\zeta_{Air}^{Res} + \zeta_{Air}^{Ref} + \zeta_{Air}^{RTM}) + \Delta + N_0 \quad (2)$$

Where, ζ_{Ter}^{Res} is the residual height anomaly computed from the residual terrestrial gravity anomaly Δg_{Ter}^{Res} (Eq. 6), ζ_{Ter}^{Ref} is the reference height anomaly synthesized from the reference gravity model using spectral weights of terrestrial gravity. The reference gravity model is used to account for the contribution outside the local gravity data coverage. ζ_{Ter}^{RTM} is the RTM effect on height anomaly for terrestrial gravity and to be added to restore the topographic effect which has been removed when forming Δg_{Ter}^{Res} in Eq. (6). ζ_{Air}^{Res}, ζ_{Air}^{Ref} and ζ_{Air}^{RTM} are the counterparts of airborne gravity, respectively.

The satellite gravity derived height anomaly contribution ζ_{Sat} can be computed from potential coefficients of satellite gravity model by spectral weighting

$$\xi_{Sat} = \frac{GM}{r\gamma} \sum_{n=2}^{N_{Sat}} W_{Sat}(n) (\frac{a}{r})^n \sum_{m=0}^{n} \left(\delta \bar{C}_{nm}^{s} \cos m\lambda + \bar{S}_{nm}^{s} \sin m\lambda \right) \bar{P}_{nm}(\sin\phi) \quad (3)$$

Where, *GM* is the geocentric gravitational constant of the Earth, *a* is the semimajor axis of the reference ellipsoid, r, ϕ, λ are geocentric coordinates of the computation point, $\left\{ \delta \bar{C}_{nm}^{S}, \bar{S}_{nm}^{S} \right\}$ are fully normalized coefficients of the disturbing potential, $\bar{P}_{nm}$ are fully normalized Legendre functions, γ is normal gravity at the computation point, N_{Sat} denotes the truncation degree of satellite gravity model, and $W_{Sat}(n)$ stands for the spectral weight for degree *n* of the satellite gravity model.

The terrestrial gravity derived residual height anomaly ζ_{Ter}^{Res} is computed from residual terrestrial gravity anomalies Δg_{Ter}^{Res} via the degree weighted Stokes' integral, harmonic continuation is performed to reduce Δg_{Ter}^{Res} from the ground to the level surface of the computation point (Heiskanen & Moritz 1967, p. 312)

$$\begin{aligned}\zeta_{Ter}^{Res} &= \frac{R}{4\pi\gamma}\iint_{\sigma}\left[\Delta g_{Ter}^{Res} - \frac{\partial \Delta g_{Ter}^{Res}}{\partial H}(H-H_P)\right]K_{Ter}(\psi)d\sigma \\ &= \frac{R}{4\pi\gamma}\iint_{\sigma}\Delta g_{Ter}^{Res}K_{Ter}(\psi)d\sigma + \delta\zeta_{HC}\end{aligned} \quad (4)$$

Where, *R* is the mean Earth radius, H_P and *H* are the orthometric heights at the computation point and the integration point, $\delta\zeta_{HC}$ stands for the height anomaly correction due to harmonic continuation. $K_{Ter}(\psi)$ denotes the degree weighted Stokes' kernel, given by

$$K_{Ter}(\psi) = \sum_{n=2}^{N_{Ter}} W_{Ter}(\mathrm{n})\frac{2n+1}{n-1}P_n(\cos\psi) \quad (5)$$

In Eq. (5), ψ is the spherical distance between the computation point and the integration point, N_{Ter} is the maximum degree corresponding to the resolution of gridded terrestrial gravity data, $W_{Ter}(n)$ denotes the spectral weight for degree *n* of terrestrial gravity.

