Evaluation of the EGM2008 Gravity Field by Means of GPS-Levelling and Sea Surface Topography Solutions

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Abstract

The new EGM2008 global gravity field model is evaluated by comparisons of geoid heights computed from the model with those available at GPS levelling stations in various regions and by computing sea surface topography solutions from the difference between the mean sea surface and the geoid from this model. In order to identify how good the model performs the same tests also are performed for other recent global gravity field models. The evaluation method, in particular, has to take into account the omission error when truncating global gravity field models at chosen degrees and orders and when comparing them with observed quantities. The procedures applied for the computation of the omission error as well as the general methods applied for the evaluation are described in the paper. For testing the models there are available GPS levelling heights in six different regions (Europe, Germany, USA, Japan, Canada, Australia). Some of these data sets seem not to be adequate for evaluation purposes, because they exhibit long wavelength structures in the geoid height differences. Nevertheless, from the results of the geoid height comparisons one can conclude that the EGM2008 model performs best in most of the regions. Similar results are derived from the sea surface topography solutions based on different global gravity field models. The dynamic ocean topography determined with the EGM2008 geoid seem to show more realistic and more detailed features than solutions based on other global gravity field models. In summary, from the tests performed one can state that EGM2008 represents a significant improvement of the global gravity field in terms of quality and resolution.

1 Introduction

In spring 2008 the new ultra high resolution gravity field model EGM2008 has been made available by the National Geospatial-Intelligence Agency (NGA) (see Pavils et al, 2008). The Inter-Commission Working Group between Commission 2 of the International Association of Geodesy (IAG) and the International Gravity Field Service (IGFS) has been asked to perform an independent evaluation of this new model. For a more detailed description of the purpose and tasks of this working group it is referred to:

In the context of this working group an extensive analysis of the EGM2008 gravity field by comparing model derived geoid heights against independently observed geoid heights at GPS levelling stations in various areas of the world and by the determination of ocean topography solutions in different ocean basins has been performed. The following chapters provide a description of the evaluation technique as well as selected results. In particular, chapter 2 describes the technique and data sets applied for gravity field evaluation specifically taking into account the problem of omission. Evaluation results are shown in chapters 3 and 4.
applying the techniques described in chapter 2. Finally, in chapter 5 a summary is given and final conclusions about the quality of the EGM2008 are drawn.

Comparing quantities of global gravity field models with other gravity field observations per definition is a chicken-and-egg problem. First of all, gravity field modellers try to use the best available data sets in their solutions, which implies, that these data cannot be used anymore for comparison purposes in order to warrant independence. Second, any gravity field quantity as observed on the Earth’s surface contains the full spectral signal power, while any global model is limited by its spectral resolution, i.e. the maximum degree and order of the spherical harmonic series of the model. When comparing such data we have to take into account this problem, which commonly is related to the “omission error”. In order to determine the omission error one would need perfect knowledge of the global gravity field. If we would know it perfectly, there would be no further need to perform gravity field modelling by spherical harmonics.

A discussion of potential techniques to be applied for the evaluation of global gravity fields and their limitations is provided in Gruber (2004) and Gruber et al (2006). In the quality analysis of EGM2008 (it is referred to the following chapters) it is tried to reduce the effect of these limitations by using independent data sets, which have not been used in the model determination, and by filtering the terrestrial and altimetric data sets in order to make them spectrally consistent. We will see later that this can only be done to some extent, because we do not have a reduction model up to infinity. In other words one can state that at least a part of the omission error remains a problem when comparing model derived quantities with observed data.

2 Evaluation Technique and Data Sets

As discussed in the introductory section, for evaluating the EGM2008 model, we compare it with independent geoid heights on GPS levelling stations and we determine sea surface topography solutions by subtracting the model geoid from a mean sea surface. In the sequel both approaches are explained in detail. For comparison, the same procedures are applied to other global gravity field models in order to identify, which of them performs better or worse.

2.1 Basic Relations

The evaluation approach with GPS-levelling data requires geoid heights computed from the model under test as well as independently observed geoid heights, while for the evaluation with the sea surface topography (dynamic ocean topography) apart from the model geoid heights a mean sea surface from altimetry is needed. The relation between the quantities involved is illustrated in Figure 1 and explained in the following.

