Colorado geoid computation – GSI/DTU solution

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1. Output files

(1) "geoid_colorado_grid.out":
Geoid heights on 1' x 1' grid for the area with latitude 36-39N and longitude 251-257E

(2) "qgeoid_colorado_grid.out":
Quasi-geoid heights (height anomalies) on 1' x 1' grid for the area with latitude 36-39N and longitude 251-257E

(3) "n_zeta_wp_gsvs17.out":
Geoid heights, height anomalies, and geopotential values at GSVS17 BM points

(4) "geoid_gpsbm.out":
Geoid heights at historical GPS/Leveling BM points

2. Summary

- Basic agreements according to ''Colorado_Experiment_Basic_req_V0.5_Oct30_2018.pdf"
- UNB Stokes-Helmert scheme [Vaníček et al., 2013]
- Remove-Compute-Restore method with XGM2016 geopotential model [Pail et al., 2018]
- Indirect use of Residual Terrain Model (RTM) [Omang & Forsberg, 2000]
- Combination of terrestrial and airborne gravity data by Least Squares Collocation (LSC) using the planar logarithmic covariance model [Forsberg, 1987]
- Stokes' integration of residual helmert gravity anomalies with the hybrid-Meissl-Molodensky modified spheroidal Stokes' kernel [Featherstone et al., 1998]

3. Data

- XGM2016 geopotential model up to degree/order 719
- Terrestrial gravity data of 59303 input points with 1 mGal error
- Airborne gravity data of 56743 input points (5 Hz) with 3 mGal error
- SRTM v4.1 digital elevation model on 90 m-grid
- Earth2014 global terrain model up to degree/order 719

4. Brief description of computational scheme

The Remove-Compute-Restore method with the UNB Stokes-Helmert scheme is employed.

The basic equation is given as follows:

$$N = N_{HGGM} + N_{IDE} + \frac{R}{4\pi\gamma_Q}\iint_0^{\psi_0}\left(\Delta g_{helmert}^{grid} - \Delta g_{HGGM}^{grid} - DWC\right)\tilde{S}(\cos\psi)d\sigma$$

N : Geoid height

N_{HGGM} : GGM-derived helmert geoid height [Huang & Veronneau, 2013]

N_{IDE} : Indirect effect [Heck, 2003]

R : Mean radius of the spherical Earth

γ_Q : Normal gravity on GRS80

ψ_0 : Integration radius

$\Delta g_{helmert}^{grid}$: Gridded helmert gravity anomaly

Δg_{HGGM}^{grid} : Gridded GGM-derived helmert gravity anomaly [Huang & Veronneau, 2013]

DWC : Downward continuation for residual helmert gravity anomaly ($\Delta g_{helmert}^{grid} - \Delta g_{HGGM}^{grid}$) by gravity gradient method [Heiskanen & Moritz, 1967]

$\tilde{S}(\cos\psi)$: Modified Stokes' integral kernel [Featherstone et al., 1998]

ψ : Angular distance between the computation point and gravity point

$d\sigma$: Surface element

$\Delta g_{helmert}^{grid}$ is expressed as follow:

$$\Delta g_{helmert-RTM} = g_{obs} - \gamma(H_p) + AC + TC + STC - RTM$$

$$\Delta g_{helmert-RTM} \to \Delta g_{helmert-RTM}^{grid}$$

$$\Delta g_{helmert}^{grid} = \Delta g_{helmert-RTM}^{grid} + RTM^{grid}$$

$\Delta g_{helmert-RTM}$: RTM reduced helmert gravity anomaly

g_{obs} : Observed gravity by terrestrial and airborne survey

$\gamma(H_p)$: Normal gravity at orthometric height H_p

AC : Height-dependent atmospheric gravity correction [Wichiencharoen, 1982]

TC : Spherical terrain correction [Heck, 2003]

STC : Secondary indirect effect [Vaníček et al, 2013]

RTM : Residual terrain model [Forsberg, 1984]

RTM^{grid} : Gridded residual terrain model [Forsberg, 1984]

$\Delta g_{helmert-RTM}^{grid}$: Gridded RTM reduced helmert gravity anomaly by LSC [Forsberg, 1987]

The geoid heights obtained are converted into quasi-geoid height (height anomaly) using the following equation:

$$\zeta = N - \frac{\Delta g_{BA}}{\gamma_Q} H_p$$

ζ : Quasi-geoid height

Δg_{BA} : Bouguer gravity anomaly

The geopotential value is computed using the following equation:

$$W_p = W_0 - H_p(g_p + 0.0424 H_p \times 10^{-5})$$

W_p : Geopotential value at surface point P

W_0 : 62 636 853.4 m^2s^{-2}

g_p : Gravity value at surface point P

5. Statistics of geoid comparison

The geoid heights obtained are compared with GPS/leveling geoid heights at 509 points (35-40N and 250-258E) and 194 points (36-39N and 251-257E).

GPS/leveling BM	Mean dif. (m)	SD (m)
509 points	0.852	0.082
194 points	0.853	0.055

Reference

Featherstone WE, Evans JD, Olliver JG (1998) A Meissl-modified Vanicek and Kleusberg kernel to reduce the truncation error in gravimetric geoid computations. Journal of Geodesy 72, 154-160.

Heck B (2003) On Helmert's methods of condensation. Journal of Geodesy 77, 155-170.

Heiskanen WA, Moritz H (1967) Physical Geodesy. W.H. Freeman and Co., San Francisco.

Huang J, Véronneau M (2013) Canadian gravimetric geoid model 2010. Journal of Geodesy 87(8), 771–790.

Forsberg R (1984) A Study of Terrain Reductions, Density Anomalies and Geophysical Inversion Methods in Gravity Field Modelling. Reports of the Department of Geodetic Science and Surveying, No. 355, Ohio State University, Columbus, Ohio.

Forsberg R (1987) A new covariance model for inertial gravimetry and gradiometry. Journal of Geophysical Research 92, 1305–1310.

Omang OCD, Forsberg R (2000) How to handle topography in practical geoid determination: three examples. Journal of Geodesy 74 (6), 458-466.

Pail R, Fecher T, Barnes D, Factor JF, Holmes SA, Gruber T, Zingerle, P (2018) Short note: the experimental geopotential model XGM2016. Journal of Geodesy 92 (4), 443—451.

Vaníček P, Kingdon R, Kuhn M, Ellmann A, Featherstone WE, Santos MC, Martinec Z, Hirt C, Avalos D (2013) Testing Stokes-Helmert geoid model computation on a synthetic gravity field: experiences and shortcomings. Studia Geophysica and Geodetica 57 (2), 369-400.

Wichiencharoen C (1982) Fortran program for computing geoid undulations from potential coefficients and gravity anomalies. Internal Report, Department of Geodetic Science and Surveying, Ohio State University, Columbus, USA.