The residual terrestrial gravity anomaly Δg_{Ter}^{Res} in Eq. (4) is given by

$$\Delta g_{Ter}^{Res} = \Delta g_{Ter} - \Delta g_{Ter}^{Ref} - \Delta g_{Ter}^{RTM} \quad (6)$$

Where, Δg_{Ter} is the terrestrial gravity anomaly, Δg_{Ter}^{Ref} is the gravity anomaly synthesized from the reference gravity model without spectral weights, Δg_{Ter}^{RTM} is the gravity anomaly computed from topographic data by the RTM method.

The airborne gravity derived residual height anomaly ζ_{Air}^{Res} can be computed through the generalized Hotine's integral by degree weighting from residual gravity disturbances δg_{Air}^{Res} at the mean flight altitude

$$\zeta_{Air}^{Res} = \frac{R}{4\pi\gamma}\iint_{\sigma}\delta g_{Air}^{Res}K_{Air}(\psi, r, h_M)d\sigma \quad (7)$$

In Eq. (7), $K_{Air}(\psi, r, h_M)$ denotes the degree weighted Hotine's kernel given by

$$K_{Air}(\psi, r, h_M) = \sum_{n=0}^{N_{Air}} W_{Air}(\mathrm{n})(\frac{R}{r})^{n+1}(\frac{R+h_M}{R})^{n+2}\frac{2n+1}{n+1}P_n(\cos\psi) \quad (8)$$

Where, h_M is the mean flight altitude (ellipsoidal height), N_{Air} is the maximum degree of airborne gravity contribution, which is usually smaller than N_{Ter} due to the flight altitude, $W_{Air}(n)$ denotes the spectral weight for degree *n* of airborne gravity. Notice that Eq. (7) and (8) combines the downward continuation of airborne data and the quasigeoid computation in one step.

Similar to Eq. (6), the residual gravity disturbance δg_{Air}^{Res} at the mean flight altitude is computed by

$$\delta g_{Air}^{Res} = \delta g_{Air} - \delta g_{Air}^{Ref} - \delta g_{Air}^{RTM} \quad (9)$$

Where, δg_{Air}^{Ref} is the gravity disturbance synthesized from reference gravity model without spectral weights, δg_{Air}^{RTM} is the gravity disturbance derived from topographic data by the RTM method. δg_{Air}

is the gravity disturbance at the mean flight altitude, which is obtained by reducing the gravity disturbance from the actual flight altitude to the mean altitude.

Similar to Eq. (3), the reference height anomalies for terrestrial (ζ_{Ter}^{Ref}) and airborne gravity (ζ_{Air}^{Ref}) are computed from the reference gravity model using spectral weights $W_{Ter}(n)$ and $W_{Air}(n)$ by spherical harmonic synthesis. The RTM effects on height anomaly for terrestrial (ζ_{Ter}^{RTM}) and airborne (ζ_{Air}^{RTM}) gravity are computed from the RTM gravity effects Δg_{Ter}^{RTM} and δg_{Air}^{RTM} via degree weighted integrals similar to Eq. (4) and Eq. (7).

The key to spectrally combine heterogeneous gravity data for quasigeoid computation is to determine the proper spectral weights of each dataset, $W_{Sat}(n)$, $W_{Ter}(n)$ and $W_{Air}(n)$. Since neither terrestrial nor airborne gravity data have been used for the determination of satellite gravity model, these three types of gravity measurements are independent with each other. The spectral weights of each dataset can be determined from the corresponding error degree variances using the condition of least squares residuals (Wenzel 1982; Sjöberg 1981). We proposed a data-driven method that directly estimate the error and error degree variance from the gravity data. The basic principle of the data-driven method is as following: (1) Comparing the terrestrial and airborne gravity data with the satellite gravity model, the spherical harmonic analysis based spectral decomposition is applied to estimate the long wavelength error and the error degree variances of terrestrial and airborne data at the low degrees (e.g., below degree 200); (2) The error degree variances at the medium and high degrees (e.g., above degree 200) are estimated directly from the gravity dataset using the method of "Leave-Out-One Cross Validation" (LOOCV). The advantage of the data-driven method is that the a prior error information of the gravity data are not needed, the assumption of a priori error information in traditional spectral weighting method is avoided, thus the obtained error degree variances and spectral weights reach good agreement with actual gravity data. For more details of the data-driven spectral weighting method, please refer to Eqs. (10 - 23) in Jiang & Wang (2016). As a summary, Fig. 1 shows the flowchart of spectral combination approach for gravimetric quasigeoid modeling.