From GPS positioning and satellite altimetry we get geometric heights above the reference ellipsoid. Over land from conventional levelling we get orthometric (normal) heights (physical heights), and by the difference between the geometric and orthometric (normal) heights we get an observed geoid (quasi-geoid) height (solid black line at land in Figure 1). From the global gravity field model we compute geoid heights referring to the same reference ellipsoid by spherical harmonic synthesis (dotted black line at land in Figure 1). Now both geoid heights can be compared and used for evaluation of the global model. Over the ocean the situation is very similar. From the differences between the mean altimetric sea surface heights and the geoid heights, computed from the global model, we get an estimate of the dynamic ocean topography (dotted black line at ocean in Figure 1). This dynamic ocean
topography can be compared to an alternatively estimated surface, e.g. from an ocean circulation model (solid black line at ocean in Figure 1).

**Figure 1:** Quantities and height systems involved in global gravity field validation by means of geoid and sea surface topography.

### 2.2 The Problem of Omission

As described in the introduction, the omission error plays a significant role for evaluating global models with independently observed data. For this reason a more detailed description of this problem is provided. Figure 2 shows an overview of the situation we have.

The horizontal axis in Figure 2 shows the spectral domain (i.e. degree of spherical harmonic series) from zero to high frequencies, while the vertical axis specifies the spatial domain from pointwise observations to block-mean values. The diagonal line from top left to bottom right shows the maximum spatial resolution, which can be represented by a spherical harmonic series up to a specific degree. The blue stars represent some examples for this. E.g. with a series up to degree and order 60 we can represent a spatial resolution or grid size of 3 degrees. Other examples are degree 360 corresponding to a 30’ grid and degree 2160 corresponding to a 5’ grid. A point observation contains the full spectral range without any limit (this is indicated by the red line in Figure 2). The green horizontal bar represents 5’x5’ block mean values as they have been used for computing the EGM2008 model. During estimation of the model one has to take care of frequencies above the maximum degree in order to avoid aliasing of the high frequencies to the estimated spectrum. This aliasing problem also can be regarded as a kind of omission problem for spherical harmonic analysis. For the model evaluation we solve the spherical harmonic series to a chosen degree and based on various spatial grids or on dedicated points by spherical harmonic synthesis. This is shown by the yellow bars and the black vertical line in Figure 2. The yellow bars show examples for solving the series up to degree and order 360 for global spatial grids with 10’, 30’ or 2 degrees resolution, respectively. In case the bar is above the diagonal line (in the white space) we can regard this as a kind of under-sampling, while the signal is over-sampled in case the bar does not reach the diagonal (in the light blue space). When we now compare both results (from the model and the observation) we suffer from the omission error, which is represented by the blue line in Figure 2.
From the situation described above we can conclude that in any conversion between space and frequency domain the omission error has to be taken into account by appropriate means. The problem is now to find the appropriate way to take the omission error into account. This in fact is not trivial, because one would need to know the global gravity field to infinity at any point in the world. In other words, as explained before, one can regard the omission error problem as a chicken-and-egg problem. For the actual evaluation of the EGM2008 model the following chapter describes the way of how the omission error is approximated in this study.

### 2.3 Model Evaluation Method

As described in the introduction two methods are applied in this study to estimate the quality of the EGM2008 global gravity field model. The first one compares model geoid heights with GPS levelling derived geoid heights and the second one estimates dynamic ocean topography solutions and checks them for consistency. The following paragraphs describe in detail how the computations are performed.

**GPS Levelling Geoid Heights**

Several GPS levelling geoid height data sets are available (see chapter 2.4). All data sets provide geoid heights for points on the Earth surface. We assume that all geoid heights refer to the tide free system, i.e. we use the tide free version of the EGM2008 coefficient data set. Then the following processing steps are performed:

1. The EGM2008 tide free model is solved for geoid heights at the location of the GPS levelling point (latitude, longitude and orthometric height) up to a selection of maximum degrees and orders $n_{\text{max}}$. The applied normal field corresponds to the one used for the GPS levelling geoid heights.

2. The omission error is approximated per point from a solution of the EGM2008 tide free spherical harmonic series for degree and order $n_{\text{max}}+1$ to 2190 (note: EGM2008
contains the full coefficient set up to degree 2160 and a selected set of coefficients for degrees 2161 to 2190).

3. The omission error is subtracted from the GPS levelling geoid height in order to make them spectrally consistent (at least approximately).

4. Geoid height differences are computed between the model geoid heights and the reduced GPS levelling geoid heights. The difference is regarded as a quality estimate for the global model.

5. For each regional GPS levelling data set a mean value of the differences is computed and subtracted in order to take into account inconsistencies in the height system definitions (or zero potential definition).

6. Finally the RMS of the “un-biased” geoid height differences is computed for each region and shown in chapter 3 for various models.

7. In addition geoid height slope differences between all points in a region are computed and a RMS per distance class is computed (for a more detailed description see Gruber et al, 2006). Results of this test are shown in chapter 3, too.