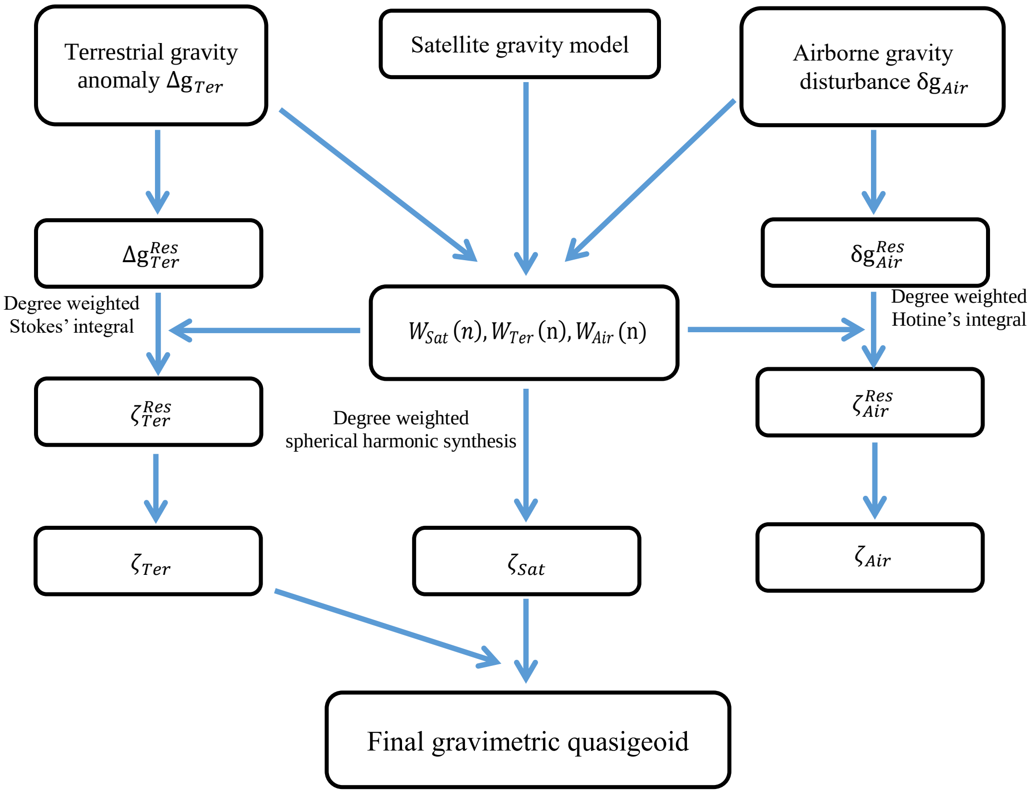

Fig. 1 Flowchart of spectral combination for gravimetric quasigeoid modeling

(2) Geoid–quasigeoid correction

The geoid-quasigeoid correction is computed by (Flury & Rummel, 2009):

$$\Delta = N - \zeta = \Delta g^{BO} \frac{H}{\bar{\gamma}} + \frac{1}{\bar{\gamma}}(V_{P_0}^{TOP} - V_P^{TOP}) \quad (10)$$

$$\Delta g^{BO} = \Delta g - 2\pi G \rho_0 H + g_P^{TC} \quad (11)$$

Where Δg^{BO} is refined Bouguer gravity anomaly, *H* is the orthometric height, $\bar{\gamma}$ is the mean normal gravity, V_P^{TOP} is the gravitational potential of the topographic masses evaluated at computation point, $V_{P_0}^{TOP}$ is the gravitational potential of the topographic masses evaluated at projected point on the geoid of the computation point, G is the constant of gravitation, $\rho_0 = 2670\ \mathrm{kg/m^3}$ is the average density of topographic masses, and g_P^{TC} is the terrain correction evaluated at computation point.