**Dynamic Ocean Topography (Sea Surface Topography) Solutions**

In order to compute sea surface topography solutions the geoid is subtracted from an altimetric mean sea surface (geodetic approach of sea surface topography determination). Again the tide free version of the EGM2008 model is used for this procedure.

1. The mean sea surface is converted to the tide free system. Note: Sea surface heights from altimetry are always given in a mean tide system, because sea level includes the contribution of permanent tides.

2. The mean sea surface model is converted to spherical harmonics up to very high degree and order applying numerical quadrature (degree and order 1800 for this study). Land areas are filled with geoid data from a global model (EGM96) in order to reduce strong leakage at coastal areas.

3. Mean sea surface heights and geoid heights are solved for a geographical grid at sea level by spherical harmonic synthesis up to degree $n_{\text{max}}$, which again is varying for different test cases. The same reference ellipsoid is used for both. By this approach both data sets are filtered in the same way, which means in other words, that the omission error can be reduced significantly when building the difference.

4. The difference between the filtered mean sea surface and geoid heights are computed for various ocean basins. Coastal areas are blended and not taken into account because of possible leakage from land.

5. Sea surface topography solutions are inspected visually in order to check the solutions for plausibility. Note: For the evaluation of EGM2008 we did not compare these results with oceanographic solutions, because from preliminary analyses it was found that the oceanographic model results deviate significantly. This needs further investigation. The results from these computations are shown in chapter 4.

**2.4 Data Sets Applied**

GPS levelling and dynamic ocean topography comparisons are performed for a set of gravity field models. The following models have been used:

- EGM96: Combined model complete to degree and order 360 (Lemoine et al, 1998). This is the well known fore runner of EGM2008.
• GGM02C: Combined model complete to degree and order 200 (Tapley et al., 2005). This represents one of the earlier GRACE based combined solutions and mostly applies the surface data sets as they have been provided by the EGM96 project.
• EIGEN-GL04C: Combined model complete to degree and order 360 (Förste et al., 2007). This model is based on a recent GRACE satellite solution, some new surface and altimetry data sets (e.g. Arctic gravity, GFZ in house mean sea surface), but also data from the EGM96 project.
• EIGEN-5C: Most recent combined model from GFZ/CNES complete to degree and order 360 (Förste et al, 2008).
• ITG-GRACE03S: Satellite-only GRACE model, which was applied as basis for the EGM2008 model (Mayer-Gürr, 2007). Note, this model is the only satellite-only model applied in the present tests. Therefore, we will experience some different behaviour as compared to all others.
• PGM2007A: This is the preliminary EGM2008 model complete to degree and order 2160 with some additional coefficients up to degree 2190, which was released in summer 2007 for evaluation purposes. It is based on a different satellite-only model and some preliminary surface data compared to EGM2008 (Pavlis et al, 2007). The model is included here in order to find out, if the final EGM2008 model shows differences to this preliminary solution. Note, this model is not published.
• EGM2008: This is the final combined model complete to degree and order 2160 with some additional coefficients up to degree and order 2190 (see Pavlis et al, 2008). This model represents the state-of-the-art of global high resolution modelling and is evaluated against all other models mentioned above in order to identify its performance.

Six different regional GPS levelling data sets are applied for comparison purposes. The point distributions for the different regions are shown in Figure 3. The following data sets have been used:
• Australia (Johnston & Manning, 1998): 197 points.
• Germany (Ihde & Sacher, 2002): 675 points in Germany.
• Europe (Kenyeres et al, 2006): 1233 data points from the EUVN-DA project (European Vertical Network – Densification A).
• Canada (Veronneau, 2007): 430 points from new Canadian network
• Japan (Nakagawa, 2003): 837 points along first order levelling lines.
• USA (NGS, 1999): Geoid on benchmark data set edited for some points exhibiting large deviations (finally 5168 points are taken).

For the dynamic ocean topography tests the Goddard Space Flight Center mean sea surface model was used (Wang et al, 2001). The model has been augmented with EGM96 geoid heights over land and transformed into a spherical harmonics up to degree and order 1800.
Figure 3: Distribution of geoid data points of the six regional GPS levelling Data sets. Australia (top left), Canada (top right, Germany (mid left), Europe mid right), Japan (bottom left), USA (bottom right).

3 EGM2008 Evaluation by Means of GPS Levelling Data

The results of the comparison of model derived geoid heights with GPS levelling geoid heights applying various degrees and orders of model truncation are shown in the following figures.
Figure 4 shows the RMS of geoid height differences (after subtracting the mean difference) for the models under test truncated at degree and order 30, 60, 90, 120, 150, 180, 360 and 2160 respectively. Each square in the plots represents the RMS geoid height difference for the color coded model (see legend in each plot) for a region under test after eliminating a bias. Note: As explained above the omission error was estimated from the EGM2008 model and
subtracted from the observed GPS levelling geoid heights. For this reason the comparison is slightly biased towards the EGM2008 model, which is visible by the identical RMS values for each degree of truncation for this model.

Generally, one can conclude that all models exhibit a significant improvement with respect to the EGM96 model (coded in yellow). EGM96 shows much larger deviations to the GPS levelling geoid heights than any other model under test (for some figures RMS values are even outside the axis range). Another common feature in all figures is that the satellite-only GRACE model (ITG-GRACE03S coded in orange), which forms the basis for EGM2008, up to degree and order 90 is very close to the EGM2008 solution, while for higher degrees RMS values increase drastically. This indicates that the EGM2008 model is dominated by the satellite-only information up to this degree, while surface and altimeter data start to contribute more and more for higher degrees. Nevertheless the satellite-only model performs quite well up to degree 90 and represents one of the best models available from GRACE-only information. The other combined models applied for comparisons show some different behavior (green, pink and blue curves). All three models are based on a GRACE satellite-only model and contain surface and altimeter data from various sources (many of them different to those used for EGM2008). For these models RMS of geoid height differences start to increase at degree 60 or 90 and are finally significantly above the EGM2008 curve. From this we can conclude that EGM2008 is based on significantly better surface data and that the transition from the satellite information to the surface data contribution is smooth. From the improvements from the pink to the blue curve one can see that the transition between the two data sources plays an important role, because both models are based mostly on the same input data, but for the more recent model (blue curve) the modeling has been improved. Finally when comparing the preliminary EGM2008 model (red curve) with the final one (black curve), one can identify, that for most regions the final model is slightly better than the preliminary one. Only for the German data set the preliminary model is slightly better, but on a very low level (2 mm RMS). This indicates that the final model is slightly superior to the preliminary model.

Regarding the level of RMS differences one can see that some GPS levelling data sets probably are not good enough for evaluation purposes. RMS differences for EGM2008 vary between 3.8 cm (for Germany) and 33.4 cm (for USA). In this sense, the global model can be applied for identifying problems in the GPS levelling geoid heights, specifically when regarding to long wavelength patterns in geoid differences. For this reason, for the both regions mentioned above the height differences are shown geographically. Figure 5 shows the results for the US GPS levelling data set for two models truncated at degree and order 360. From the differences one can see that both models show similar behavior in terms of the general structures. This is an indicator, that the GPS levelling data set is contaminated by some long term feature, which does not agree well with the model results. Excellent results exhibit the German data set. For this, we have RMS differences on the level of a few centimeters. Looking at the results in Figure 6 we can clearly see, that from left to right we get significant improvement. While for EGM96, there is a strong long wavelength feature in the differences, the result for EIGEN-5C is significantly better, but still shows some pattern in the differences. Finally, applying the EGM2008 model only some small systematic feature from North-West to South-East is left, which could be either a problem in the GPS levelling data or the EGM2008 model. From these very small differences, we can also conclude that the data set is well suited for evaluating global models (in contrast to the results from the US data set). For the other regions results vary (figures are not shown). Some data sets seem to be applicable for global model evaluation, some of them show problems like the US data set. In principle one can identify the level of suitability of the data sets from the level of the RMS
differences shown on the vertical axes in Figure 4. The sequence from best to worst comparison results per region is: Germany – Japan – Canada – Europe – Australia – USA.

Figure 5: Geoid height differences for US GPS levelling data set: left: EIGEN-5C model, right: EGM2008. Both models are truncated at degree and order 360. All differences are given in [m].

Figure 6: Geoid height differences for the German GPS levelling data set: left: EGM96, middle: EIGEN-5C model, right: EGM2008. All models are truncated at degree and order 360. All differences are given in [m].