3. Input data

(1) Satellite gravity model: GOCO05S
(2) Reference gravity model: EGM2008
(3) Terrestrial gravity data (provided by NGS)
   1) Point file with terrestrial gravity data in the form of free-air anomalies based on the NGA/NGS gravity dataset. This is a 'cleaned' dataset. Number of points = 59,303.
   2) Geographic Limits: 35°N - 40°N, 250°E - 258°E
(4) GRAV-D airborne gravity data (provided by NGS)
   1) GRAV-D block MS05 .
   2) Full field gravity values along survey lines with mean altitude of 6186 m (ellipsoidal height).
   3) Number of points = 283,716.
   4) Geographic Limits: 34.51°N – 38.88°N, 250.84°E – 258.65°E
(5) Digital Elevation Model (DEM) (provided by NGS)
   1) Based on the SRTM v4.1 data from http://www.cgiar-csi.org/data/srtm-90m-digital-elevation-database-v4-1.
   2) Geographic Limits: 33°N - 42°N, 248°E - 260°E.
   3) Grid Spacing: 3″ (0.00083333°).
(6) GPS leveling Data (provided by NGS)
   1) Point file with GPS Leveling data from Colorado and surrounding states. 509 total GPSL marks but very sparse due to mountainous region. 467 marks are from the NGS Integrated Database (IDB) and 42 marks from NGS OPUS-Share Tool (https://www.ngs.noaa.gov/OPUS/).
   2) Geographic Limits: 35°N - 40°N, 250°E - 258°E
(7) GSVS17 GPS levelling points (provided by NGS)
   Ellipsoidal coordinates of GSVS17 GPS levelling points (latitude, longitude, ellipsoidal height).

4. Computation

(1) General constants
   1) Constant of gravitation (G) 6.674 28 × 10^{-11} $\mathrm{m^3kg^{-1}s^{-2}}$
   2) Geocentric gravitational constant (GM) 3.986 004 415 × 10^{14} $\mathrm{m^3s^{-2}}$ (including the Mass of the Earth's Atmosphere)
   3) Nominal mean angular velocity of the Earth (ω) 7.292 115 × 10^{-5} rad s−1
   4) Conventional reference potential value (W_0) 62 636 853.4 $\mathrm{m^2s^{-2}}$
   5) Average density of topographic masses (ρ_0) 2670 kg $\mathrm{m^{-3}}$.
(2) For the reference ellipsoid, GRS80 – the parameters published in Moritz H.: Geodetic Reference System 1980, J Geod 74: 128-133, 2000 are used.
(3) Atmospheric reduction has been applied on the terrestrial gravity and the GRAV-D airborne data using EGM96 method.
(4) The computation is performed in tide-free system.
(5) Zero-degree term

$$N_0 = \frac{(GM - GM_{GRS80})}{r_{P_0} \cdot \gamma_{Q_0}} - \frac{\Delta W_0}{\gamma_{Q_0}}$$

$$\Delta W_0 = W_0 - U_0 = 62636853.4 m^2s^{-2} - 62636860.85 m^2s^{-2} = -7.45 m^2s^{-2}$$