In order to identify how the models perform over different distances, geoid slopes from the models and the GPS levelling data have been computed. These slope differences have been put into distance classes and RMS values of slope differences per distance class have been computed. Some results of this test are shown in Figure 7, for two regions and different degrees of truncation of the global models. Figure 7 supports the conclusions from the height differences tests described above. Up to degree 90 nearly no differences are visible for the GRACE based models. Only EGM96 exhibits much larger geoid slope differences (Figure 7 top left). By increasing the degree and order of truncation, differences between the solutions become more and more visible. The earlier GRACE model EIGEN-GL04C (pink curve) performs worse than all other GRACE based models for Canada (up to degree 150) and for Germany (up to degree 120 and 360) (see Figure 7 top right, bottom left and bottom right). Major improvements could be reached with the EIGEN-5C model (blue curve), which is computed by an improved combination technique, but which is based on more or less the same data as EIGEN-GL04C. This again shows that the proper combination technique of satellite and surface information is crucial for the final model performance. Also the GGM02C model performs partly better than EIGEN-GL04C, which probably has the same reason. Remarkable well performs the satellite-only GRACE model also up to degree and order 150 (orange curve) for the geoid slope differences. Finally, from Figure 7 we can conclude that the EGM2008 model (black curve) performs slightly better than the preliminary model (red curve), specifically for the longer distances, which are somehow related to the lower degrees and orders. This could be a hint to the effect of the replaced satellite-only model on the final solution.

Figure 8, Figure 9 and Figure 10 show solutions for the dynamic ocean topography computed by the procedure as described in chapter 2.3. For comparison purposes the North Atlantic was selected, because there the structure of the sea surface topography is better known than in other regions. Figure 8 shows the results for the global gravity field models available up to degree and order 360, while Figure 9 shows the dynamic ocean topography computed from the full resolution of PGM2007A and EGM2008.
Figure 8: Sea surface topography solutions for the North Atlantic based on the GSFC00.2 mean sea surface and different global gravity field models up to degree and order 360 (full resolution of gravity models): top: EGM96 solution; middle: EIGEN-GL04C solution; bottom: EIGEN-5C solution.

From Figure 8 we can identify that from top to bottom the structure of the dynamic ocean topography models becomes more and more realistic. This supports the results from the GPS levelling comparisons. Nevertheless, also for the bottom figure there is still some ringing pattern visible, which could be caused by truncating the spherical harmonic series of the mean
sea surface at degree and order 360. Even more realistic solutions are computed from the EGM2008 preliminary and final models (see Figure 9). There the ring structures disappear and well known ocean features become visible. The final EGM2008 solution (bottom figure) shows a slightly rougher structure than the preliminary solution. The reason for this is unknown, but has probably to do with some different altimeter data sets applied for the final model. From this we can not conclude, which of the two solutions is superior to the other. Finally for completeness, Figure 10 shows the global sea surface topography solution for the final EGM2008 model. In general, from the sea surface topography solutions one can draw similar conclusions than for the geoid height tests. There is visible a clear improvement with respect to the previous global gravity field models, which can be addressed to new data sets, to a better satellite-only solution, and partly also to the higher resolution of the global model.
5 Summary and Conclusions

The purpose of this paper is the evaluation of the EGM2008 global gravity field model by means of comparisons with GPS levelling geoid heights and sea surface topography solutions. In order to make such comparisons one has to take into account the problem of the omission error when comparing point observations with band limited information from global models (by truncation of the spherical harmonic series). In the approach applied, the very high degree harmonics of the EGM2008 solution are taken in order to compute the omission error. This may cause all comparisons to be slightly biased towards the EGM2008 model. For all other models in principle coefficient sets from two different models are mixed (model under test up to the degree of truncation, EGM2008 from truncation degree up to degree and order 2190). Nevertheless, it is assumed that the computations performed provide a rather fair information about the quality of the EGM2008 model.

The comparisons with the GPS levelling data clearly show that the new EGM2008 solution (also the preliminary model) performs significantly better than any other model, which has been used in the tests. This is true for all truncation degrees above 90. Up to degree and order 90 the solution is dominated by the satellite-only GRACE model, which also seems to perform very well in this spectral range. Not all data sets seem to be adequate to perform such an evaluation, because the GPS levelling data themselves, seem sometimes to have systematic errors. Therefore, for the final conclusions it is important to select adequate data sets. In our case we assume that the German, Canadian and Japanese data sets are suitable for such tests. The sea surface topography solutions computed by subtracting the geoid from a mean sea surface show more realistic features for the EGM2008 solution than for the others. This could be partly caused by effects of truncating the mean sea surface topography at degree 360 for the global gravity field models available with this resolution. Nevertheless, the solutions based on EGM2008, at least in the region investigated (North Atlantic), shows new, realistic and more detailed features than any other model. For this reason, also from this evaluation one can conclude that EGM2008 provides the best high resolution geoid.
Looking into the future, GOCE will provide more detailed satellite derived information for the global gravity field. Therefore we can expect with future updates of EGM2008, which will incorporate GOCE data as well, that the model can be further improved specifically in the range of wavelengths between degree 90 and 250.

References


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