The computed result is

$$N_0 = -0.177\ \mathrm{m}.$$

(6) The radius of spherical integration cap of the Stokes' and Hotine's formula is chosen as 1°.
(7) Program GEOGRID is used for the gridding of terrestrial and airborne gravity data.
(8) The maximum degree N_{Ter} in Eq. (5) should be 10800 corresponding to the 1′ grid spacing of terrestrial gravity data.
(9) The RTM gravity effects are computed by flat-top prism integration from 3″ DEM and 5′mean topography with a 100 km integration radius using the program TC (Forsberg 1984).
(10) $V_{P_0}^{TOP}$, V_{P}^{TOP} in Eq. (10) are computed by flat-top prism integration from 3″ DEM with a 100 km integration radius.
(11) g_P^{TC} in Eq. (11) is computed by flat-top prism integration from 3″ DEM with a 100 km integration radius using the program TC (Forsberg 1984).
(12) A quasi-geoid model grid and a geoid model grid at a resolution of 1′×1′ are computed.
Geographic Limits: 35.5°N – 39.5°N, 250.5°E – 257.5°E.
Grid point number: 101461.

Table 1. Statistics of quasigeoid and geoid grid model (m).

Model	Min.	Max.	Mean	Std.
Quasigeoid	-25.454	-11.424	-18.179	2.922
Geoid	-25.615	-12.185	-18.674	2.732

5. Model validation

(1) NGS historic GPS levelling derived geoid heights at 194 points are used to validate the geoid model.

Geographic Limits: 36°N – 39°N, 251°E – 257°E.

(2) Geoid heights at the 194 points are interpolated from the geoid grid model using spline method, geoid height differences are then computed by subtracting the GPS levelling derived geoid heights from the interpolated geoid heights.

Table 2. Statistics of the differences between geoid model and GPS levelling derived geoid heights (m).

Point No.	Min.	Max.	Mean	Std.
194	0.709	1.038	0.859	0.053

6. Computation of values at the GSVS17 GPS levelling test points

(1) Height anomalies at the 223 GSVS17 test points are interpolated from the quasigeoid grid model using spline method.
(2) Geoid heights at the 223 GSVS17 test points are interpolated from the geoid grid model using spline method.
(3) Potential value W(P) at the 223 GSVS17 test points are computed from the height anomalies by:

$$W(P) = U(P) + \gamma(P) \cdot \zeta(P) + \Delta W_0 \quad (12)$$

Reference

[1] Flury J. & Rummel R. On the geoid–quasigeoid separation in mountain areas. J Geod (2009) 83:829–847, DOI 10.1007/s00190-009-0302-9
[2] Forsberg, R., 1984. A study of terrain reductions, density anomalies and geophysical inversion methods in gravity field modelling, Reports of the Department of Geodetic Science and Surveying, #355, The Ohio State University, Columbus, Ohio.
[3] Heiskanen, W.A. & Moritz H., 1967. Physics geodesy, Freeman, San Francisco.
[4] Jiang, T. & Wang, Y.M., 2016. On the spectral combination of satellite gravity model, terrestrial and airborne gravity data for local gravimetric geoid computation, Journal of Geodesy, 90, 1405--1418.
[5] Jiang T. (2018). On the contribution of airborne gravity data to gravimetric quasigeoid modelling: a case study over Mu Us area, China. Geophysical Journal International, Volume 215, Number 2, Page 1308, DOI: 10.1093/gji/ggy346
[6] Sjöberg, L.E., 1981. Least-squares combination of satellite and terrestrial data in physical geodesy, Ann. Geophys., 37, 25--30.
[7] Wang Y.M., Becker C., Mader G., et al (2017). The Geoid Slope Validation Survey 2014 and GRAV-D airborne gravity enhanced geoid comparison results in Iowa. Journal of Geodesy, Volume 91, Issue 10, pp 1261–1276, DOI: 10.1007/s00190-017-1022-1
[8] Wang Y.M., Huang J.L., Jiang T. and Sideris M.G. (2016). Local Geoid Determination. In E.W. Grafarend (ed.): Encyclopedia of Geodesy, Springer International Publishing Switzerland
[9] Wenzel, H.G., 1982. Geoid computation by least-squares spectral combination using intergral kernels, In: Proceedings of the General IAG Meeting, Tokyo, pp 438--453.